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GEAR CUTTING

IN THEORY AND PRACTICE

By

JOSEPH G. HORNER,

A.M.I. MECH. E.

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PREFACE

When offering a new book on Gear Cutting to the public, the reasons for doing so may be stated.

Many books have been written on the subject of gears, dealing with the designs of teeth and giving some account of the shop practice connected with the formation of the tooth shapes. But none that I know of covers all the later developments in a reasonably comprehensive manner. Either theory is treated apart from practice, or the work of the shops is dealt with in a too summary fashion.

During say ten years past, shop practice has been profoundly modified by the growth of form planing, and generating planing methods chiefly applied to the bevel gears, and by the rapid extension of the mode of generating spur gears by hobbing. New machines have been developed and come into assured favour. These are illustrated and treated at length in this book by the help both of detailed drawings and of photographs.

The growth of automobile practice and the developments of the all-geared machine tools have brought the high carbon steels and the alloy steels to the front. These and the necessity for hardening of the gears have had a far-reaching influence on the methods not only of the gear cutter, but also on the whole sequence of the production of gears from the selection of the materials to the final testing. A condensed account has been given of these details.

As it is not possible to understand the method of operation of a machine from photographs alone, examples of all the great types of gear-cutting machines are illustrated by detailed drawings. In some instances also full examples are given of the sizing and cutting of gears from shop operation sheets, in association with the machines to which they have reference. It is hoped that these drawings and examples will prove of special value to the men in the machine shop.

The large number of illustrations—all specially prepared for the work—will be helpful to the student.

Portions of the work have appeared previously in articles of mine in the *Mechanical World*, and the section on double-helical gears was written originally for the *Engineering Review*.

My acknowledgments are due to the firms whose names appear in connection with the accounts of their machines. I am specially indebted to the Brown & Sharpe Manufacturing Company for the use of formulæ and tables which were originally worked out for and printed in their handbook.

JOSEPH HORNER.

BATH, 1914.

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GEAR CUTTING

IN THEORY AND PRACTICE

SECTION I.—PRINCIPLES OF DESIGN

CHAPTER I

ELEMENTS OF TOOTH FORMS

Historical.—More progress has been made in the cutting of toothed gears during twenty or twenty-five years past than in the two or three centuries previous. The first men who cut gears were the horologists and astronomical-instrument makers, and their work was done in the lathe and without much regard to the correct shapes of the teeth. The book by Camus, who died in 1768, was the first in which the subject was treated at length. There is not any example in this of double curved teeth, though the construction of internal and external cycloids is illustrated and described. All the illustrations are either of involute teeth, or lantern gears, or of the vastly predominating design in which epicycloidal curves are combined with radial flanks below pitch line. Or alternatively the flanks above pitch line are semicircles like knuckle gears, but the flanks below are invariably radial. True contact, therefore, could only occur at and in the immediate vicinity of the pitch circles. Though the construction of the hypocycloid or "internal" cycloidal curves was illustrated they were not applied to the forms of the teeth shown in that work. Also, though the teeth were produced by cutting, there was a large amount of flank clearance allowed. Allusion is made to inequality and other defects in the teeth. A pinion is also said to be preferable to a lantern for driving. There is much of this kind of curious interest in this book with which the present subject is not concerned. But the mathematical bases of the two forms of teeth used to-day are explained at length, and may be studied there with advantage and interest.

As the varied designs of the gear cutters depend upon, and are derived from, the fundamental principles of the design of the teeth of gears, it is essential to give some preliminary attention to the elements of gear-tooth construction. Machines must be so designed that mathematical accuracy in all the gear elements shall be embodied in the action of the machines. This is seen at its highest development in the generating machines, in some of which no formed cutter is used at all, but a tool with a plane face or a narrow edge only does the cutting, the generating elements being embodied in the machine. The action of some of these machines can only be understood if the fundamental principles of the formation of the shapes of the gear teeth are grasped.

Definitions.—Gears derive their various names primarily from their axes of rotation and pitch surfaces. Cylinders having their axes parallel and teeth parallel with the axes are *spur gears.* They may have internal teeth. Cylinders the axes of which are not parallel, but set at an angle, have teeth which class them as *spiral* or properly *screw* gears. Racks are either spur, or spiral (screw) gears of infinite radius. *Worm* gears are a type of screw gears, a screw engaging with a wheel, in which the axes are usually, though not essentially, at exact right angles—hence often termed tangent wheels. Conic frustra having their axes in the same plane, and intersecting, have teeth the bounding lines of which would if prolonged meet at the common apex of their cones. These are *bevel* gears. The axes of skew bevels are not in the same plane. These classifications say nothing about the forms of the teeth. And whatever the sectional shapes of the teeth which are adopted, they vary in outlines and dimensions with every variation in the character and dimensions of the pitch surfaces, and with the relative dimensions of the gears in contact, and also with the direction in which the teeth are arranged on their pitch surfaces. This fact is very pronounced in the worm gears and spiral gears.

The Ideal Gears.—The pitch lines (Fig. 1) P.D, P.D are cut by the line of centres which joins the centres or axes of revolution of gears in mutual engagement. This is divided at the point where the pitch surfaces are in contact, the *pitch point, o.* This occurs at a distance between the centres of revolution inversely proportional to the radii, diameters, or circumferences of the gears in contact. The linear velocity being alike in each of a pair, the angular velocity varies inversely as the diameters. This assumes that no slip occurs between the surfaces. The fact that this might happen between smooth frictional surfaces is the reason why interlocking teeth are used, and then the ideal is, so to design the teeth that the gears will transmit motion as uniformly and smoothly as that which would be produced by the simple frictional contact of smooth surfaces. It is around this ideal that the whole practice of gear-tooth design and cutting has been developed. Practice only takes account of two simple kinds of teeth, the cycloidal and the involute. The principles which underlie those as well as all other forms of teeth are these:—

The Common Normal.—The law which governs correct tooth contact, by virtue of which gears transmit motion as though by the contact of smooth surfaces, is this: The common normal of pressure to the tooth curves in contact must pass through the pitch point. This permits of the choice of several possible curves, as well as straight or curved faces in the case of racks. Based on this law, several systems of gear-tooth curves have been proposed, and demonstrations given showing how, having the curve of one tooth given, that of its fellow can be constructed. Though interesting, they are of little practical value. But the fundamental fact is of first importance, because if it is not embodied in gear-wheel teeth the engagement will not be mutual, regular and correct. In the older teeth perfect contact was ignored, as in the knuckle gears and those which have straight flanks below the pitch line. Some slipping occurs in these cases which does not happen when the teeth are made to fulfil the condition just now stated.

A templet can be made of wood (Fig. 2) to demonstrate the accuracy of the statement that the common normal must always pass through the pitch point. Cut the edges of two pieces of wood to represent arcs of base circles of involute teeth, and bore a hole at the centre of each to pivot them around. Cut two pieces with involute curves struck from the base circles, using centres *a*, *b*, and screw them on the first pieces. On centres *a*, *b* pivot a link having a centre line connecting the pivots. Then on moving the tooth edges in contact, the centre line of the link in its movement will always cut the pitch point *o* in the line of centres.

The curves of teeth may be of any form so long as they fulfil the conditions just stated. Practical considerations of manufacture limit these to regular curves such as can be struck from centres with compasses or which can be readily generated.

In order to ensure smoothness of engagement it is essential that there shall be no break of contact at any stage of the action. In other words, the path of contact must be continuous, and from every point in the path of contact normals carried therefrom from both curves must pass through the pitch point. This fact lies at the basis of tooth profiles. The path of contact may travel either along curves or along a straight diagonal path. But in any case the same curves, or the same diagonal path, will be traversed by the teeth of each gear in engagement. This is a most important fact, because it renders the interchangeability of gears a simple matter. It is only necessary first to determine the nature of the path of contact, and then all teeth of the same pitch shaped by that will engage correctly. Or having the tooth curves of one wheel laid down, those of others to gear with it can be described. It is possible, though not workable, to use other curves besides those of the circle for generating tooth curves. The generating elements may be of segmental or of

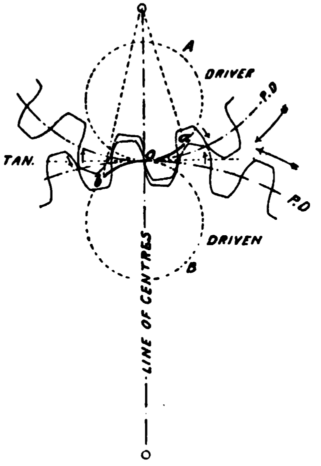


Fig. 1.

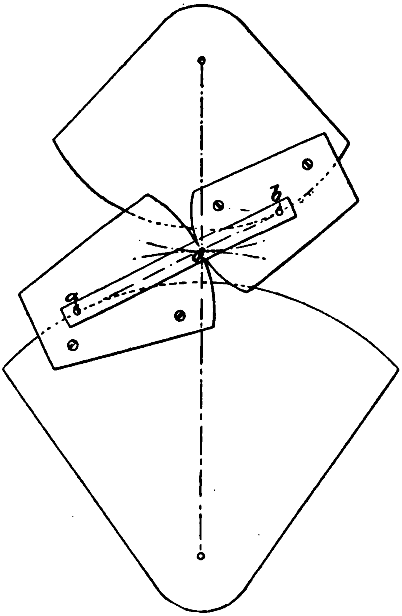


Fig. 2.

parabolic form, but the difficulties of applying them would be too great.

Friction of Teeth.—The obliquity of action of teeth is considered objectionable, due not only to the variation in the angle of pressure from the tangent to the line of centres, but also because of the greater amount of sliding friction which occurs. The action along the tangent, Fig. 1, when contact takes place at the pitch point is rolling simply. Elsewhere it is that of rubbing or sliding. This is a most important difference which gives occasion to the designs of the helical teeth, and of trundle and knuckle gears. The friction along the arc of approach, *a o*, Fig. 1, the upper gear being the driver and the lower one the driven, is more injurious than that along the arc of recess *o b*. Hence in designing standard gears the aim is to lessen the obliquity as far as is practicable without the sacrifice of other advantages. The angles included by *o a*, *o b*, are the angles of approach and recess respectively.

In order to ensure perfect action, contact should occur both above and below the pitch line, though a few gears are made which do not fulfil this condition. The portion of the tooth above pitch line is termed the *face*, that below is the *flank*. It is not essential, in order to the correct contact of one pair of gears, that the path of contact shall follow the same curves above and below pitch line, but it is essential if all gears of the same pitch are to interchange. Bevel gears which work only in pairs might obviously have different curves of contact above and below pitch line, which is done occasionally to avoid a weak pinion. But for convenience and economy of manufacture there is usually no difference made in these.

Cycloidal Teeth.—The path of contact coincides in cycloidal teeth with the circles A, B, Fig. 1, which are used for generating the tooth curves—that is, the teeth in engagement travel for half the period along the path of contact, *a o*, defined by one circle A, to the pitch point *o*, the arc covered being the arc of approach, and during the second half from the pitch point *o b* along the other circle B during the arc of recess. The diameters of these generating circles are most important in an interchangeable system of gears, because small circles, though permitting of the employment of small pinions, entail a high angle of obliquity and thrust, and very wide flanks in the larger wheels. The basis is commonly either a thirteen or a fifteen toothed pinion, using a generating circle having a diameter equal to its radius. Such a pinion has radial flanks. Smaller pinions will have convex flanks much undercut. All gears larger than the base will have concave flanks, the width at the base increasing with increase in diameter. The basis, therefore, of this system is the generating circle, of half the diameter of the base pinion of the series.

Although the cycloid described by the rolling of a generating circle on a base is not a regular curve, a very close approximation is struck with compasses. The difference is negligible in small gears. But there is a marked difference in teeth of coarse pitch.

Single Curve Teeth.—Single curve teeth, the involutes (Fig. 3), have no concave faces, the curves of the teeth in each of a pair being convex. The curves are produced by unwinding a cord from an arc, Fig. 4, or by rolling a strip with a needle point round the arc, Fig. 5. An approximation thereto is obtained by striking the curves from a single centre. The path of contact is a straight line which is a tangent to the base circles, Fig. 3, of wheel and pinion, and the pressure coincides with the path of contact. The base in this system is not a pinion as in the cycloidal, but a rack, Fig. 6. In determining the base, the angle of the path of contact is first selected, the object sought being, besides interchangeable gears, the employment of an angle without too high obliquity, with consequent injurious thrust on journals and bearings. The angle usually taken varies from 14½° to 20° to the tangent to the pitch circle, 14½° or 15° having been common until recently, preference being often now given to higher angles (*see* p. 40). Then this determines the angle of the faces of the rack teeth, which faces are perpendicular to them; hence the rack is the basis of this system, as the smallest pinion is that of the cycloidal system. The involute rack has straight faces (in theory); the cycloidal rack has double curves, whence important results arise when the question of cutting accurate gears is under consideration.

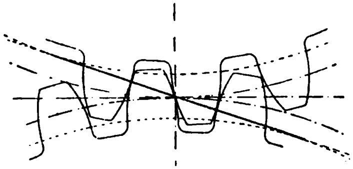


Fig. 3.

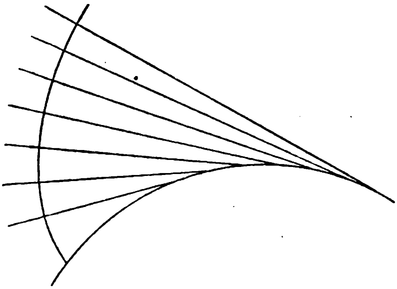


Fig. 4.

Pitch Lines.—The pitch lines in cycloidal and involute teeth are not of equal importance. In the first the pitch lines are also the base circles on which the generating circles are rolled, and therefore true tooth contact can only take place so long as the pitch lines are in contact. But in involutes the tooth curves are developed on the base circles, which are not the pitch circles. The positions of the latter have no relation except an arbitrary one to the base circles, and therefore correct tooth contact takes place irrespective of whether the pitch circles are in contact or not. In this respect, therefore, the involute is a very elastic gear.

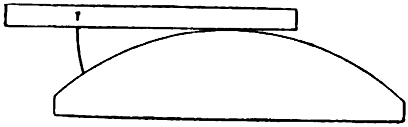


Fig. 5.

Growth of the Involute.—For fifty years these two classes of wheel teeth have occupied debatable ground. The involute has invaded much of the domain formerly held by the cycloidal, and generally with good results. When the writer was a youth, involutes were rarely seen in English shops, never in some of them; now they are common everywhere. Prejudice had something to answer for in the neglect of the involute. It was believed, and stated in print, that there was an injurious amount of oblique thrust in involute teeth which was not present in the cycloidal. That was only true of the gears as then made, and the objection has no force in present-day practice. The writer remembers, and has made, worm gears with teeth of cycloidal section, a practice that perhaps could not be found existing in any shop to-day. Then, again, it was held that the involute system was not so suitable, when small pinions were concerned, as the cycloidal. To some extent this is true, but not wholly, for twelve-toothed pinions are regularly embodied in an interchangeable involute system.

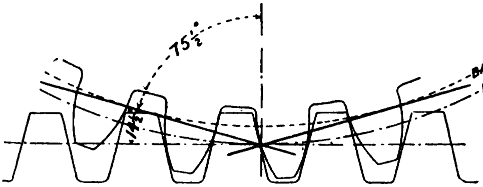


Fig. 6.

Standard Gears.—In the attempt to standardize gears a good many matters have to be considered, and well weighed. The subject concerns both cut and cast gears, and in both cases vested interests are a formidable objection to a system of uniformity. A large set of patterns and tooth blocks, or of cutters, may easily have cost several hundreds of pounds. It may be accepted as a settled fact that the question of the best standard gear will not be determined by mathematical considerations, as the metric system was, but by purely practical and working ones. The simpler the system adopted the better; and there seems no reason to entertain the thought of discarding either the cycloidal or the involute systems already established. They fulfil all requirements, and are easily understood and worked out by practical men who are not mathematicians.

CHAPTER II

TOOTH CURVES

Drawing Tooth Outlines.—The formation of one tooth curve to suit another, though possible by point construction, is not a practicable method. It would be too tedious, and error creeps in, besides which it would have to be repeated for every gear. A more practicable method is to cut a piece of sheet metal to the shape of the tooth to which it is desired to match a gear, centre this and the intended gear, roll them together, and from the edges of the metal templet mark a succession of tooth outlines from the time of entering to that of leaving gear. The bounding outlines common to all these curves give the conjugate outlines of the wheel teeth required.

But the actual marking of tooth outlines is seldom done by a development of each tooth from its mating tooth. In striking out teeth for working drawings, arcs are obtained which are as approximately correct as it is possible to get them to the mathematical tooth curves. Then a circle or circles are struck from the centre of the gear through the centre obtained for the tooth arc, and on this circle so obtained the centres of all the tooth curves will be located.

The double curved or cycloidal teeth are struck either by rolling generating circles directly, or by an odontograph, or by constructions which are based on the employment of either circles or an odontograph. The first is absolutely correct, the second two are approximately so.

Cycloidal Curves.—The usual method of marking out cycloidal tooth curves directly is shown in Figs. 7, 8 and 9. Having adopted a certain size of describing circle, templets are made with convex and concave edges cut to the same curvatures respectively as the pitch circles *a* of the wheel and *b* of the pinion. The circles A, B being provided with a needle point, will, when rolled on the external and internal templet edges of wheel and pinion, generate the tooth profiles indicated, and these will, when in contact, fulfil the conditions of accurate engagement stated in the first chapter. Radii are adapted to the curves which sufficiently approximate to the actual curves. Through the centres of the arcs thus found, circles or cars are struck concentric with the pitch circle, and these form lines of centres whence all the teeth are struck. Two such arcs are shown at *c*, *d* for the flanks in the figures. The centres for the faces happen to come on the pitch lines *a*, *b* in these cases. Fig. 9 shows the same process applied to the rack, which is a spur gear of infinite radius. Here the arcs are alike above and below pitch line.

The cycloidal curves can be drawn without using templet circles by adopting the stepping-round process. Circles or arcs are struck representing successive positions of the rolling of the generating circles, and short distances are stepped round. The curves drawn through the points thus obtained will be practically correct.

There are several methods of construction which give approximations to these curves, illustrated in works on gears. Some afford reasonably correct shapes, others do not. It seems unnecessary to employ these when the employment of rolling circles against templet curves give absolutely correct shapes. This remark applies in a lesser degree to the adoption of odontograph scales and to constructions based upon them. Although three or four such scales have been proposed, only one has been generally employed, that of Professor Willis. Its basis is the selection of a definite angle of pressure, and its principle lies in the movements of link work.

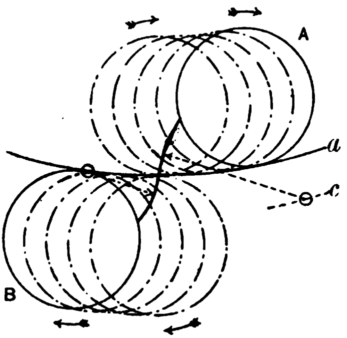


Fig. 7.

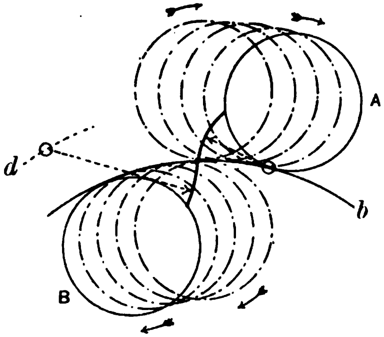


Fig. 8.

Principle of Willis's Odontograph.—If, in Fig. 10, round the fixed centres A and B the links A *c* and B *d* rotate, and are connected by the rigid link *c d*, then whatever the positions taken by A *c* and B *d*, they will be always normal or perpendicular for an instant to the urve which they describe. If these perpendiculars are prolonged they will meet at *e*, around which *c d* is for the instant moving. *e* is termed the instantaneous or virtual centre. A line dropped from *e* perpendicular to the link *c d* cuts A B in *o* where the line of centres A B is divided. The points *c* and *d* are the centres in the odontograph from which tooth curves are struck. Successive movements of the links A *c*, B *d* are indicated by the dotted lines, and in each position the link *c d* cuts the line of centres A B in points not far removed from *o*. The link *c d* forms with the other lines a series of equal triangles, by virtue of which the angular motions of A *c*, B *d* are to each other inversely as the segments into which the link *c d* divides the line of centres A B. In other words, the angular velocity of A *c* divided by the angular velocity of B *d* equals B *o* divided by A *o*. Thus, to obtain uniform ratios, the triangles on each side of the pitch point *o* must be similar, which can only occur when the link *c d* intersects the line of centres A B in the point *o* in the proportion of the required velocity ratios of the arcs A *c*, B *d*.

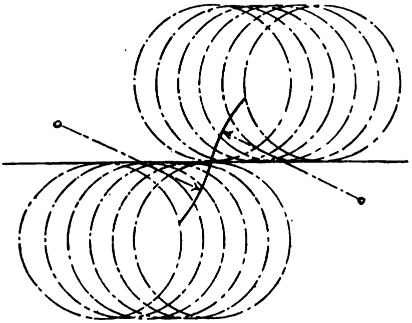


Fig. 9.

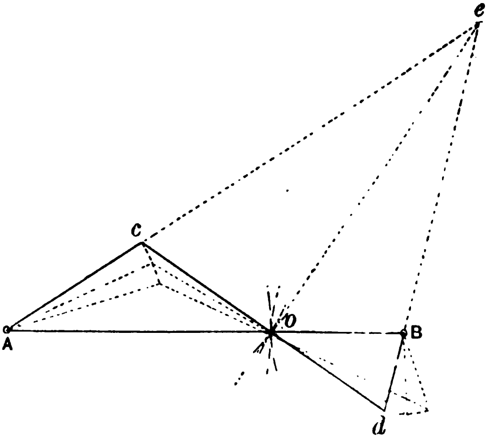


Fig. 10.

In Fig. 10 the imaginary link *c d* divides the line of centres A B exactly at *o*. If now radii be struck from any points in the line *c d*, wheels with centres at A and B will be driven precisely as by the movement of the link *c d*, absolutely correctly when at the point of intersection *o*, and nearly so in positions a little above and below that point when the perpendicular from *e* falls to one side of *o*. It is upon these facts, and the coincidence of the point *o* with the velocity ratio, that the construction of the odontograph is based.

It should be mentioned here that it is not necessary that the curves should be struck at *o*. They may be located on either side of that point, or even outside the link, and they may be both alike convex, or convex and concave. Or two curves may be employed. The essential is that the location of their centres shall be in the line of the link *c d*.

The line *c d* is the angle of pressure, and in fixing this a mean has to be struck. Willis first fixed upon this at 75°, which he selected after various trials as being one which avoids too much rounding of the teeth on the one hand, and too oblique a pressure on the other.

Again, the position of the tooth curves in relation to the point *o* may vary. In single curve teeth the best location is fixed at the point *o* which corresponds with the moment of passing the line of centres. But with double curve teeth the passing of the line of centres should correspond with the point of contact of the arcs forming the faces and flanks of the tooth. The elements of this are given in Fig. 11. Let A, B be the centres of a wheel and pinion in contact at the pitch point *o* in the line of centres A B. From *o* draw the line C *o* D at the angle determined on. From *o* draw the perpendicular *o* E. Assume a centre *a* in the line C D whence the circular arc *b* of a tooth for the wheel A shall be struck. To find the corresponding centre for the wheel B, join A *a* and produce the line to meet *o* E in E. Join E B and produce to meet *o* D in *c*. *c* will be the centre for the tooth curve of pinion B.

If in Fig. 11 the point E is supposed to be situated at an infinite distance, the points *a* and *c* not being given, then the lines A *a* E, *c* B E will be parallel with *o* E and perpendicular to C *o* D. If, therefore (Fig. 12), lines are drawn from A and B perpendicular to C D, then *a* and *b* will be centres from which arcs may be struck at *o* which will engage correctly. And any wheel of a set in which the angle A *o* C remains constant will work correctly if the arcs are struck through the point *o*. This is the basis of the single curve system. And if new radii are substituted as *c e*, *d f*, radii struck from *e* and *f* through the point *o* will engage correctly. But it is essential that the arcs be struck through the point *o*. The objection which Professor Willis made to this system was that the contact was point contact only, strictly exact at the moment of passing the line of centres. For this reason he gave the preference to the double curve teeth, the arcs of which are struck on opposite sides of the pitch point *o*. And the arcs are so set out that the points of mutual action of faces and flanks shall take place, one nearly in the middle of the arc of motion before the line of centres is reached, and the other about the middle of the arc of motion that lies beyond the line of centres and between that and the quitting of contact.

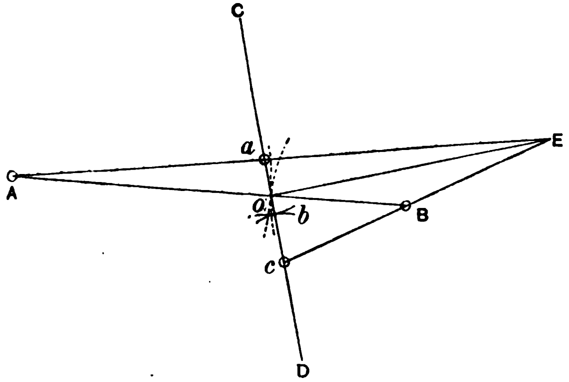


Fig. 11.

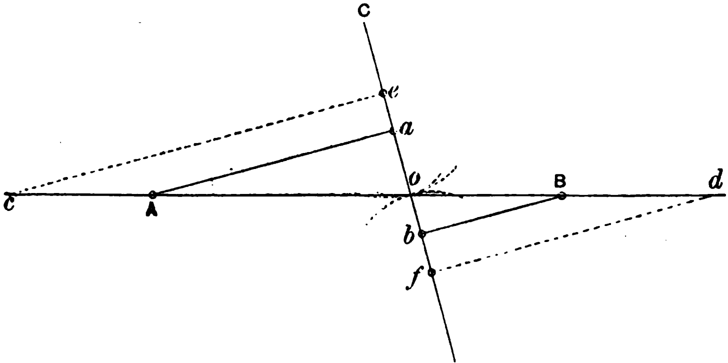


Fig. 12.

Figs. 13 and 14 show the construction for a set of gears for interchangeability. A B is the line of centres divided at the pitch point *o*.

o C is the line of action of 75°, and *c o* is perpendicular to *o* C. The two points *e*, *e*, on the lines C *o*, are set off at equal distances from *o* in each diagram. The centres of the convex arcs are found by joining the centres of each wheel in both figures—A with the point *c* which lies on the opposite side of the line C *o* of the wheel A in Fig. 13, B with the point *c* in Fig. 14. The point of intersection of these lines with C *o* is the centre of the arc. In Fig. 13, *a* is the centre of the convex arc *b*, and in Fig. 14, *a* is the centre of the convex arc *b*.

The centres for the concave arcs are found by joining the centre of each wheel with the point *c* which lies between it and the line C *o*. Thus in Fig. 13, *d* is the centre of the concave arc *e*; in Fig. 14, *d* is the centre of the concave arc *e*. The whole of these arcs are struck through a point lying beyond *o* uniformly taken at one half the pitch. This with the constant distance *o c* ensures interchangeability in a set.

In such a gear the angle of the line of pressure is not constant, as in the involute design, but varies as the teeth traverse the paths of contact. It is highest when small generating circles are at the basis of construction. The angle of 75° in the Willis system leaves 15° as its complement. These angles are only correct for an instant when one side of the tooth is at the line of centres. Twelve teeth is the basis pinion in the Willis system; fifteen teeth in the Brown & Sharpe.

Odontographs may be instruments, or calculated tables only. In each case they crystallize into a handy form the labours of the mathematician, or of the practical man who has obtained standard teeth by more laborious methods. When an instrument is used, error may creep in by its careless application to the primary lines of the wheel; when a table of dimensions is used, error may also be present in application. More than that, odontographs are not absolutely correct, because they generally make use of single circle arcs, and true tooth curves are not arcs of circles. They are extremely close approximations, and the results are often within the limits of average workshop requirements and capabilities. Willis's odontograph is an approximation only, but the same is true also of sets of cutters, each of which has to do duty for a range of teeth, greater or less, in the case of high and low numbered gears respectively.

Interferences in Involutes.—Although the involute rack produces a perfect involute curve in wheels which it generates, this curve cannot be continued below the base circle, on which lies what is termed the interference point. The line of action is tangent to the base circle, and the interference point occurs at the point of tangency on which both the working curve of a wheel and the working length of the teeth of the rack terminate. The teeth are generally lengthened beyond this, but finished with curves. Without some rounding, the rack faces would cut out concavities in the flanks of the smaller pinions (Fig. 15).

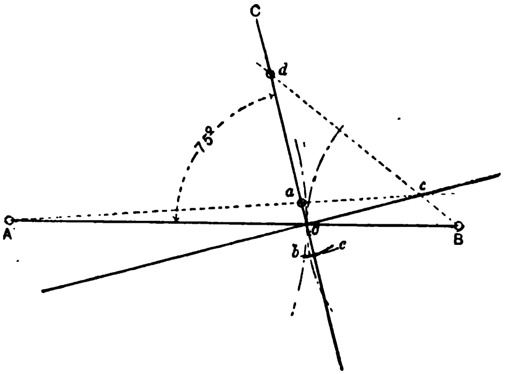


Fig. 13.

In the standard interchangeable involute series, if the smallest gear is one of 12 teeth, this gear determines the limit or interference point for any wheel of the series, below which there can be no working curve.

The 30 *Teeth Limit.*—The construction of a system of cutters for involute teeth is therefore not so simple as it would appear to be from the statement that a rack with straight diagonal faces and flanks generates all teeth correctly. It does so by using a generating machine, and a hob with a thread of rack section corrected. But in setting out teeth for the curves of which centres are used, slight errors and corrections for interference are unavoidable. Thus, in the Brown & Sharpe system a gear of 30 teeth marks a dividing limit at which the construction changes. Above 30 teeth a single curve forms the face and flank, but for numbers below that two curves are used, different for face and flank.

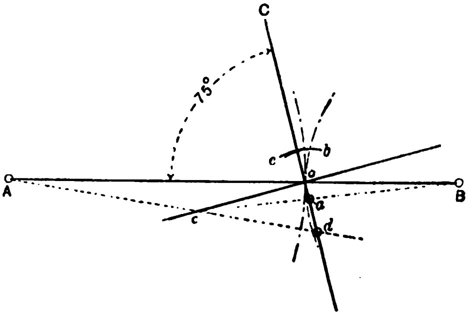


Fig. 14.

A construction given by Brown & Sharpe for single curve gear of 30 teeth and over, illustrated in Fig. 16, gives an approximate to the exact involute.

Strike the pitch circle, P.L, pitch it round and divide each pitch into equal parts to give the tooth thickness and space. Strike the lines for the addendum A, the working depth W.D, and the root line for bottom clearance, corresponding with the whole depth of tooth. From a pitch point on a line joining the point to the centre describe a semicircle B equal to the radius of the pitch diameter. Take one-fourth of this radius and step it back from the pitch point to the semicircle at *a*. Through the point of intersection describe a circle concentric with the pitch circle. This is the base circle B.C. With a radius equal to one-fourth of the radius of the pitch circle, and compass set on the point *a* in the base circle, describe the tooth arcs. The tooth fillets by which these curves merge into the bottoms of the tooth spaces are struck with a radius equal to one-sixth of the widest part of the tooth space or at the addendum circle.

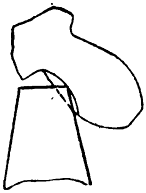


Fig. 15.

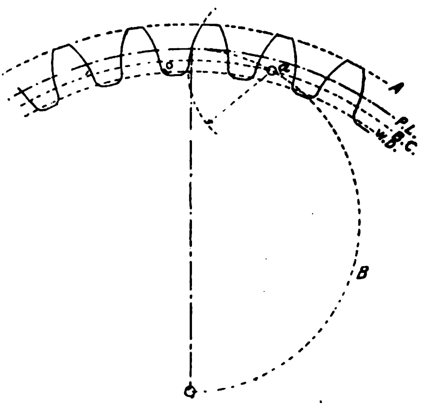


Fig. 16.

As the rack tooth has flanks and faces of 75½° of angle, this gives the construction at once for a normal rack (Fig. 17). The addendum line, the working depth, and the bottom clearance line, are set out as proportioned for a circular gear, and the pitch is set out and divided equally. A line is then drawn through any pitch point perpendicular to the pitch line. From the pitch point an arc is struck, and on it angles of 75½° are set off to right and left of the perpendicular, and diagonals drawn through these from the pitch point. These lines represent the path of contact and angles of pressure, and the tooth sides are drawn at exact right angles with these. This completes the essential tooth forms, but fillets are inserted in the bottom equal to one-sixth of the width of spaces of the rack teeth at the addendum line.

Corrections below 30 *Teeth.*—The interference of rack teeth with low-numbered gears—that is, those below 30 teeth—is such that certain corrections and adjustments have to be made both in the rack and pinion teeth. Straight-faced rack teeth will, as previously stated, cut into the flanks of the teeth of small pinions, not only below the pitch line, but to a little distance above that line. So that, although the straight rack teeth are correct in theory, they have to be modified in an interchangeable system which has to include pinions below 30 teeth. The teeth of the racks are therefore always slightly rounded off, using a cutter for 100 or 135 teeth.

In the Brown & Sharpe system the racks are struck with a slightly rounding face as in Fig. 18, so that they will gear with small pinions without interference. The teeth of the smaller pinions are designed in a manner which is based on experiments made with templets. In gears below 20 teeth a certain proportion of the tooth length below pitch line is made parallel (Fig. 18), varying in amount from one-third to one-sixth of the diametral pitch, as the gears are larger or smaller. A radius is then taken from the centre of a tooth to a parallel line, and an arc struck to meet the fillet, having a radius equal to one-sixth of the width of the space at the addendum circle, as seen in Fig. 18.

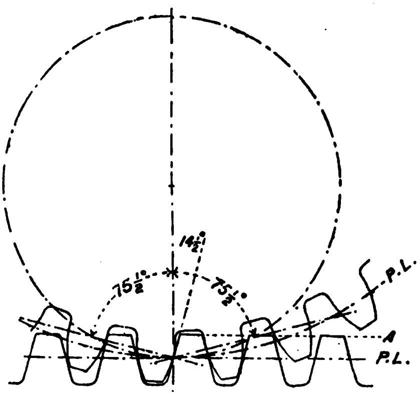


Fig. 17.

If the rule given for striking the curves of a gear above 30 teeth were adopted in low-numbered gears, the teeth would be too rounded, too narrow at the points, and would not take much bearing away from the pitch line. The curves of the faces of the teeth of the smaller gears are therefore obtained by a succession of tangent lines and radii as in Fig. 18. The construction is similar to that which would be produced by the unwinding of a cord. Four tangent lines are taken in the figures. The successive points, 1, 2, 3, 4, are centres of short radii from tangent line to tangent line, the arcs of which are a sufficiently close approximation to an involute.

The foregoing relates to a highly developed system by which good gears can be produced. It has been most extensively adopted and is the generally accepted standard. But it involves the acceptance of minute approximations which would be eliminated in a good generating system.

The Grant System.—Another system which is used to a more limited extent is that of George B. Grant, who fixed the limit of a gear without correction at 36 teeth. He compiled involute odontograph tables by striking out very large teeth of about 8 in. long.

Corrections were made by rounding the points so that they would clear the radial flanks of a 12-toothed pinion—the smallest in the set; and centres were found by trial on the base line, which would agree with the true involute up to the limit point and clear the corrected point. The radii are embodied in a table for given numbers of teeth, and these radii are available for different pitches by multiplying them by the circular pitch, or dividing them by the diametral pitch. Two radii are required up to 36 teeth, but one curve only afterwards. In this system the rack teeth are rounded off to a constant radius for each pitch, to avoid interference.

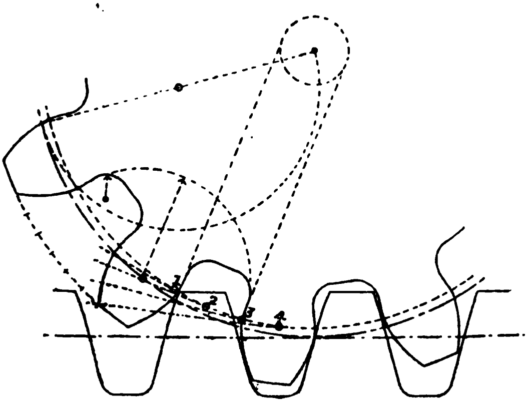


Fig. 18.

Fig. 19 illustrates the nature of a correction as applied to a 12-tooth pinion of 2 pitch, and the table is given on p. 20.

The addendum line is drawn outside the pitch line at a distance from it equal to one, divided by the diametral pitch, or to one-third of the circular pitch. The dedendum line is inside of the pitch line by the same distance. The clearance line is inside of the dedendum line by one-eighth of this distance. The base line is drawn inside of the pitch line by one-sixtieth of the pitch diameter.

Having drawn these lines and divided the pitch equally, the table is consulted. Opposite 12 teeth is found the face radius, 2.51, which divided by the diametral pitch 2 gives 1.25. With the dividers set to this face radius 1.25, draw the faces of the teeth from centres located on the base line. If the number of teeth is greater than 36, or if the pitch is small, this face radius should be continued to the base line.

In the table, at 12 teeth is found the flank radius .96, which divided by the diametral pitch 2 gives .48. With the dividers set to .48, and from centres on the base line draw in all the flanks of the teeth from the pitch line to the base line. From the base line continue the flanks of the teeth to the dedendum line by straight radial lines, and complete with a radius into the clearance line.

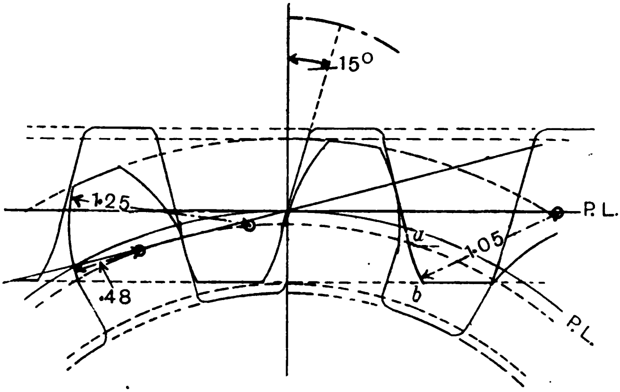


Fig. 19.

To draw the rack, draw a straight line at an angle of 15° with the radius connecting to the centre of the pinion. Round the point of the tooth from the half-way point *a* to the point *b*, with an arc drawn from a centre set on the pitch line, with the dividers set to 2.10in. divided by the diametral pitch, or .67in. multiplied by the circular pitch.

INVOLUTE ODONTOGRAPH.

Standard Interchangeable Tooth, Centres on Base Line.

(By George B. Grant.)

Teeth.	Divide by the Diametral Pitch.		Multiply by the Circular Pitch.	
	Face Radius.	Flank Radius.	Face Radius.	Flank Radius.
10	2·28	·69	·73	·22
11	2·40	·83	·76	·27
12	2·51	·96	·80	·31
13	2·62	1·09	·83	·34
14	2·72	1·22	·87	·39
15	2·82	1·34	·90	·43
16	2·92	1·46	·93	·47
17	3·02	1·58	·96	·50
18	3·12	1·69	·99	·54
19	3·22	1·79	1·03	·57
20	3·32	1·89	1·06	·60
21	3·41	1·98	1·09	·63
22	3·49	2·06	1·11	·66
23	3·57	2·15	1·13	·69
24	3·64	2·24	1·16	·71
25	3·71	2·33	1·18	·74
26	3·78	2·42	1·20	·77
27	3·85	2·50	1·23	·80
28	3·92	2·59	1·25	·82
29	3·99	2·67	1·27	·85
30	4·06	2·76	1·29	·88
31	4·13	2·85	1·31	·91
32	4·20	2·93	1·34	·93
33	4·27	3·01	1·36	·96
34	4·33	3·09	1·38	·99
35	4·39	3·16	1·39	1·01
36	4·45	3·23	1·41	1·03
37—40	4·20		1·34	
41—45	4·63		1·48	
46—51	5·06		1·61	
52—60	5·74		1·83	
61—70	6·52		2·07	
71—90	7·72		2·46	
91—120	9·78		3·11	
121—180	13·38		4·26	
181—360	21·62		6·88	

Draw the rack tooth by the special method (p. 19).

By methods such as these approximations to correct shape, the cutters for systems of gears are shaped. Similar approximations are made to the double-curve or cycloidal teeth. It does not seem necessary to describe these, in view of the fact that they are used to a constantly lessening amount. Moreover, the interest in these diminishes because of the growth of generating practice. Having disposed thus of the subject of tooth curves, we will go on to consider the pitches, proportions, and various dispositions of teeth on their primary or pitch surfaces.

CHAPTER III

PITCHES

Pitches or Tooth Centres.—The settlement of so simple a matter as the measurement of the centres of the teeth of gear wheels is not done without variations in practice which, to say the least, complicate calculations. We find four pitches in use—the circular, the diametral, the module and the chordal; to which may be added that termed the diameter pitch. Each is of value in its own sphere; but each must be capable of translation into its equivalents in the other terms. The following are the meanings of these definitions, and the special utilities of the different systems:—

Circular Pitch.—This, also sometimes termed arc pitch, is the division of the circumference of the pitch circle by the number of teeth in the gear. This measurement, therefore, takes place around the curve of the arc, while chordal pitch, also based on the circular pitch, is taken along the chord or shortest distance between the pitch centres. Chordal pitch is rarely used as a basis of calculation; but the chordal thickness of a tooth is an essential measurement in gear cutting, and is taken account of in measuring tooth thicknesses.

Arc measurement therefore being understood—that taken round the curve and not along the chord—a circular pitch equals the pitch diameter multiplied by 3.14159, and divided by the total number of teeth, or:—

$$p = \frac{p\,d \times 3.1416}{n};$$

where p=circular pitch, n=number of teeth, $p\,d$=pitch diameter. Also:—

$$p\,d = \frac{n \times p}{3.1416},$$

$$n = \frac{p\,d \times 3.1416}{p}.$$

For convenience, all circular pitches are given in even numbers and vulgar fractions, as 1, 2, 3, 1¼, 1½, 1¾, ¼, ½, ¾in., and so on. The result is that the diameters involve the use of fractions. This is due to the relation 3.1416, between circumference and diameter.

DIAMETRAL PITCH COMPARED WITH EQUIVALENT CIRCULAR PITCH.

Diametral Pitch.	Circular Pitch.	Thickness of Tooth on Pitch Line.	Diametral Pitch.	Circular Pitch.	Thickness of Tooth on Pitch Line.
1	3·1416	1·5708	10	·3142	·1571
1¼	2·5133	1·2566	11	·2856	·1428
1½	2·0944	1·0472	12	·2618	·1309
1¾	1·7952	·8976	14	·2244	·1122
2	1·5708	·7854	16	·1963	·0982
2¼	1·3963	·6981	18	·1745	·0873
2½	1·2566	·6283	20	·1571	·0785
2¾	1·1424	·5712	22	·1428	·0714
3	1·0472	·5236	24	·1309	·0654
3½	·8976	·4488	26	·1208	·0604
4	·7854	·3927	28	·1122	·0561
5	·6283	·3142	30	·1047	·0524
6	·5236	·2618	32	·0982	·0491
7	·4488	·2244	36	·0873	·0436
8	·3927	·1963	40	·0785	·0393
9	·3491	·1745	48	·0654	·0327

CIRCULAR PITCH COMPARED WITH EQUIVALENT DIAMETRAL PITCH.

Circular Pitch.	Diametral Pitch.	Thickness of Tooth on Pitch Line.	Circular Pitch.	Diametral Pitch.	Thickness of Tooth on Pitch Line.
3″	1·0472	1·5000″	⅞″	3·5904	·4375″
2¾″	1·1424	1·3750″	13/16″	3·8666	·4062″
2½″	1·2566	1·2500″	¾″	4·1888	·3750″
2¼″	1·3963	1·1250″	11/16″	4·5696	·3437″
2″	1·5708	1·0000″	⅝″	5·0265	·3125″
1⅞″	1·6755	·9375″	9/16″	5·5851	·2812″
1¾″	1·7952	·8750″	½″	6·2832	·2500″
1⅝″	1·9333	·8125″	7/16″	7·1808	·2187″
1½″	2·0944	·7500″	⅜″	8·3776	·1875″
1⅜″	2·2848	6875″	5/16″	10·0531	·1562″
1¼″	2·5133	·6250″	¼″	12·5664	·1250″
1⅛″	2·7925	·5625″	⅛″	25·1327	·0625″
1″	3·1416	·5000″	1/16″	50·2655	·0312″
15/16″	3·3510	·4687″			

COMPARISON OF THE DIAMETRAL, MODULE AND CIRCULAR PITCHES.

Diametral Pitch.	Module.	Circular Pitch.		Diametral Pitch.	Module.	Circular Pitch.	
		Inch.	Mm.			Inch.	Mm.
1	25·400	3·141	79·795	11	2·309	0·285	7·254
1¼	20·320	2·513	63·837	12	2·116	0·261	6·646
1½	16·932	2·094	53·197	14	1·814	0·224	5·700
1¾	14·514	1·795	45·597	16	1·587	0·196	4·986
2	12·700	1·570	39·897	18	1·411	0·174	4·432
2¼	11·288	1·396	35·465	20	1·270	0·157	3·990
2½	10·160	1·256	31·917	22	1·154	0·142	3·627
2¾	9·236	1·142	29·016	24	1·058	0·130	3·325
3	8·466	1·047	26·598	26	0·976	0·120	3·068
3½	7·257	0·897	22·799	28	0·907	0·112	2·850
4	6·350	0·785	19·949	30	0·846	0·104	2·659
5	5·080	0·628	15·959	32	0·794	0·098	2·494
6	4·233	0·523	13·299	36	0·705	0·087	2·217
7	3·628	0·448	11·399	40	0·635	0·078	1·994
8	3·175	0·392	9·974	48	0·529	0·065	1·661
9	2·822	0·349	8·867	60	0·423	0·052	1·331
10	2·540	0·314	7·981	80	0·317	0·039	0·998

As it is troublesome to have fine fractional diameters, the nearest even diameter is generally taken, and the pitches are worked in to suit. This often causes difficulty in utilizing exact pitches when centres are fixed, so that it sometimes happens that odd or bastard pitches, full or bare to exact dimensions, have to be tolerated.

Diametral Pitch.—This is the number of teeth which corresponds to 1in. of diameter of the pitch circle. Hence, if the pitch diameter is divided by the number of teeth in the gear, the quotient is the diametral pitch. It is also the module.

It is not a dimension stated in inches or parts of an inch, but a ratio. Being the number of teeth to an inch of diameter, diametral pitches are fractions.

$$pd=\frac{n}{dp};$$

$$dp=\frac{n}{pd};$$

$$n=pd\times dp;$$

where pd=pitch diameter, n=number of teeth, dp=diametral pitch.

So that while circular pitch is given in inches and parts, as 2in. pitch, 2½in. pitch, etc., the diametral is given by the number which represents the ratio, or number of teeth per inch of diameter. 2 pitch means 2 teeth per inch. A wheel 6in. diameter having 12 teeth would be of 2 pitch.

The Module.—The diametral pitch and the module are each quotients obtained by dividing the diameter of the pitch circle by the number of teeth. But while the diametral pitch is stated as a ratio, the module is given as a dimension. Also, as the latter is most commonly applied to metric pitches, it has become generally identified with diametral pitch measurements in millimetres. *See* the Table, p. 24. This, however, is not an essential distinction, since the module is also expressed in parts of an inch, and is thus embodied in some formulæ. The difference between the diametral pitch, which is a ratio, and the module, which is both a ratio and dimension, is, therefore, a real difference; but only one of terms. Thus, if a gear has, say, 48 teeth, and the pitch diameter measures 12in., dividing 12in. by 48 teeth gives ¼in. as quotient. The module, therefore, is ¼in. But the diametral pitch is 4 pitch, because it is convenient to designate diametral pitches by the denominator of the fraction, or the reciprocal of the fraction.

Metric Pitches.—The reason why the module is used much more extensively on the Continent than the diametral pitch is that measurements are in millimetres, and the millimetre being a fine dimension, the module is very conveniently expressed in millimetres and fractional parts of the same. Therefore, in forming mental images of these pitches, one may work from the circular pitch, of a definite dimension—say in inches—to the diametral pitch, which is a ratio only, to the circular pitch, and to the module, which is a definite dimension of the diametral pitch, and related like it to the circular pitch.

These metric pitches are given in modules, the equivalents of the British diametral pitches, only that they are based on the millimetre instead of on the inch. A module is the pitch diameter in millimetres divided by the number of teeth in the gear. Also the addendum is one module in length, and the depth below pitch line is equal to one module plus the bottom clearance. Modules are even numbers, and even fractional ones, as 1mm., 1.25mm., 1.5mm., 1.75mm., 2mm., etc. But the diametral pitches which correspond with them all give uneven fractions, so that the two are not practically translatable. The relations for module gears are therefore similar to those for diametral pitches (p. 24). Thus:—

$$m = \frac{pd}{n};$$
$$pd = n \times m;$$
$$n = \frac{pd}{m};$$

where:—

m = module;
pd = pitch diameter;
n = number of teeth.

A study of the figures in the Table on p. 24 will show that neither the diametral pitch, the module, nor the circular pitch in inches or millimetres agree, each entailing fractions, so that they are not translatable into whole numbers, for which the cutters for diametral pitches are made.

The Pitches compared.—The relations of the circular and diametral pitches being exactly those of the ratio of circumference to diameter, or 3.1416 to 1, a circular pitch of 3.1416in. is the equivalent of 1 diametral pitch, or 1 P. And a circular pitch of 1in. has as its equivalent 3.1416 diametral pitch. Hence, the coarser the circular pitch the smaller is the number which represents the ratio of the diametral pitch, because the coarser pitches will not subdivide so much into parts of 1in. of diameter as the finer pitches will. Thus, a circular pitch of 4in. corresponds with 0.7854 diametral pitch, and a circular pitch of ¼in. with a diametral pitch of 12.5664. Hence, we can usefully remember that high numbers of the latter correspond with fine pitches.

But as the ratio 3.1416/1 is such that even numbers cannot be obtained by the translation of one pitch into the terms of another, therefore it is almost impracticable to work the two in conjunction. If the dimensions of one are even, then those of the other must be fractional. Both diametral and circular pitches are given in integers and in even fractional numbers as halves and quarters; but neither can be translated into similar even numbers in the other system. So that cutters made for either system cannot be used for the other. The selection, then, becomes one of greater or less convenience. The choice is in favour of the diametral, because in this, diameters are all even, being calculated directly in terms of pitch and numbers of teeth. In the circular system, on the other hand, diameters almost invariably entail small fractional dimensions, being calculated from even circumferences on the basis of pitch and number of teeth. This is the real explanation of the fact that the diametral system is rapidly displacing the other. But further, the diametral system simplifies the proportioning of the teeth, because the pitch is made a basis for proportioning. The length of addendum is made equal to one diametral pitch, and the dedendum also, plus a bottom clearance equal to one-tenth of a pitch.

With regard to the ratio 1/3.1416, dividing by 3.1416 is the same as multiplying by 0.3183, because 1/3.1416=.3183. The latter is, therefore, often preferred as a multiplier. If the circumference of the pitch circle is multiplied by .3183, the product is the pitch diameter. And if the circular pitch is multiplied by .3183, the product has the same relation to the circular pitch that the diameter bears to the circumference. Hence, the circular pitch multiplied by .3183 equals one *diameter pitch*—another term which requires explanation. The diameter pitch expresses the modulus or module, or ratio of the diameter to the circumference. It is the diameter of the pitch circle divided by the number of teeth.

Manchester Pitch.—The old Manchester diametral pitch used fifty or sixty years ago was not the same thing as the diametral pitch of the present. That was what we now term the module. If the diameter of the pitch circle be divided into as many equal parts as there are teeth in the wheel, one of these parts was termed the diametral pitch (module); thus:—

$$\frac{\text{Diameter}}{\text{Number of teeth}}=\text{diametral pitch, or module.}$$

A gear in which the relations were 2 teeth to the inch of diameter was called 2 pitch; one having 4 teeth to the inch 4 pitch, and so on. In the present diametral system the reciprocal of the old or module system is used. Thus, for example, taking 2 diametral pitch, which is ½in. module, the ½in. is dropped and the denominator only is retained. ½in. × 3.1416=1.5708 circular pitch.

The following formulæ will afford examples of calculation. The diametral pitch, as we have said, is denoted by the letter P. If there are 4 teeth to each inch of diameter of the pitch circle, the diametral pitch is 4P. To obtain the circular pitch from P=3.1416/P, hence 3.1416/4P=.7854 for the equivalent circular pitch (C.P.). Therefore, P=4, C.P.=.7854in.; and 1in./4P=¼in. for the module, or one tooth, or diameter pitch, which is usually designated as *s*. And, conversely, 1in./s=P, or 1in. divided by the module equals the number of teeth to 1in., or the diametral pitch. To obtain diametral pitch from circular pitch, divide 3.1416 by the circular pitch, 3.1416/C.P.=P.

The simplicity of the diametral-pitch system has not only enabled it to take the lead easily of the circular-pitch system, but it has reacted on the latter. The old scales of tooth proportions have given way to the simplicity of the diameter pitch or module for getting the outside and root diameters. The addendum is equal to this, and the dedendum also, but plus a root clearance.

Chordal Pitch.—This is hardly ever used now for obtaining the pitch-circumferences. It is evident that gears, in order to imitate the action of the smooth frictional peripheries of cylinders, must have their pitches measured round the arcs of the pitch cylinders, and not along the chords. The difference is negligible in large gears, but it is very apparent in the smaller wheels and pinions, and especially in those of coarse pitches. But the chord measurement is applied to the tooth thickness in cut gears in which, as there is no flank clearance at all, very slight inexactitude in thicknesses would either produce engagements too slack or too tight. It is the practice, therefore, to correct the thicknesses for chord measurement, and either set a vernier caliper to the thickness, or use a solid gauge.

Looking at Fig. 20, the chord distance C E is made less than half the circular pitch. This must be so, because the chord thickness K I measured straight across the tooth is less than that around the arc K D I of the pitch circle; and also because the points C and E will touch the sides of the teeth above the pitch circle, because F C=G D, and C E is, therefore, made still smaller than the actual chord thickness. Obviously, too, this thickness C E is different even for gears of the same pitch, but having different numbers of teeth, owing to differences in the curvature of their arcs. It would not, therefore, be strictly correct to use the same gauge for all gears of the same pitch from a rack to a pinion. Tables are calculated and are available for the chordal thicknesses of teeth of different pitches and heights of arc. But practically perfect results can be secured by using one solid

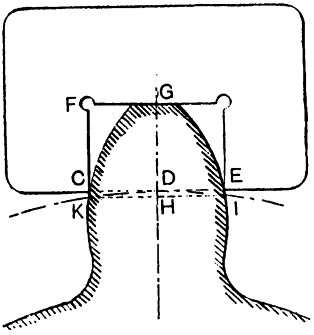


Fig. 20.

gauge made for a gear midway between a rack and a small pinion. This makes the small pinions with slightly thicker teeth than standard, and the larger gears with teeth correspondingly thinner.

Fig. 21 illustrates the gauges used at the Gleason Works for a range of gears from 2 D.P. to 16 D.P. They are made from a gear about half-way between a pinion and a rack tooth. If more precise measurements are required, a vernier, as made by Messrs Brown & Sharpe, must be set to each different gear. The Tables on pp. 30 and 31 are given by the firm for calculating the chordal thicknesses of teeth by, both for diametral and circular pitches.

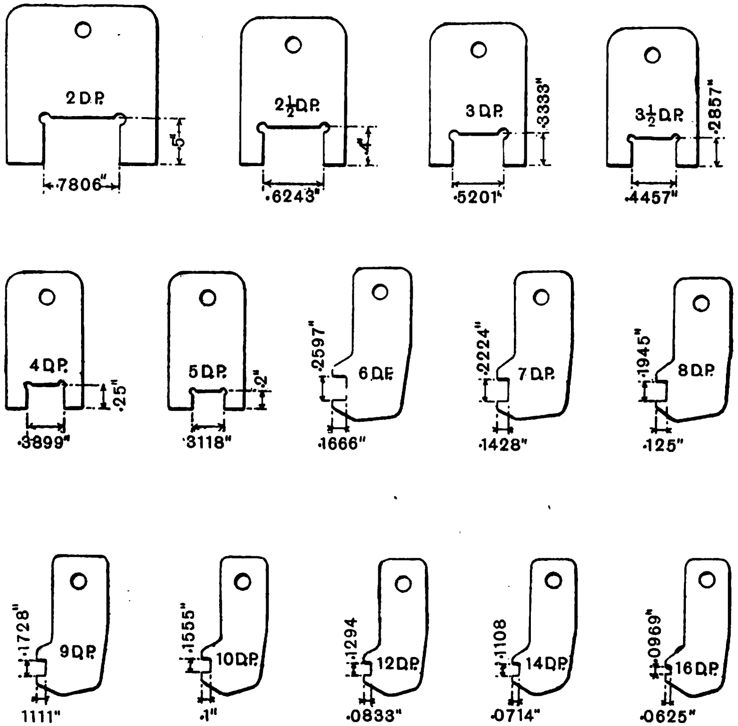


Fig. 21.

To calculate the chordal thickness K I of half the circular pitch and the length to addendum plus H D, Fig. 20, to set the vernier by, Fig. 22, let:—

T=Chordal thickness of tooth, K I in Fig. 20.

H=Height of arc or versed sine, H D in Fig. 20.

D=Pitch diameter.
R=Pitch radius.
A=Angle=90° divided by the number of teeth in the gear.
Then:—$T = D \times \text{sine of angle } A$.
$H = R \times (1 - \text{cos of angle } A)$.

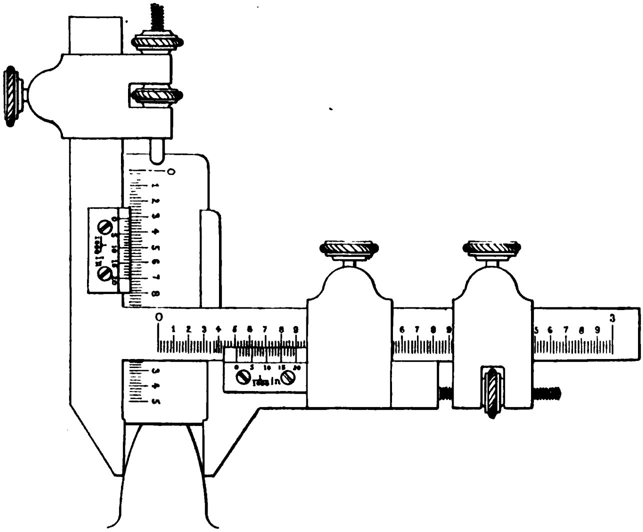


Fig. 22.

CHORDAL THICKNESS OF TEETH FOR SPUR GEARS.

On a Basis of 1 Diametral Pitch.

To obtain chordal thickness of teeth and corrected depth of Pitch Line for any diametral pitch less than 1, divide figures in table by the diametral pitch required.

The Gleason Works, Rochester, N. Y.

Number of Teeth	Chordal Thickness of Teeth.	Corrected Depth of Pitch Line.	Number of Teeth.	Chordal Thickness of Teeth.	Corrected Depth of Pitch Line.
8	1·5607″	1·0768″	21—22	1·5694″	1·0294″
9	1·5628″	1·0684″	23—25	1·5696″	1·0268″
10	1·5643″	1·0616″	26—29	1·5698″	1·0237″
11	1·5654″	1·0559″	30—34	1·5701″	1·0208″
12	1·5663″	1·0514″	35—41	1·5702″	1·0176″
13	1·5670″	1·0474″	42—54	1·5704″	1·0147″
14	1·5675″	1·0440″	55—79	1·5706″	1·0112″
15—16	1·5679″	1·0411″	80—134	1·5707″	1·0077″
17—18	1·5686″	1·0362″	135	1·5708″	1·0046″
19—20	1·5690″	1·0324″			

CHORDAL THICKNESS OF TEETH FOR SPUR GEARS.

ON A BASIS OF 1″ CIRCULAR PITCH.

To obtain chordal thickness of teeth and corrected depth of Pitch Line for any circular pitch, multiply figures in table by the circular pitch required.

The Gleason Works.

Number of Teeth.	Chordal Thickness of Teeth.	Corrected Depth of Pitch Line.	Number of Teeth.	Chordal Thickness of Teeth.	Corrected Depth of Pitch Line.
8	·4968″	·3428″	21—22	·4996″	·3277″
9	·4975″	·3401″	23—25	·4997″	·3268″
10	·4979″	·3379″	26—29	·4997″	·3258″
11	·4983″	·3361″	30—34	·4998″	·3249″
12	·4986″	·3347″	35—41	·4998″	·3239″
13	·4988″	·3334″	42—54	·4999″	·3230″
14	·4990″	·3323″	55—79	·5000″	·3219″
15—16	·4991″	·3314″	80—134	·5000″	·3208″
17—18	·4993″	·3298″	135	·5000″	·3198″
19—20	·4995″	·3286″			

CHAPTER IV

TOOTH PROPORTIONS

Tooth Proportions.—If the diametral pitch is simpler than the circular, it also simplifies the proportioning of the teeth. The millwrights and patternmakers have long employed scales of proportions based upon a division of the pitch into a certain number of equal parts, and taking certain numbers of these to represent lengths and clearances. The old rules made the teeth longer than present practice recognizes. Short teeth are now in favour, not only as being more in harmony with the conditions of strength advocated by Mr Longridge, but also as being better adapted to the practice of cutting. Also the adoption of the Brown & Sharpe system of cutters and proportions has greatly favoured the movement. All these proportions, both for circular and diametral pitch, are tabulated in columns to facilitate the sizing of blanks, setting up of machines, and selection of cutters. The proportions on which these Tables are based are as follows, illustrated by Figs. 23, 24, and reference letters.

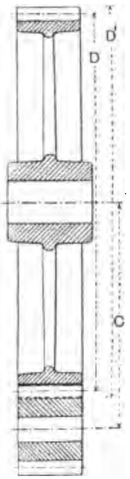


Fig. 23.

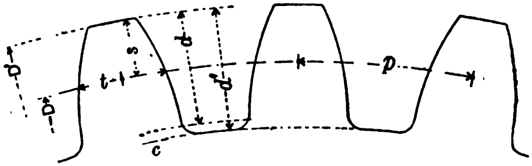


Fig. 24.

In Fig. 23 D is the pitch diameter and D^1 the outside diameter. In ordinary gears it must be stated which diameter is intended. The pitch diameter is the proper one to give. The centres C of a pair of gears, in conjunction with their diameters, give the ratio between the two, both as diameters and numbers of teeth. In Fig. 24 the circular pitch is p, the tooth thickness t. The distance between the

SIZING AND CUTTING OF GEARS.

"Nuttall."

DIAMETRAL PITCH.

To Get	Having	Rule	Formula
The Diametral Pitch	The Circular Pitch	Divide 3.1416 by the Circular Pitch........................	$P=\frac{3.1416}{C}$
The Diametral Pitch	The Pitch Diameter and the Number of Teeth.....	Divide Number of Teeth by Pitch Diameter................	$P=\frac{N}{D}$
The Diametral Pitch	The Outside Diameter and No. of Teeth.........	Divide Number of Teeth plus 2 by Outside Diameter..........	$P=\frac{N+2}{D}$
Pitch Diameter	The No. of Teeth and the Diametral Pitch........	Divide Number of Teeth by the Diametral Pitch...............	$D'=\frac{N}{P}$
Pitch Diameter	The No. of Teeth and Outside Diameter...........	Divide the product of Outside Diameter and No. of Teeth by No. of Teeth plus 2............	$D'=\frac{DN}{N+2}$
Pitch Diameter	The Outside Diameter and the Diametral Pitch	Subtract from the Outside Diameter the quotient of 2 divided by the Diametral Pitch....	$D'=D-\frac{2}{P}$
Pitch Diameter	Addendum and the No. of Teeth	Multiply Addendum by the Number of Teeth..............	$D'=s\,N$
Outside Diameter	The No. of Teeth and the Diametral Pitch........	Divide Number of Teeth plus 2 by the Diametral Pitch........	$D=\frac{N+2}{P}$
Outside Diameter	The Pitch Diameter and the Diametral Pitch...	Add to the Pitch Diameter the quotient of 2 divided by the Diametral Pitch................	$D=D'+\frac{2}{P}$
Outside Diameter	The Pitch Diameter and the Number of Teeth.....	Divide the No. of Teeth plus 2 by the quotient of No. of Teeth divided by Pitch Diameter....	$D=\frac{N+2}{\frac{N}{D'}}$
Outside Diameter	The No. of Teeth and Addendum..	Multiply the Number of Teeth plus 2 by Addendum..........	$D=(N+2)s$
Number of Teeth	The Pitch Diameter and the Diametral Pitch...	Multiply Pitch Diameter by the Diametral Pitch...............	$N=D'P$
Number of Teeth	The Outside Diameter and the Diametral Pitch	Multiply Outside Diameter by the Diametral Pitch and subtract 2	$N=DP-2$
Thickness of Tooth	The Diametral Pitch	Divide 1.5708 by the Diametral Pitch..........................	$t=\frac{1.5708}{P}$
Addendum	The Diametral Pitch	Divide 1 by the Diametral Pitch or $s=\frac{D'}{N}$....................	$s=\frac{1}{P}$
Root	The Diametral Pitch	Divide 1.157 by the Diametral Pitch............................	$R=\frac{1.157}{P}$
Working Depth	The Diametral Pitch	Divide 2 by the Diametral Pitch	$W\,D=\frac{2}{P}$
Whole Depth	The Diametral Pitch	Divide 2.157 by the Diametral Pitch..........................	$D'=\frac{2.157}{P}$
Clearance	The Diametral Pitch	Divide .157 by the Diametral Pitch..............................	$f=\frac{.157}{P}$
Clearance	Thickness of Tooth	Divide Thickness of Tooth at pitch line by 10................	$f=\frac{t}{10}$

CIRCULAR PITCH.

"Nuttall."

To Get	Having	Rule.	Formula.
The Circular Pitch.	The Diametral Pitch.	Divide 3.1416 by the Diametral Pitch	$P' = \frac{3.1416}{P}$
The Circular Pitch.	The Pitch Diameter and the Number of Teeth	Divide Pitch Diameter by the product of .3183 and Number of Teeth	$P' = \frac{D'}{.3183\ N}$
The Circular Pitch.	The Outside Diameter and the Number of Teeth	Divide Outside Diameter by the product of .3183 and Number of Teeth plus 2	$P' = \frac{D}{.3183\ N+2}$
Pitch Diameter.	The Number of Teeth and the Circular Pitch	The continued product of the Number of Teeth, the Circular Pitch and .3183	$D' = NP'.3183$
Pitch Diameter.	The Number of Teeth and the Outside Diameter	Divide the product of Number of Teeth and Outside Diameter by Number of Teeth plus 2 . . .	$D' = \frac{ND}{N+2}$
Pitch Diameter.	The Outside Diameter and the Circular Pitch	Subtract from the Outside Diameter the product of the Circular Pitch and .6366	$D' = D-(P'.6366)$
Pitch Diameter.	Addendum and the Number of Teeth.	Multiply the Number of Teeth by the Addendum	$D' = N\ s$
Outside Diameter.	The Number of Teeth and the Circular Pitch	The continued product of the Number of Teeth plus 2, the Circular Pitch and .3183 . . .	$D = (N+2)P'.3183$
Outside Diameter.	The Pitch Diameter and the Circular Pitch	Add to the Pitch Diameter the product of the Circular Pitch and .6366	$D = D'+(P'.6366)$
Outside Diameter.	The Number of Teeth and the Addendum	Multiply Addendum by Number of Teeth plus 2	$D = s\ (N+2)$
Number of Teeth.	The Pitch Diameter and the Circular Pitch	Divide the product of Pitch Diameter and 3.1416 by the Circular Pitch	$N = \frac{D'\ 3.1416}{P'}$
Thickness of Tooth.	The Circular Pitch.	One-half the Circular Pitch . .	$t = \frac{P'}{2}$
Addendum.	The Circular Pitch.	Multiply the Circular Pitch by .3183, or $s = \frac{D'}{N}$	$s = P'.3183$
Root.	The Circular Pitch.	Multiply the Circular Pitch by .3683	$s + f = P'.3683$
Working Depth.	The Circular Pitch.	Multiply the Circular Pitch by .6366	$D'' = P'.6366$
Whole Depth.	The Circular Pitch.	Multiply the Circular Pitch by .6866	$D'' + f = P'.6866$
Clearance.	The Circular Pitch.	Multiply the Circular Pitch by .05	$f = P'.05$
Clearance.	Thickness of Tooth.	One-tenth the Thickness of Tooth at Pitch Line	$f = \frac{t}{10}$

TOOTH PROPORTIONS.

(DIAMETRAL PITCH.)

Brown & Sharpe Manufacturing Company.

Diametral Pitch.	Thickness of Tooth on Pitch Line.	Addendum.	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.
½	3·1416	2·0000	4·0000	2·3142	4 3142
¾	2·0944	1·3333	2·6666	1·5428	2·8761
1	1·5708	1·0000	2·0000	1·1571	2·1571
1¼	1·2566	·8000	1·6000	·9257	1·7257
1½	1·0472	·6666	1·3333	·7714	1·4381
1¾	·8976	·5714	1·1429	·6612	1·2326
2	·7854	·5000	1·0000	·5785	1·0785
2¼	·6981	·4444	·8888	·5143	·9587
2½	·6283	·4000	·8000	·4628	·8628
2¾	·5712	·3636	·7273	·4208	·7844
3	·5236	·3333	6666	·3857	·7190
3½	·4488	·2857	·5714	·3306	·6163
4	·3927	·2500	·5000	·2893	·5393
5	·3142	·2000	·4000	·2314	·4314
6	·2618	·1666	·3333	·1928	·3595
7	·2244	·1429	·2857	·1653	·3081
8	·1963	·1250	·2500	·1446	·2696
9	·1745	·1111	·2222	·1286	·2397
10	·1571	·1000	·2000	·1157	·2157
11	·1427	·0909	·1818	·1052	·1961
12	·1309	·0833	·1666	·0964	·1798
13	·1208	·0769	·1538	·0890	·1659
14	·1122	·0714	·1429	·0826	·1541
15	·1047	·0666	·1333	·0771	·1438
16	·0982	·0625	·1250	·0723	·1348
17	·0924	·0588	·1176	·0681	·1269
18	·0873	·0555	·1111	·0643	·1198
19	·0827	·0526	·1053	·0609	·1135
20	·0785	·0500	·1000	·0579	·1079
22	·0714	·0455	·0909	·0526	·0980
24	·0654	·0417	·0833	·0482	·0898
26	·0604	·0385	·0769	·0445	0829
28	·0561	·0357	·0714	0413	·0770
30	·0524	·0333	·0666	·0386	·0719
32	·0491	·0312	·0625	·0362	·0674
34	·0462	·0294	·0588	·0340	·0634
36	·0436	·0278	·0555	·0321	·0599
38	·0413	·0263	·0526	·0304	·0568
40	·0393	·0250	·0500	·0289	·0539
42	·0374	·0238	·0476	·0275	·0514
44	·0357	·0227	·0455	·0263	·0490
46	·0341	·0217	·0435	0252	·0469
48	·0327	·0208	·0417	·0241	·0449
50	·0314	·0200	·0400	·0231	·0431
56	·0280	·0178	·0357	·0207	·0385
60	·0262	·0166	·0333	·0193	·0360

pitch diameter D and the outside of the tooth is the addendum *s*, or face. Twice the addendum equals the working depth *d* of tooth, and the working depth inside or below the pitch circle is the flank. The clearance *c* at the bottom, added to the working depth, equals the whole depth d^1 of the tooth, which is the depth to which the cutter is set to work. The addendum *s* is a very useful number in the diametral-pitch system, being equal to one diametral pitch, so that to get the outside diameter it is only necessary to add two to the number of teeth and divide the sum by the diametral pitch, which is simpler than consulting a table of proportions of tooth parts. Messrs Brown & Sharpe call the addendum a *diameter* pitch. It is equal to a circular pitch multiplied by 0.3183. The clearance is proportional to the pitch, being $\frac{1}{10}$ of the thickness *t* of the tooth at the pitch line. Having ratio given, to find the pitch diameters:—

Divide the centres by the sum of the terms of the ratio. Twice the quotient is multiplied by each term separately, and the two products will be the pitch diameters of the two wheels.

The two tables on pp. 33 and 34 give both in words and formulæ the relations of the standard (B. & S.) tooth proportions for diametral and circular pitches. To save the labour of calculation, tables are provided by the gear-cutting firms for all the ordinary diametral and circular pitches. That on p. 35 is for the first named.

The groups of Figs., 25-27, illustrate to actual size, teeth of the commonest diametral pitches to the Brown & Sharpe proportions.

Gear Rules.—To lessen the chance of error in sizing blanks is the object of these rules. They are divided out like scales in several lines of divisions to correspond with diametral pitches, Fig. 25 showing one line or division only for 5 pitch. Thus, a rule by the Brown & Sharpe Manufacturing Co. has four lines of graduations on each side—namely, 18, 20, 22, 24, 26, 28, 30, and 32 parts of an inch running along a length of 12in. As the addendum diameter of a wheel is equal to two diametral pitches, all that is necessary is to measure off the number of teeth + 2 on the line of divisions which corresponds with the diametral pitch. Thus, if a wheel is to have 20 teeth of 18 pitch, 22 parts will be measured off on the rule, being 20 for the pitch diameter, and 2 for the addenda.

Another rule of the same length includes a larger number of divisions, by graduating 1in. only from each end, the intermediate 10in. containing 1in. divisions ruled right across. These numbers

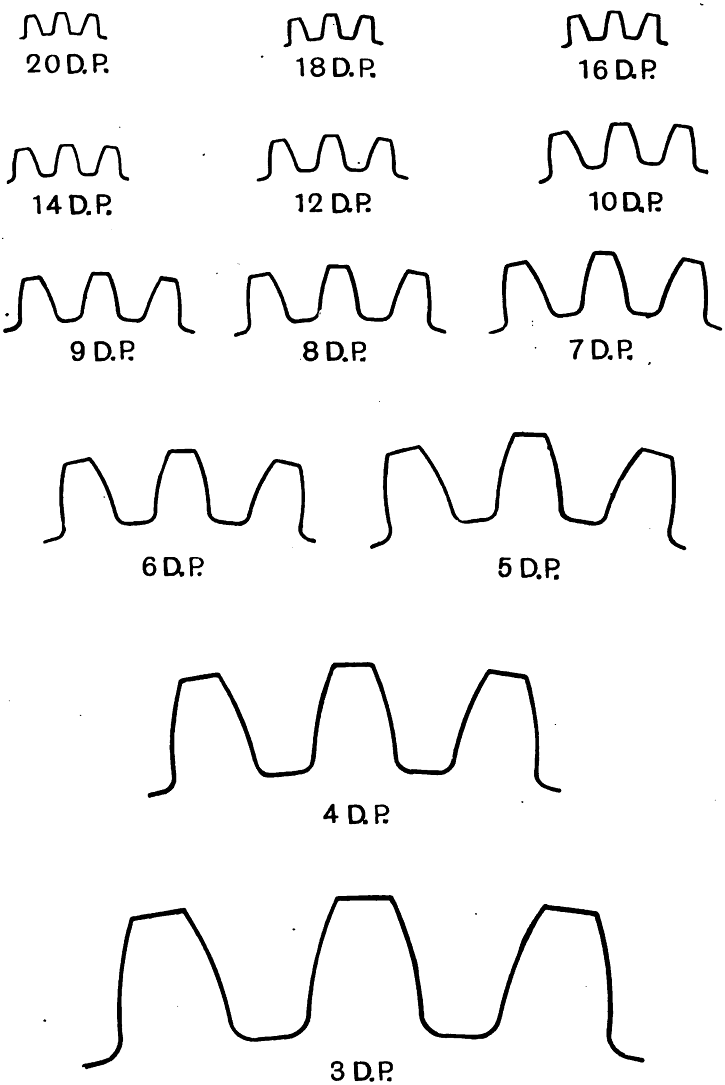
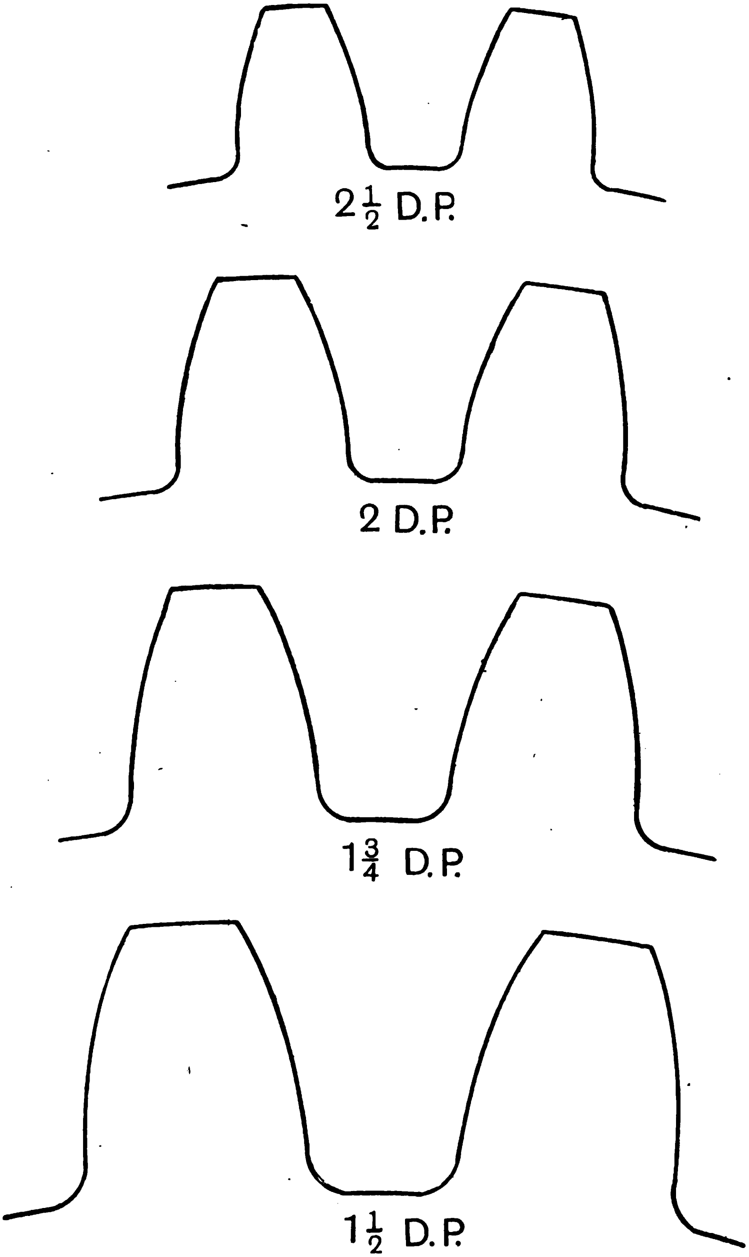


Fig. 25.



$2\frac{1}{2}$ D.P.

2 D.P.

$1\frac{3}{4}$ D.P.

$1\frac{1}{2}$ D.P.

Fig. 26.

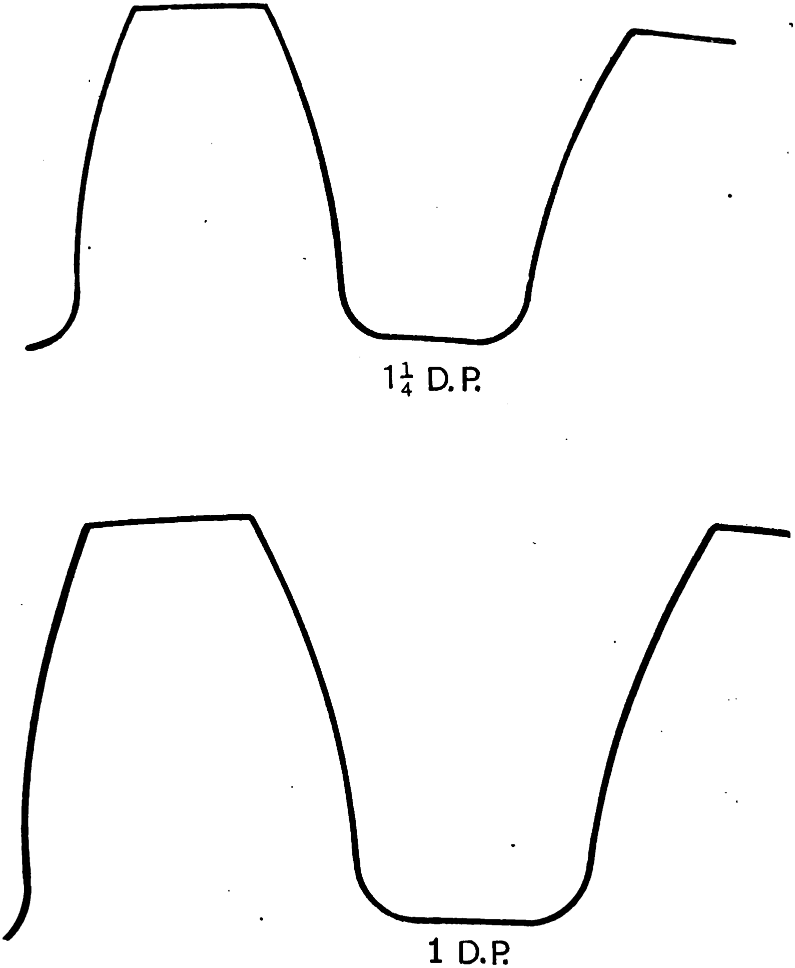


Fig. 27.

include 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, and 38 parts of an inch. The large numbers are taken on the intermediate inches, and the small ones on the divided inches, similarly to draughtsmen's scales. Thus a 62-tooth wheel of 10 pitch would have 60 measured on 6 plain inches, and the two parts on the divided inch, on the line divided into tenths of an inch. Other rules are also made with various other graduations.

Steel gear gauges (Fig. 29) are made, in which the trouble of calculating depth and thickness of tooth is saved. The gauge resembles a disc wire gauge, the place of the notches being taken by teeth cut around its edges, in sets of two or three of each diametral pitch, so that there are several pitches in the circle of each separate gauge. It is a convenient arrangement, because the depth and width of tooth can be marked on a blank; and a wheel when cut, or partly cut, can be tried with the teeth on the gauge, when any inaccuracy will be apparent.

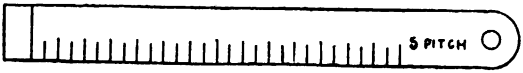


Fig. 28.

Changes Proposed.—The ordinary or standard proportions give one module as the length of addendum and one for the dedendum. The standard angle of pressure is 14½°. Out of this to some extent arises the question of length of tooth. The longer the addendum the more prolonged the contact, but the greater the influence of the undercut in small pinions (Fig. 30). To lessen this is the object of the stub tooth proposed by Mr Fellows, with a 20° pressure angle, and an addendum equal to three-fourths of the module. Mr Wilfred Lewis proposes a 22½° pressure angle, and an addendum of seven-eighths the modulus.

What the present 14½° of angle entails in small pinions has been shown in Chapter II. The shape of the teeth of every rack and every small pinion involves a departure from strict theory, a compromise made to get either a smooth gear or a stronger tooth than the rigidly generated form would yield. Rack teeth are rounded slightly, the addenda of pinions are lengthened, and numbers of modifications n, and departures from the theoretical forms arise. A higher angle of pressure avoids these makeshifts, permitting correctly generated forms to be adopted without exceptions. It may be increased to 18, 20, 22½, or 25 degrees. But the higher the angle the stronger the roots of the small pinions become. This feature is one of much value in most feed and speed change gears. If there were no small pinions no trouble would occur. But as they are in regular demand a sound reason exists why the basis should be such as to include them legitimately instead of retaining an unsatisfactory base and making corrections for the small pinions.

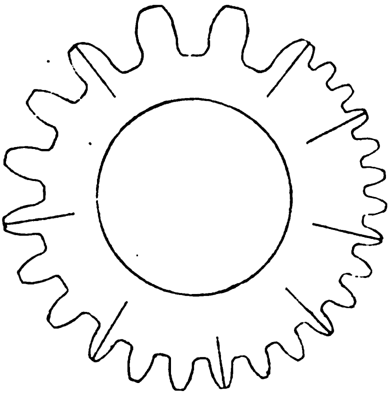


Fig. 29.

Alterations in Pressure Angle.—The 14½° angle system can be retained without the modifications in construction for gears below 30 teeth by increasing the diameter of the blank. This takes advantage of the fact stated, p. 7, that the essential basis in the involute curve is not the pitch circle, as it is in the curves of the cycloidal gears, but the base circle, since on this the curves are generated. Hence the relative positions of the base and the pitch circles can be varied without affecting the accuracy of the engagement. And thus by enlarging the blank the length of addendum is increased and that of the dedendum diminished. In other words, the tooth faces are lengthened and the flanks are shortened. This is sometimes done of necessity, but it is another case of a compromise inseparable from a low angle of pressure rigidly fixed. Fig. 31 shows for comparison the undercut of a small gear A, of 14½° pressure angle, the effect of increasing the addendum on the same gear at C, and the result of

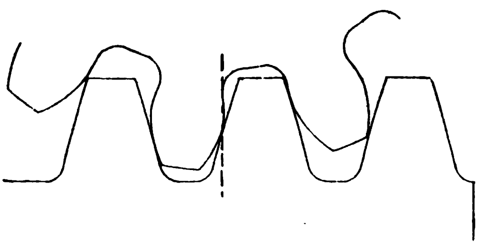


Fig. 30.

altering the pressure angle to 20° at B, without altering the addendum.

Shortening of Teeth.—If, in addition to the alteration in angle, the teeth are shortened, the difficulties inseparable from the present standard system vanish. The case is not perhaps so strong for the short tooth as for the change in angle, but it has many advocates and is adopted rather extensively in special practice. The difference between the two is seen in Figs. 32 and 33. In Fig. 32 the pressure angle is 14½°, and the teeth are of ordinary length. The arc of contact A is long, extending from *a* to *b*, being nearly as long as the arc of action B, and the sliding and rubbing action which occurs during approach is prolonged. In Fig. 33 the pressure angle is 20° and the teeth are shortened. The results are that the arc of contact is reduced to A, extending from *a* to *b* only, while the arc of action extends to B. Gears cut on this system may still utilize the tables of standard tooth proportions, but the pitch depth is selected from a diametral pitch different from that of the pitch to be cut. Thus a 10-pitch gear is turned as for a 12-pitch gear, being one of two pitches smaller.

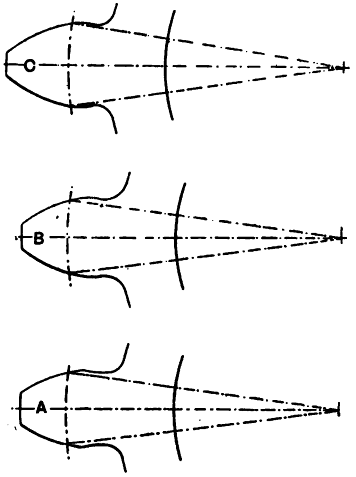


Fig. 31.

Hence:—

Diametral pitch	4	5	6	7	8	9	10	12
Pitch depth	5	7	8	9	10	11	12	14

Some abnormal spur pinions cut on the Fellows gear shaper illustrate in an instructive way the fact that the length of addendum need not be constant in wheels with involute teeth gearing together correctly, as it must be in cycloidal teeth. Three blanks were cut: one of the correct diameter for 24 teeth, one for 26, and one for 23. Each, however, was cut in the generating gear cutter with the index mechanism set for 24 teeth, and each, therefore, when in gear revolved at the same speed, and each geared correctly with the other.

The explanation is to be found in the difference between pitch surface and base line. The length of addendum was different in each. In the largest wheel it was greatest, the teeth being nearly all "faces"; in the smallest it was least, the teeth being nearly all "flanks."

Objections to Change.—Much discussion has taken place in recent years on the relative values of longer or shorter teeth. The advantages, on the whole, seem to be in favour of adopting teeth shorter than those which are at present standardized. But the great objection to change in this respect, as in pressure angles, is the immense vested interests sunk in the enormous stocks of cutters carried in the shops, as well as the necessity for renewals of gears to engage with those already existing in thousands of machines.

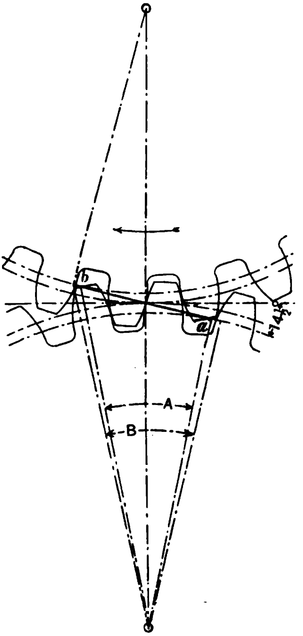


Fig. 32.

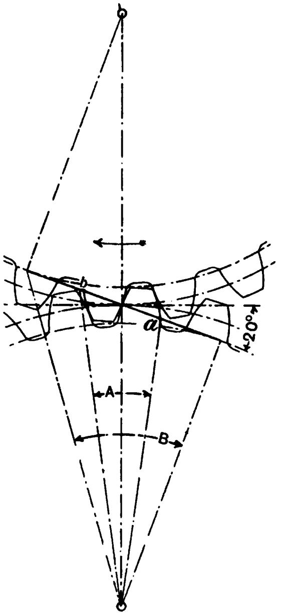


Fig. 33.

The stock of single disc cutters carried in a large establishment is immense. Every separate pitch must have its set of cutters, and for some pitches more than one set when the work done is extensive.

Besides these there are the numerous stocking or roughing cutters. Moreover, the 14½° pressure angle has been adopted for so long a period that the substitution of another angle would for a long while entail the use of both the old and the new. New gears would often be required to engage with existing ones, and until these were worn out or scrapped the two systems would have to be run side by side. The attempt therefore to substitute another standard for gears would not only fail to command general approval from the manufacturing point of view, but it would cover a wide debatable field. It is even open to discussion whether a universal system is desirable. The subject is not on a parallel with that of screw threads or rail sections or of many other products which are standardized. Standardization is desirable in some things, but it may not be so in all. Perhaps the teeth of gears are an exception.

CHAPTER V

GEARS RELATED TO THE SPURS

Gears which are related to the spurs include racks, internal gears, spirals, helicals, and worms. The teeth of each of these are derived directly from the straight rack tooth, and each has teeth of parallel section throughout.

Internal Gears.—These are designed as though the teeth were those of an external wheel, but the difference is that the metal is just reversed. That is (Fig. 34), the teeth of the cycloidal spur wheel A would fill the tooth spaces in the internal wheel B, of the same size, and *vice versa.* The only modification is that the point and root clearances are reversed. The drawing is made as for the wheel A. For the internal gear B the flanks of B would be deepened and the faces shortened in relation to the pitch line.

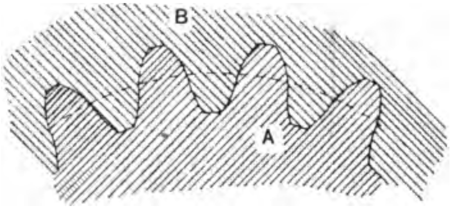


Fig. 34.

The Tooth Forms.—In the case of an internal involute wheel (Fig. 35), the points of the teeth cannot act beyond the interference limit line of the pair, and they may therefore be either cut off entirely, as in the figure, or well rounded off to clear, and the pinion teeth be deepened. But as there can be no mesh beneath the limit line, this alternative is merely a matter for taste and good appearance. In practice the concave form is often departed from, the teeth being cut with the flattest cutter of the set (No. 1), used for wheels of 135 teeth to a rack.

These general principles are not of universal application excepting to gears in which there is a large variation in the sizes of wheel and pinion, as there is in the greater number of instances. Internal cycloidal standard gears are made upon the same basis as spur gears of the same size, only so long as there is a difference of 15 teeth or more between the wheel and pinion. Thus, a pinion of

15 teeth would engage with an annular ring of 30 teeth, but a pinion of 20 teeth would not, on the standard basis of a 12-toothed pinion. It is necessary, therefore, to take a new base, as small as the difference between the wheel and its pinion; in the case of the 30 and 20 teeth pair, a base of 10 teeth or less. The tooth curves would therefore be obtained by a generating circle of the same diameter as, or less than, the radius of a 10-toothed pinion. This is a general statement of the principle suitable for practical work, though the subject itself would admit of wider treatment. Actually, the interferences which have to be guarded against in the case of large pinions are sometimes got over neatly and fairly efficiently by adopting the knuckle gear, which only gives true contact on the pitch line.

These internal gears, therefore, stand alone, and the rules by which the construction of spur gears are governed are not regarded in these. Outside the case of pinions and wheels which are largely disproportionate in dimensions, every pair of gears must be dealt with individually on its own merits.

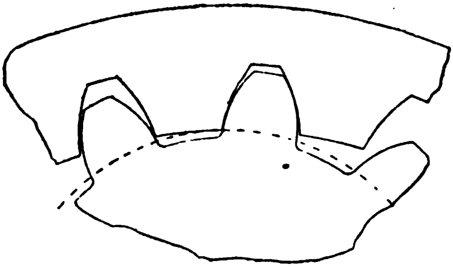


Fig. 35.

There is no real difficulty in designing teeth for gears of low ratios; but it is usually better to start *ab initio*. In double-curved gears a basis can be taken by employing a generating circle having a definite relation to the pinion, as, say, one having a diameter equal to its radius, or larger, if spreading flanks are desirable. Then the tooth curves of both pinion and gear can be obtained directly and accurately. In a pair of involutes the standard pinion being taken as a base, if the difference in the number of teeth of pinion and gear equals the number of teeth in the base pinion, the gear teeth curves can be those which correspond with the standard cutters. Thus though a 15-toothed pinion would mesh with a 30-toothed internal gear, if the shape of the teeth corresponded with the tooth spaces of a 30-toothed spur made to a standard cutter for that number, if the pinion has a larger number of teeth, then a construction must be made specially to suit it. The pinion teeth as shaped with standard cutters may be taken, and the teeth of the gear similarly, and templets be made and corrected until suitable shapes are found. As the discrepancy in dimensions increases, a more drastic method must be adopted. The teeth of pinion and gear can be struck out with reference to a very low angle of pressure, and the curves be obtained from the base lines of pinion and gear. It will soon be found that the teeth must be very short and that the pinion must have all points, and the gear all roots. Or else the knuckle gear must be adopted. In each instance the idea of rolling action between flanks and faces must be sacrificed, for we have to deal with freak gears in the extreme cases. Fig. 36 shows an extreme case, a pair of gears cut by Messrs Arnold Pochin & Bros.

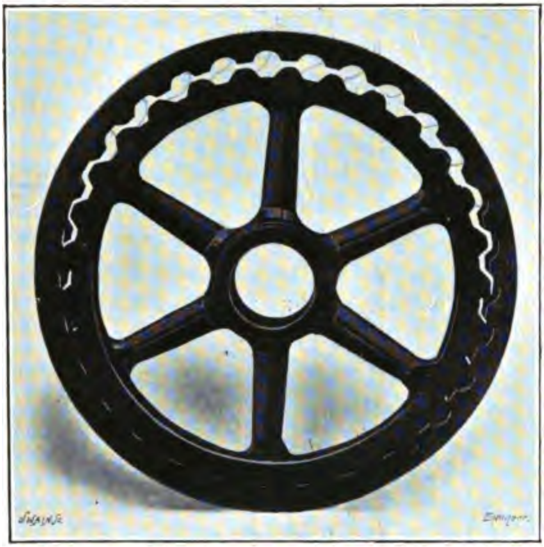


Fig. 36.—Pair of Internal Gears Cut by Messrs Pochin.

The Spiral Gears.—Of late years the spiral, helical, and worm gears have taken a new lease of life. The demand for accurate gears capable of high efficiency at high speeds, and the response to this demand made by the machinist explain this fact. The designs are old, but they are revivified in regard to their applications and their methods of manufacture. We first consider the spiral gears, one of the great group of screw gears to which the helical and worm types are related.

This gear is used chiefly in cases where motion has to be transmitted through shafts which are not in the same plane, and for which bevels would be unsuitable. They may be regarded as spur gears with shafts at an angle, the teeth being twisted to suit the angle. Generally the shafts are at right angles, and the numbers of teeth are equal. But the numbers are also often unequal, and the shafts may be varied in position from 90°. They are preferred to worm gears when the velocity ratios are alike, or not very dissimilar, being less difficult to construct than worm gears of high angles and many threads. Since the special gear-cutting machines have come into general use, the manufacture of spiral gears has been established on a better commercial basis than hitherto, and large numbers of these are therefore employed. Many are now hobbed instead of being milled with single rotary cutters.

Spiral gears can be marked out and dimensions obtained graphically, or the various relations can be got by calculations. The first is the older, the second the almost general practice now. The proportions of the teeth are the same as those used in standard spur gears, the tooth proportions and cutters being selected for the normal pitch.

The elements in a spiral gear (Fig. 37) are the circular or real pitch, *c.p.*, which, multiplied by the number of teeth, gives the pitch circumference of the gear, the angle θ of the teeth or screw which screw in making one turn round the cylinder gives the advance or lead L, the normal circular pitch, *n.p.*, or shortest distance between adjacent teeth or threads, for the section of which the cutter is selected, and the "divided" lead or axial pitch, *a.p.* The relations of the circular and normal pitches and the angle included between them are the most important elements. Normal pitches are calculated on a basis either of circular pitches or of diametral pitches.

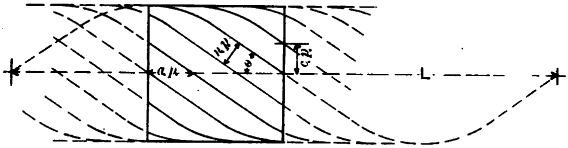


Fig. 37.

The Angles of Spirals.—In the majority of instances spiral gears are used to transmit equal rates of revolution at right angles, in which case the angles are 45° (Fig. 38) and the numbers of teeth are equal in both gears. But ratios up to 2 and 3 to 1 are frequent. Then the diameters of the gears and the leads of the screws will be in the same ratios. If the ratio is 2 to 1 (Fig. 39), the diameters and the leads will be as 2 to 1. The normal pitches are alike, and the circular pitches are alike, because the tangent line T makes equal angles with the axes of the gears. In Fig. 40 the elements of the two gears are shown developed. *c* is the circumference of the wheel in Fig. 39, c^1 that of the pinion, *a.p.* the total axial pitch or lead of the wheel, $a.p.^1$ that of the pinion. The angles are similar in each, as are the circular pitches, *c.p.*, and also the normal pitches, *n.p.* Cutters will be selected to correspond with the numbers of teeth in each.

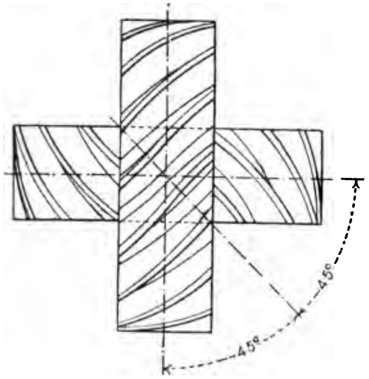


Fig. 38.

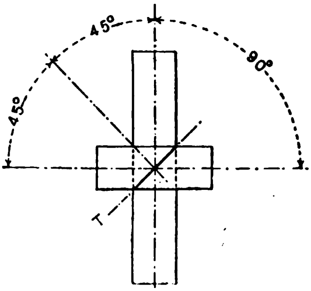


Fig. 39.

Velocity Ratios.—Such gears transmit unequal velocity ratios; but the axes also of spiral gears for imparting either equal or unequal velocity ratios may be either at right or at other angles. The angle included between the axes, whether of 90° or less, must be bisected in the ratio of the gears, equally for equal ratios, unequally for other ratios.

Unequal velocity ratios are obtained by varying the numbers of teeth. The diameters may or may not be altered correspondingly. A condition of gearing is that the tangent passing through the pitch point common to the helices of both gears corresponds with the line of bisection, which corresponds with the ratios of the gears. If this is at an angle of 45°, or makes an equal bisection in the case of any other angle than that of 90°, the diameters will vary exactly as the numbers of teeth, as in Figs. 39 and 40. But if the diameters are the same, but not the numbers of teeth, the tangent line and line of bisection of the angle will be in the same ratio as the numbers of teeth. The pitches of the spirals or their leads will also be in proportion to the numbers of teeth. Thus, gears of 2 to 1 would have the angles of their axes bisected in the proportion of 2 to 1 (Fig. 41), and

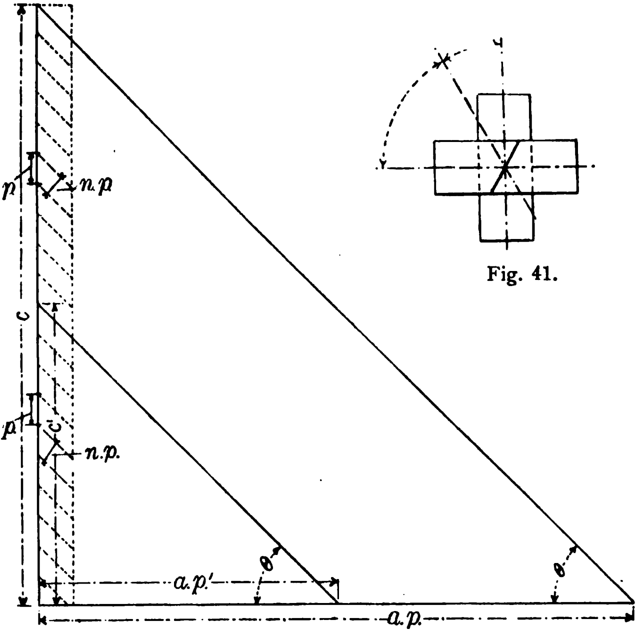


Fig. 41.

Fig. 40.

the axial pitch or lead of the first, *a.p.* (Fig. 42), would be twice the length of that of the second, *a.p.*[1]. In Fig. 42 the angles θ of the spirals correspond with those in Fig. 41, and the circumferences *c*, *c* of the two gears are alike. The normal pitches, *n.p.*, are also alike, but the circular pitches, *c.p.* and *c.p.*[1], are in the ratio 2 to 1. In such cases the gear which has the shorter lead and the coarser circular pitch drives.

Diameters.—The angles of the spirals are governed by the diameters as well as by the leads. For, given equal lead, the smaller the diameter the less is the angle. For equal diameters the longer the lead the less is the angle. These are the relations which are useful variables in obtaining velocity ratios, while normal pitches and their tooth proportions remain unaffected.

The spiral gears are thus an example of gears where the velocity ratios do not, except in the case of those with 45° of angle, depend on diameter, but on numbers of teeth only. We have seen that gears of the same diameter can have velocity ratios of 2 to 1. Following the analogy, a spiral gear may be regarded as a coarsely pitched worm, as the worm, too, may be considered a short spiral of fine lead. And, again, the diameter of a worm has no influence on the velocity of its worm wheel. The number of its threads is the determining element, as is the number of teeth in a spiral gear.

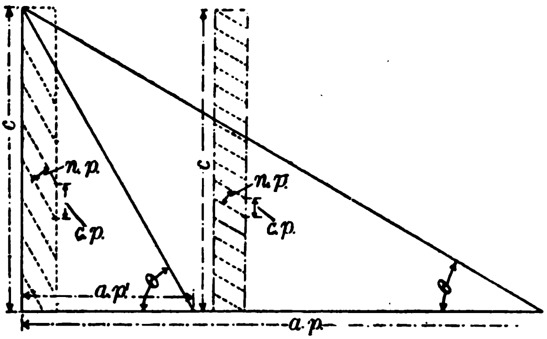


Fig. 42.

Relations with the Rack.—Another relation is that of the rack which, whether straight or skew, is a spur or spiral gear respectively unrolled and laid out in a plane. Hence, either form of rack will gear with a spiral (Fig. 43). In these cases the common tangent passes along the teeth of the rack and of the tooth of the wheel in contact, through the pitch point. A rack and a spiral gear, both of 45° of angle, will work at 90° with normal pitches alike, and circular pitches alike, and the rack is a 45° gear laid out flat. When a square rack engages with a spiral gear the common tangent must make unequal angles with 90°. These gears then come under the class in which the circular pitches are unlike.

Graphic Methods.—Graphic methods of obtaining the relations of spiral gears are adopted. When calculations are made it is safer to supplement them with a proportional diagram in order to check the figures. In the foregoing developed diagrams the lead or total pitch of the spiral is the base, the circumference of the cylinder round which the spiral is laid is the perpendicular, and the hypotenuse is the angle of the spiral.

The perpendicular divided by the number of teeth gives the circular pitch. The base divided by the number of teeth gives the pitch measured along the axis, the divided lead, or axial pitch. The normals to these divisions give the normal pitch. Hence from two elements the third can be obtained, thus:—

To find the angle of spiral for a given axial pitch or lead and perpendicular (circumference): Draw the axial pitch as a horizontal and the circumference as a perpendicular, and connect with a hypotenuse. The angle which the latter makes with the horizontal is the angle of spiral. If this angle is given, and the circumference, draw the perpendicular equal to the circumference, and the hypotenuse to the angle with the horizontal, and the length where the horizontal meets the hypotenuse is the axial pitch or lead. Also the circumference or perpendicular divided by the base gives the tangent of the angle of spiral. The perpendicular divided by the tangent of the angle gives the axial pitch or lead of the spiral. And the tangent of the angle multiplied by the pitch or lead of the spiral gives the circumference or perpendicular.

Spiral gears having axes which are not at right angles can have equal or unequal ratios, as seen in Figs. 44 and 45 respectively. In the first the angle included between the axes is equally bisected, forming a tangent line common to both gears. Here the circular pitches are alike. In the second the angle included between the axes is bisected in the unequal ratios desired. Here the spirals are of unequal angles, and the circular pitches differ.

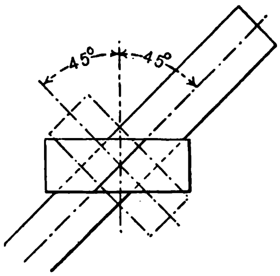


Fig. 43.

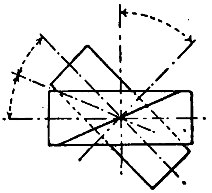


Fig. 44.

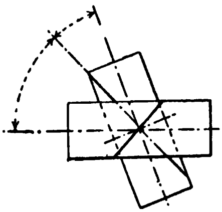


Fig. 45.

General Formulæ: For Circular Pitches.—Having the real circular pitch and angle of spiral given, to obtain the normal circular pitch: Multiply the circular pitch by the cosine of angle of spiral, or:—

$$cp \times \cos\theta = ncp.$$

Note.—The angle of spiral with *axis* is always understood.

Having the normal pitch and angle of spiral given, to obtain the circular pitch: Divide the normal pitch by the cosine of the angle, or:—

$$\frac{ncp}{\cos\theta} = cp.$$

To obtain pitch diameter:—

$$\text{Pitch diameter} = \frac{\text{Number of teeth} \times \text{Circular pitch}}{3.1416}$$

To obtain the whole diameter of blank:—

$$\text{Whole diameter} = \text{Pitch diameter} + 2\frac{\text{Normal circular pitch}}{3.1416}$$

Also when tables are available for the addenda of teeth, from these the whole diameter is obtained, thus:—

Whole diameter = Pitch diameter + 2 addendum.

For Diametral Pitches.—Having the normal diametral pitch and cosine of angle of spiral, to obtain the circular diametral pitch: Multiply the normal diametral pitch by the cosine of the angle, or:—

$$cdp = ndp \times \cos\theta.$$

Having the circular diametral pitch and the angle of spiral, to obtain the normal diametral pitch: Divide the circular diametral pitch by the cosine of the angle of spiral, or:—

$$ndp = \frac{cdp}{\cos\theta}$$

To obtain pitch diameter:—

$$\text{Pitch diameter} = \frac{\text{Number of teeth}}{\text{Circular diametral pitch}}$$

$$\text{or,} \quad \frac{\text{Number of teeth}}{ndp \times \cos\theta}$$

To obtain the whole diameter of blank:—

$$\text{Whole diameter} = \text{Pitch diameter} + \frac{2}{\text{Normal diametral pitch}}$$

$$\text{or,} \quad \frac{\text{Number of teeth}}{ndp \times \cos\theta} + \frac{2}{ndp}$$

$$\text{Depth of tooth} = \frac{2.157}{ndp}$$

The circular diametral pitch, as in spurs, is the number of teeth divided by the pitch diameter. And conversely, the pitch diameter multiplied by the circular diametral pitch gives the number of teeth.

The normal diametral pitch in 45° spirals is always:—

$$n\,d\,p = \frac{\text{Circular diametral pitch}}{0.70711}$$

0.70711 being the cosine of the angle 45°.

The normal diametral pitch for any other angle, with the axes at right angles, which occurs in gears of equal sizes, is:—

$$n\,d\,p = \frac{\text{Number of teeth}}{\text{Pitch diameter} \times \text{Cosine of angle}}$$

or:—

$$\frac{\text{Circular diametral pitch}}{\text{Cosine of angle}}$$

In a 45° angle spiral the lead is the same as the circumference. Hence:—

$$\text{Lead} = \text{Pitch diameter} \times 3.1416.$$

In gears of any other angle, the axes being at right angles:—

$$\text{Lead} = \frac{\text{Pitch diameter} \times 3.1416}{\text{Tangent of angle}}$$

(In an angle of 45° the tangent of the angle is 1.)

The relations may be summarized:—

If N=number of teeth; $N \times c\,p$=circumference of pitch circle.

$$\frac{N \times c\,p}{3.1416} = p\,d.$$

$$\frac{N \times n\,c\,p}{3.1416 \times \cos\theta} = p\,d.$$

$$\frac{N}{3.1416 \cos\theta} \times \frac{3.1416}{n\,d\,p} = \frac{N}{n\,d\,p \cos\theta}$$

Another method of calculating is to use a Table of Secants, as below:—

$$\text{Circular pitch} = \frac{\text{Secant} \times 3.1416}{\text{Normal diametral pitch}}$$

$$\text{Normal diametral pitch} = \frac{\text{Secant} \times \text{Number of teeth}}{\text{Pitch diameter}}$$

$$\text{Pitch diameter} = \frac{\text{Secant} \times \text{Number of teeth}}{\text{Normal diametral pitch}}$$

$$\text{Number of teeth} = \frac{\text{Pitch diameter} \times \text{Normal diametral pitch}}{\text{Secant}}$$

$$\text{Angle of spiral, Secant of} = \frac{\text{Pitch diameter} \times \text{Normal diametral pitch}}{\text{Number of teeth}}$$

To facilitate calculations and selection of cutters, tables are sometimes given for the angles of spiral gears in most common use. Two of these are given in the Appendix, by Messrs David Brown & Sons, Ltd., and by the Garvin Machine Company.

The Brown & Sharpe formulæ are:—

Let:—

$\left.\begin{matrix} N_a = \\ N_b = \end{matrix}\right\}$ Number of teeth in gears $\left\{\begin{matrix} a \\ b \end{matrix}\right.$.*

C=centre distance.
P=diametral pitch.
cp=circular pitch.
ndp=normal diametral pitch.
ncp=normal circular pitch.
A=angle of axes.
L_1=exact lead of spiral on pitch surface.
L_2=approximate lead of spiral on pitch surface.
T=number of teeth marked on cutter to be used when teeth are to be cut on milling machine.
D=pitch diameter.
B=blank diameter.
$\left.\begin{matrix} \theta_a = \\ \theta_b = \end{matrix}\right\}$ angle of teeth with axis.
t=thickness of tooth.
s=addendum.
$D'' + f$=whole depth of tooth.

General formulæ for all cases that arise:—

$$A = \theta_a + \theta_b$$

$$ncp_a = ncp_b$$

$$D = \frac{cpN}{\pi}; \quad \text{or} = \frac{ncpN}{\pi \cos \theta}; \quad \text{or} = \frac{ND}{N + 2\cos\theta}$$

$$B = D + 2s; \quad \text{or } D + \frac{2}{ndp}$$

---

* Letters *a* and *b* occurring at bottom of notation refer to gears *a* and *b*.

$$cp=\frac{D\pi}{N};\ \text{or}=\frac{ncp}{\cos\theta}$$

$$ncp=cp\cos\theta$$

$$ndp=\frac{\pi}{ncp}\ \text{(pitch of cutter)}$$

$$s=\frac{ncp}{\pi};\ \text{or}=\frac{1}{ndp};\ \text{or}\ \frac{B-D}{2}$$

$$t=\frac{ncp}{2}$$

$$D''+f=2s+\frac{t}{10}$$

$$T=\frac{N}{\cos^3\theta}\ \text{(cutters of regular Brown \& Sharpe involute system)}$$

$$L_1=\frac{Ncp}{\tan\theta};\ \text{or}=\frac{N\pi}{P\tan\theta};\ \text{or}=\frac{N\,ncp}{\tan\theta\cos\theta}$$

The cutters in the Brown & Sharpe system are given at p. 80.

Helical Gears.—These are essentially spiral gears of large diameter and long lead. In the helical pinions the spiral form is very obvious. The relation is clear in Fig. 46, in which the teeth of the double helical pinion of width A, if prolonged, are true spirals of lead L. In helical gears the axes are parallel and the action of the teeth wholly rolling; in spirals the axes are at an angle and the action wholly sliding. Comparing helical gears with the spurs which they supplant, rolling contact of the teeth is substituted for the contact which is mainly of a sliding character in common spur gears. In these the only rolling contact which occurs is at and near the pitch line. The intensity of the sliding action increases in proportion to the distance of the parts in contact away from the pitch line.

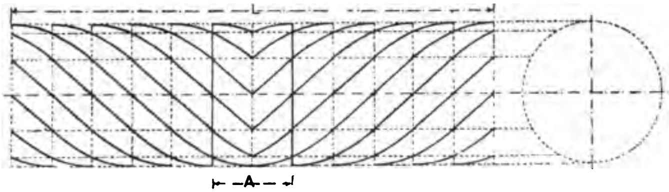


Fig. 46.

At the basis of the helical teeth is the old stepped gear in which the length of a tooth is cut into steps in order to reduce the amount of sliding action by dividing it between the steps. Obliterating the angles of the steps gives the spiral form of tooth, which will engage with a gear of the same angle of spiral and pitch, but of opposite hand. Theoretically any angle will be suitable, but in practice high angles are objectionable for several reasons. Fig. 47 shows a gear with teeth at angles of 45° with the axis, Fig. 48 one with 30° of angle, and Fig. 49 one with 23° of angle, each of which accords with common practice. In the high-angled gears the difficulties of ensuring correct contact are greater than in those having lower angles, and the longitudinal thrusts must be more nicely balanced. A large angle weakens the normal section of the teeth and produces a more wedge-like action.

Essentially the great advantages which the helical teeth have over the straight teeth of spur gears are that the friction of sliding contact is eliminated, and that the whole pressure due to the load on the gears never comes on one edge of the tooth, but is distributed over other portions.

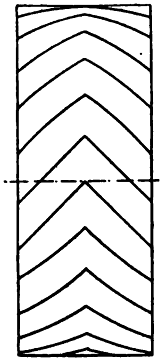


Fig. 47.

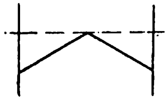


Fig. 48.

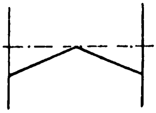


Fig. 49.

The action being continuous, pinions can be used much smaller than for spur gears. The action being that of rolling mainly instead of sliding combined with rolling, the load is transferred from tooth to tooth without shock. Even if the teeth are not stronger than those of similar pitches in spur gears, the elimination of shock and the fact that a portion of the load is always being borne at and near the pitch, relieving the pressure on the points, averages the pressure on the teeth.

A result is that teeth of the same pitch as spurs can be more severely loaded and run at higher speeds, with less vibration and noise. If accurately cut, the fullest use may be made of small pinions of hard steel to ensure equal wear with the wheel. This is impracticable with spur gears, in which small pinions produce shocks, vibrations, and variations in pressure in the line contacts from points to roots. This is the reason why raw-hide gears are freely used to reduce noise and lessen shock. Well-made double helical gears will secure equally good or better results with smaller pinions.

Generally, when helical gears are now mentioned, the double helical form is understood. The single gears are used extensively, especially on some Continental machine tools. But there is an endlong thrust on these which is objectionable, and which is absent from the double form when correctly made, in which the thrust in one direction is neutralized by that in the opposite way. But here the difficulties of manufacture come in, which are discussed at p. 85.

The Wüst Gears.—Dimensions proposed for an interchangeable system for the Wüst gears are as follows:—

Tooth shape	Involute.
Pressure angle	20 deg.
Spiral angle	23 deg.

$$\text{Pitch diameter (20 teeth and over)} = \frac{\text{Number of teeth}}{\text{D.P.}}$$

$$\text{Blank diameter (20 teeth and over)} = \frac{\text{Number of teeth} + 1.6}{\text{D.P.}}$$

$$\text{Pitch diameter (under 20 teeth)} = \frac{(0.95 \times \text{Number of teeth})}{\text{D.P.}} + 1$$

$$\text{Blank diameter (under 20 teeth)} = \frac{(0.95 \times \text{Number of teeth})}{\text{D.P.}} + 2.6$$

Addendum	$\frac{0.8}{\text{D.P.}}$
Dedendum	$\frac{0.96}{\text{D.P.}}$
Full depth	$\frac{1.76}{\text{D.P.}}$
Working depth	$\frac{1.6}{\text{D.P.}}$

Standard face width for gears with pinions of not less than 25 teeth=6 times circular pitch.

Face widths for high ratio gears with small pinions=6 to 12 times circular pitch.

When a pinion of less than 20 teeth is used with a standard gear, the centre distance must be slightly increased to suit the enlargement of the pinion. If it is desired to keep the centre distance to the standard dimensions, the gear diameter may be reduced by the amount of the enlargement given to the pinion. For example:—If a pinion of 10 teeth, 5 D.P., is to mesh with a gear of 90 teeth at 10in. centres.

Pitch diameter of pinion $= \frac{(0.95 \times 10) + 1}{5} = 2.1$in.

Enlargement over standard pinion $= 0.1$in.

Pitch diameter of standard gear $= \frac{90}{5} = 18.0$in.

Reduced pitch diameter of gear $= 18.0 - 0.1 = 17.9$in.

Centre distance $= \frac{17.9 + 2.1}{2} = 10$in.

Worm Gears.—The efficiency of these gears has been increased vastly in recent years. Formerly unsatisfactory and neglected, they have now displaced bevels and spurs in large numbers of cases. This is due to a better design and improved systems of cutting, and to their enclosure in gear cases with oil baths. There is no hard-and-fast division between spiral and worm gears. A spiral is a worm of long lead, and some worm gears might be regarded as spirals of quick lead. Some spiral gears are cut with concave faces just as worm gears are. Multiple-threaded worms approach to the spiral form. The points in favour of worm gears are that two or three teeth are in contact between the worm and wheel at one time, while in spurs but two can be in contact at once, and often only one when pinions are small. The load is divided in the first but concentrated in the second. Also while the wholly sliding contact which takes place between the worm and its gear is regarded as objectionable, it has an advantage. The contact is smoother and the load is taken more easily between successive contacts than in the direct pressure of spur-gear teeth. When in addition suitable materials are used, and ample lubrication afforded, the worm gear is the most perfect type for economical reduction and transmission. In the best gears the worm is of forged steel of high tensile strength and the wheel is of hard phosphor-bronze fastened to a centre of cast iron. End thrusts are taken on ball bearings. Hollow-faced spiral gears are cut for gas and oil engines, including the Diesel type.

Worm Gear Elements.—The teeth of involute worms and worm wheels are proportioned similarly to those of spurs and spirals. The present standard angle of the sides of the worm threads is 75½° with the axis of the worm, and 14½° with the perpendicular to the axis,

Fig. 50. The pitch is commonly called linear pitch in worms and racks, although it is the equivalent of circular pitch in other gears. The number of threads in one linear inch is the reciprocal of the linear pitch. If the number of threads to one inch linear is multiplied by 3.1416, the product is the diametral pitch of the worm wheel.

The lead of a worm as of a spiral gear is the axial pitch, or the linear advance which a thread makes per revolution. The pitch is the distance from the centre of one thread to that of the one adjacent. In a single-threaded worm this is identical with the lead. But in a double-threaded worm the lead is twice the pitch, in a treble-threaded worm it is three times the pitch, and so on. In worm gears the normal pitches of worm and wheel must be alike. But the circular or linear pitches are taken as in Fig. 50. The section of the hob thread being identical with that of the worm, it develops the teeth of the wheel in cutting.

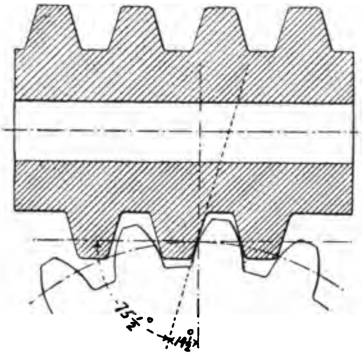


Fig. 50.

The angle of a worm thread is obtained by drawing a diagonal on the pitch circumference from the pitch or lead. This may be embodied in a diagram for a large range of leads. The angle is important as affecting efficiency and reversibility. It must be large for high efficiency, but the gear ceases to be self-sustaining if the angle exceeds about 6° or 7°. The angle of thread is usually less than 10°, or from 6° to 9° when single-threaded. It is readily obtained by laying down the circumference of the pitch circle as a perpendicular, the pitch as a base, and drawing the hypotenuse, which gives the required angle.

In the section Fig. 50 the pitch diameters are set out as for spurs, and also the addenda and bottom clearance. From the centre of the worm, radii are struck (Fig. 51) for the bottom and top of thread both of the worm and of its wheel, and for the rim thickness. The ends of the wheel teeth are drawn to the worm centre. The diameter of a worm usually ranges from three to five times the circular pitch, but it is sometimes less, or more.

In the circular pitch system:—

Pitch diameter = Number of teeth × Circular pitch × .3183.

Throat diameter = (Number of teeth + 2) × Circular pitch × .3183.

$$\text{Centre distance} = \frac{\text{Pitch diam. of worm} + \text{Pitch diam. of wheel}}{2}$$

$$\text{Circular pitch} = \frac{3.1416}{\text{D.P.}}$$

A Table in the Appendix, by Messrs Bardons & Oliver, gives proportions for the teeth of worm gears.

Undercut.—The undercutting which occurs in the teeth of spur pinions having less than 30 teeth when the flanks of the generating rack teeth are straight, occurs also in worm wheels which have a less number of teeth than 30. The corner of the worm thread hollows out the flanks of the wheel, and bears only by its corner against the undercut flanks. This may be avoided by rounding off the addendum as in racks. But as it is more usual to make worm-thread tools with straight sides, another device is adopted—that of enlarging the blank to bring nearly the whole of the teeth outside the pitch line (Fig. 52). The Brown & Sharpe rule is: Multiply the pitch diameter of the wheel by 0.937, and add to the product four times the addendum. The sum will be the outside diameter of the throat of the blank. The largest part of the diameter is obtained by drawing the curve from the centre of the worm. Messrs David Brown & Sons increase in addition the pitch diameter by twice the addendum.

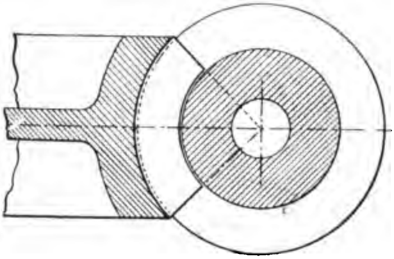


Fig. 51.

Thus:—

$$\text{Pitch diameter} \times 0.937 + 4\,\frac{\text{Circular pitch}}{3.1416} = \text{Enlarged outside diameter.}$$

$$\text{Pitch diameter} \times 0.937 + 2\,\frac{\text{Circular pitch}}{3.1416} = \text{Enlarged pitch diameter.}$$

Another way is to alter the angle of the worm-thread section from 29° to 40° for gears with wheels of less than 30 teeth.

Hollow-faced Worm Gears.—With reference to the enveloping curves of the usual design of worm wheel (Fig. 51), this design is adopted because it spreads the contact of the worm and wheel over larger surfaces, and therefore the gears wear very much longer. They are now generally used, because the methods of cutting them by means of hobs in special hobbing machines have been improved and developed. In old practice the teeth were generally roughed-out with a rotary cutter and finished by a hob of a crude form, not backed off, the hob driving the worm wheel in the absence of change gears. Unless wheel teeth are hobbed there is little or no advantage in having them curved to envelop the worm. Cast worm wheels were often used in preference to cut ones, as they are now sometimes, because it is possible to obtain more accurate results by casting than by cutting and correcting without hobbing, and hobs are costly. Worm-wheel patterns are cut and corrected by using the pattern worm or the cut worm as a hob is used, by setting the two up at temporary centres and running them around, and easing off the teeth with the chisel until perfect contact is obtained.

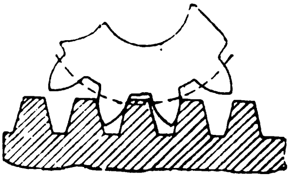


Fig. 52.

The difficulties of getting accurate wheel teeth were also very much greater under the old crude system whenever multiple threads were in question, than they are under the present. Their accurate cutting or casting was practically impossible. Now, with a machine in which hob and gear are driven by independent mechanisms, and the hob is backed off, wheels can be hobbed correctly for worms of two, three, four, or even more threads.

Worm Gears with Straight Teeth.—These are produced with single rotary cutters (Fig. 53) on a gear-cutting machine, or on a milling machine, the blank being turned like that of a spur wheel. They are easily cut, but the worm contact is that of points only along the plane of the worm axis, and therefore such gears are not suited for heavy service. For light duties they are used extensively, and for dividing mechanisms. Another design used for dividing is a modification of each form. The blank is turned flat, but the teeth are gashed in, slightly concave only, so that points of the worm thread alone are in engagement. The straight diagonal tooth form of worm wheel has the advantage over the other that it can be used to gear with a rack, similarly to a spiral gear.

Although usually the axis of the worm lies at an angle of 90°

with the axis of its wheel, it may be arranged at some other angle. This makes no difference in the principle and methods of cutting, but only to the angle at which the wheel teeth are cut.

Sizing Blanks.—Rules for sizing worm-wheel blanks and finding numbers of teeth often take, instead of the pitch, the number of threads per inch, and use the reciprocal of 3.1416, or 0.3183. Thus:—

To find the diameter of a worm wheel at the throat (or smallest part of the outside diameter), when the number of teeth in the worm wheel and the number of threads per inch in the worm are given. Add 2 to the number of teeth in the worm wheel and multiply by 0.3183, and divide the result by the number of threads per inch in the worm, and the result will be the diameter of the worm wheel at the throat.

To find the number of teeth in a worm wheel, when the diameter at the throat and the pitch of the worm are given: Multiply the diameter at the throat by the number of threads per inch in the worm, and multiply the product by 3.1416, then subtract 2, and the result will be the number of teeth.

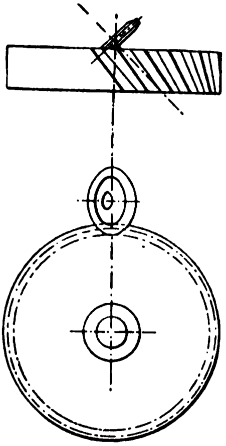


Fig. 53.

The Hindley Gear.—There is one special application of the worm gear to which reference should be made, the so-called hour-glass gear. It has been an object of curiosity for half a century. But it was clearly an impracticable gear as drawn in books, and not until the machinist took it in hand has it been rendered of service. Known now as the Hindley or the "globoid" gear, it is cut in large numbers. It is superior in bearing surface to the concave worm gear, just as this is superior to the straight diagonal toothed type, because, instead of the worm being a tangent to the wheel, it becomes an envelope to it, following its circular form.

Fig. 54 shows the worm as now made, by which it will be seen that while undercutting is avoided, the hour-glass shape of the worm section is retained. The method of construction in theory is that the worm is cut, not by a tool moved parallel with the axis of the worm, but by tools shaped like and representing sections of worm-wheel teeth taken on the middle plane, and rotated in a circle on their axis in unison with the worm blank by means of suitable gearing, precisely as the finished worm and wheel would revolve. The idea is to cut a worm with a cutter head, each tool in which represents a tooth, and to convert the worm so cut into a hob for cutting the actual worm wheel. The worm used is cut in the same way as the hob.

Looking at Fig. 54, it will be noted that the thread of the worm in following round the curve corresponding to that of its wheel leans over as it approaches the ends, so that the sectional view shows radial sections. Also, that the inner faces of the last turn of the screw are parallel with each other. Further, unlike the common worm, the distance from centre to centre of adjacent teeth is slightly less at the points than at the roots. These peculiarities produce interesting developments during the early stages of the process of hobbing the wheel, taken in conjunction with the difference in pitch at the commencement and termination of the hobbing of a worm wheel.

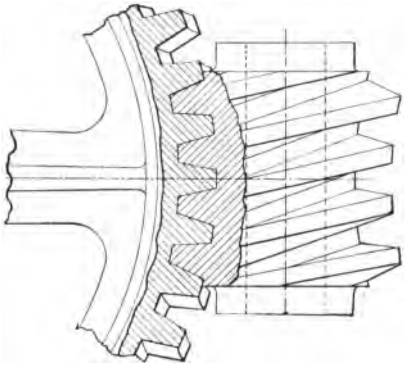


Fig. 54.

It has been found impossible to carry out the theoretical method of cutting by an exact interchange of worm and wheel used as cutters, because the concavity of the worm wheel is not present in the head or disc of flat cutters, which is the equivalent of the worm wheel.* The result is that the worm and wheel will not enter into perfect contact. The case is similar to that of forming common concave teeth with a milling cutter instead of with a hob. The difficulty is got over by using two hobs to cut the worm threads by, roughing-out with one having a diameter equal to that of the worm wheel on the centre plane, followed by another a little larger, but driven by the same gear at the same rate. The centre of this is set back so that it takes nothing more out of the centre tooth space of the worm, but cuts away some portions of the inner face of the outer turns of the thread. The contact of the wheel and its worm in the centre plane is seen in Fig. 54. The result of this double hobbing is that there is contact on both sides of one tooth only * (the middle one);

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* But see note on p. 66.

on the others there is contact only on one side; on the other side there is clearance. The driving action is therefore confined to about one-half the worm thread, but this is more than compensated for by the large amount of surface in action. There is no bottom clearance in this gear, and the length of tooth is made greater than that in the common worm and wheel.

The recent rapid growth of this gear is a remarkable fact, and it is clearly due to the perfection of modern hobbing machines, apart from which the difficulties inherent in this peculiar construction would probably still be, as they have been for so many years, insuperable.

The object in adopting this gear is to increase the surfaces in contact and so reduce the friction and wear per unit of surface. The friction being proportionate to pressure, the larger the surface over which it is spread the less the pressure per square unit of surface. This perhaps has been overrated because the thread of the worm can only be in partial contact with the wheel teeth away from the central portion, because the angle of thread changes as the diameter increases, so that there is a difference between the angle of thread at the centre and that at the end, due to the difference in diameters.

If the wear is reduced by the substitution of this gear for the ordinary worm there are counteracting disadvantages. It is more difficult to make and set, and end thrust cannot be allowed for. Neither apparently can the advantages of multiple-threaded worms be secured in this type of gear.

The Globoid worm gearing as made by the Maschinenfabrik Pekrun, of Dresden, is designed to diminish friction and increase efficiency. The worm is a hollow-cut design, following the curve of the wheel, but the teeth of the latter are conical rollers which revolve on steel pins, hardened and ground and fixed in the rim. In this way rolling friction is substituted for the sliding friction of the ordinary worm gear. In addition the gears are enclosed in an oil bath with the worm underneath. End clearance is provided to the rollers on their pins, so that they are thrown outwards on their pins by the rotation of the worm leaving a space into which oil enters and prevents risk of the rollers sticking on their studs. The worm thrust is taken by ball races and that of the wheel on steel discs hardened and ground. The efficiency is from 90 to 95 per cent.

Variable or Intermittent Gears.—These, though used to but a slight extent in engineers' machine tools, are adopted in many other types of machines for producing variable intermittent or reversing motions. The principal forms used are square, star-shaped, triangular, elliptical, and scroll gears, in each of which variable motions are produced, varying in number and in maximum and minimum rates with the shapes and the proportions or ratios of the pitch lines. In each of these the shapes imparted to the teeth are those which correspond with the curvatures of each particular section. In a square gear there will be four straight sides with rack-shaped teeth and four curved corners corresponding with a gear of the same radius. In triangular gears there will be three straight and three curved. In elliptical gears two curves of short radii correspond with the transverse diameter, and two of longer radii suit the conjugate diameter. The mutilated gears provide the same kind of movements—variable but with a break or pause in the motion, and often too with a reversal of direction. There are many forms of these with external and internal teeth to which the statement just made applies. But in cases where the teeth would be liable to leave gear the teeth are generally lengthened beyond standard, or pins and catches are inserted.

The Hindley Gear Co., of Philadelphia, U.S.A., who make the gear as illustrated in Fig. 54, has kindly offered the following observations on the remarks made on p. 64:—

"The statement that 'the concavity of the worm wheel is not present in the head or disc of flat cutters, which is the equivalent of the worm wheel,' is not strictly accurate. The end faces of the discs forming the cutters are ground away to give a cutting edge, the latter being in a radial plane. A circle having the same diameter as the points of the worm wheel teeth can, therefore, be passed through the points of the cutters, which take the place of the wheel in cutting the worm, from which it would appear that the concavity of the worm wheel *does* exist in the head.

"The fact that the worm and wheel will not fit together when first cut must be due to other reasons. One of these reasons is probably the decrease in the angle of the helix, due to the increased diameter of the outer turns of the worm.

"As to the tooth contact, the effect of the clearance cut is probably to destroy the contact at the end of the outer turns of the worm thread only. Near the bottom of the thread the contact is still likely to be on both sides of the worm wheel tooth instead of on one side as stated."

CHAPTER VI

BEVEL GEARS

Bevel Gears.—These are more difficult to cut by any process than the spurs, because the sectional forms of the teeth change constantly. The principle on which they are designed has for its basis the cone; but properly the cone should be considered as included in a sphere on the surfaces of which the tooth shapes are formed. Actually the tooth shapes are always marked on planes tangential to the surfaces of the pitch cones. The difference between this system and the spherical basis is a matter of no practical importance, because the shapes of the teeth in one gear are always, by the methods which are adopted, made conjugate to those of the other gear. Thus, in Fig. 55, where the relations of three bevel gears to the sphere are shown, it can make no practical difference whether the teeth are marked out on the curved surface or on the tangential planes. It is therefore only necessary to bear in mind the fact that any bevel gear in axial section is a frustum of a cone; the breadth of thc tooth face the portion of a cone which is terminated by the ends of the teeth lying at right angles with the pitch planes; and that all bounding lines or planes in the teeth, in flanks, and points, and roots, terminate in the apex of the cone, which is a vital fact in considering methods of manufacture. It explains why rotary cutters are the most incorrect, and why planing processes are the most accurate.

Relation to Spur Gears.—From one point of view it is convenient also to connect the spur gears with the bevels on the basis of the rack. A crown bevel gear is imagined as being that of a bevel rack, or a rack rolled into a circle with the teeth flanks, roots, and points, all terminating at a common centre in the axis of the pitch plane (Fig. 56). It is an essential and convenient basis in studies relating to the planing of teeth by a generating process.

Elements.—Fig. 57 illustrates the elements of a pair of bevel gears. The sizing of blanks is done by such a diagram, or by calculation. The accuracy obtained by each depends on the degree of care exercised. The relations of the elements of these gears will be clear from the diagram without the necessity for inserting numerous reference letters.

The diameters of a pair are as their angular velocity ratios, as in spur gears. The axes may be at right or other angles. The pitch is reckoned on the major or larger diameters of the wheel and pinion. The pitch surfaces of both are cones, the common apex of which intersects the centres or axes of both the gears. All the lines or cone surfaces of faces, roots, and flanks of wheel and pinion would, if prolonged, meet in this common apex. These relations are shown in the diagram.

Tooth Proportions.—The tooth proportions (always measured on the major diameter) are the same as for spurs—that is, the length of the addendum, the dedendum, root clearance, and tooth thickness, are the same as for spurs (*see* pp. 35 and 36). The systems and shapes of the teeth, the pressure angles, are as in spurs. The tooth curves are either of cycloidal or involute form. In the latter the pressure angle may be 14½° or more, and the same interferences have to be avoided as in small spur pinions.

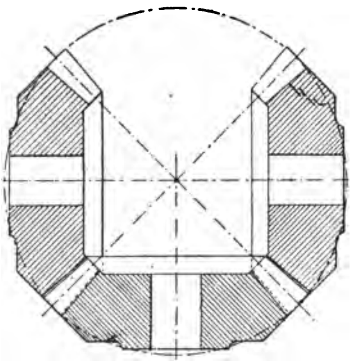


Fig. 55.

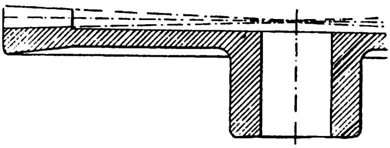


Fig. 56.

The pitch diameters being taken on the major diameter, lines are drawn thence to the common apex, to which root and point lines, set off in the same proportions as spur gears, are drawn. The ends of the teeth being on planes at right angles with the pitch surfaces, these planes, when projected to the centre lines or axes of the gears, termed the " back cone distance," give the radii on which the teeth are drawn. That is, the radius of the pitch circle for marking the teeth on is not half the diameter of the gear, but one of larger diameter—very much larger the more the gears approach to the crown form, as is the case in the wheel in Fig. 57. It is on these developed curves that the tooth shapes are obtained as though they were spur gears of corresponding radius, so that the flatter the gear and the larger the radius, the more nearly the tooth forms approximate to those of rack teeth. The small ends of the teeth may be marked with equal accuracy on either of the radii indicated, directly on the inside or inner back cone distance, from centres *a* and *b* ; or by transference from the centres A and B, from which the tooth dimensions at the large ends are struck (Fig. 57). This can make no difference whatever in the relative proportions at the larger and smaller ends.

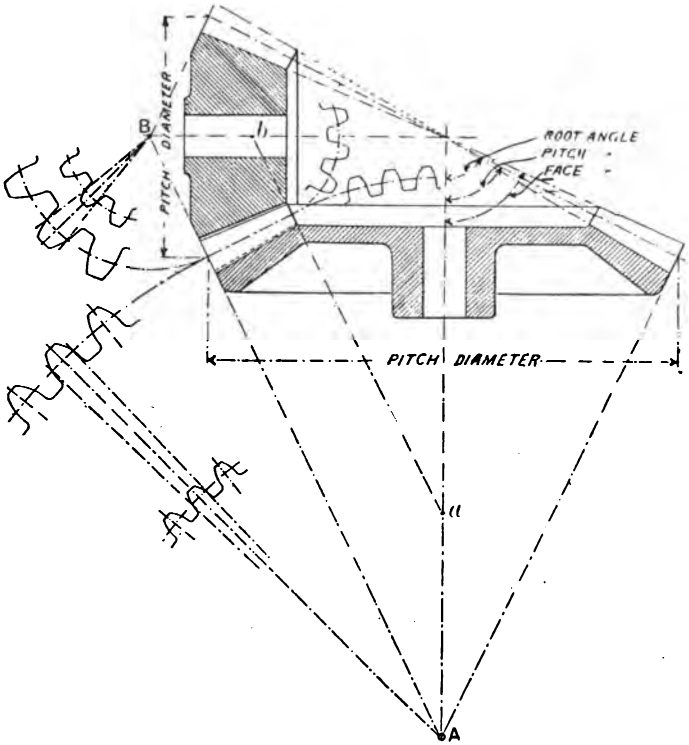


Fig. 57.

It is most usual to have bevel gears with axes at right angles, but that is not an essential feature. They may be at acute or obtuse angles without affecting the principles of construction. The one feature which is essential is that the axes and the various tooth elements must converge to a common centre or apex.

The elements of bevel gears if drawn out in the manner shown in Fig. 57 can be measured from, but the elements of cut gears are usually calculated. Certain names are given to the parts, as in the diagram Fig. 58, some of which are common to spurs, some inherent in the bevel form. The " outside diameter " is measured at the large end of the gear, as is also the diameter at the root, obtained respectively from the addendum and dedendum, which are the same as in spur gears. The angles which the pitch line, point or face line, and root lines make with the axis are termed respectively the " pitch angle," the " face angle," and the " root angle." The distance from the large end of the tooth to the apex of the cone is termed the " cone distance." The distance from the pitch line on the large diameter to the axis taken at right angles to the pitch cone

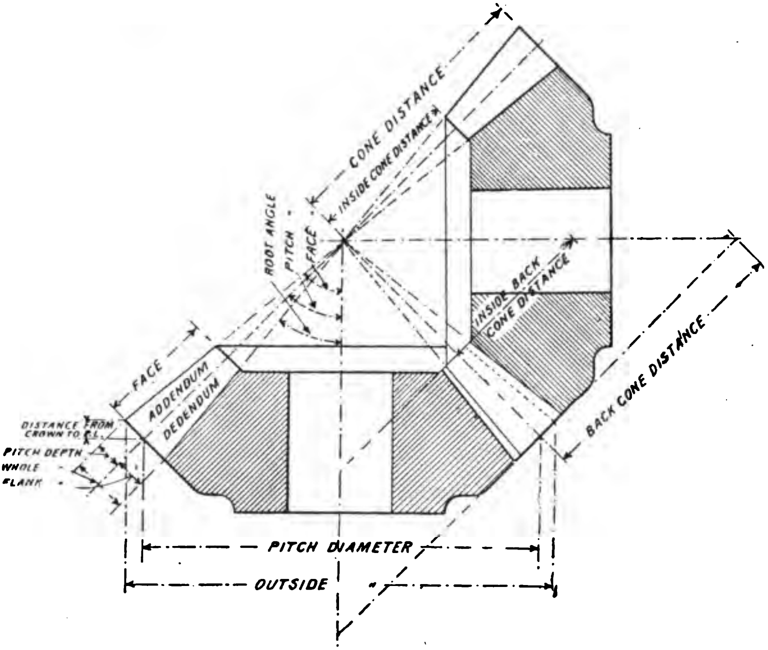


Fig. 58.

is the “ back cone distance.” The following formulæ are for bevel gears with axes at right angles (Fig. 58):—

To get outside diameter of wheel=Pitch diameter of wheel + (2 module × cos. pitch angle of wheel).

To get outside diameter of pinion=Pitch diameter of pinion + (2 module × cos. pitch angle of pinion).

To get distance from crown to pitch line=Pitch depth × sine of pitch angle.

To get pitch angle of wheel=

$$\frac{\text{No. of teeth in wheel}}{\text{No. of teeth in pinion}} = \text{tan. pitch angle of wheel.}$$

To get pitch angle of pinion=

$$\frac{\text{No. of teeth in pinion}}{\text{No. of teeth in wheel}} = \text{tan. of pitch angle of pinion.}$$

To get the face angle=Pitch angle + addendum or face angle.

To get the root angle=Pitch angle - dedendum or root angle.

To get the pitch on the small diameter=The pitch at outside is to pitch at inside as cone distance is to (cone distance) - face.

To get the tangent of the addendum angle $= \frac{\text{Pitch depth}}{\text{Cone distance}}$.

To get the tangent of the dedendum angle $= \frac{\text{Flank depth}}{\text{Cone distance}}$.

To get the cone distance at root angle $= \frac{\text{Cone distance at pitch line}}{\text{Cosine of dedendum angle}}$.

To get the back cone distance=Cone distance × tan. of pitch angle.

To get the inside back cone distance=Cone distance - face × tan. of pitch angle.

To get the tooth angle tangent=Half the thickness of tooth at pitch line $+ \frac{\text{Flank depth} \times \text{tangent pressure angle)}}{\text{Cone distance}}$.

To get radius of 14½° base circle=Back cone radius × cosine 14½°.

To get radius of 20° base circle=Back cone radius × cosine 20°.

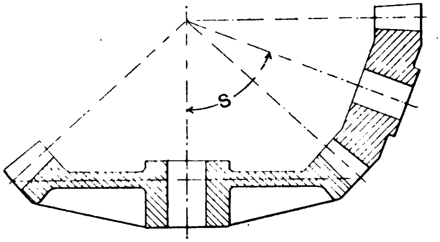
Though the teeth are shown marked out on the back cone distance in Fig. 57, it is not necessary so to mark them when they have to be shaped by rotary cutters. Cutters are selected to suit gears having radii equal to those of the projected radii, or back cone radii. In the case of mitres, one cutter obviously suffices, but in all bevel gears having unequal ratios a separate cutter must be selected for the wheel and the pinion. Therefore, the actual numbers of teeth in the two are ignored, the numbers which would correspond with the projected pitch circles are taken, and the bevel-gear cutters are selected for those approximately. And here, too, the question of the proper length of face to give to bevel gears comes in.

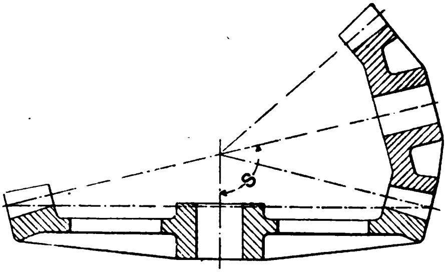
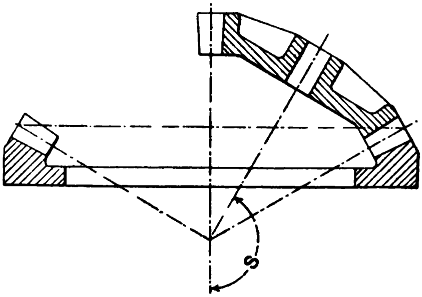
Face Width.—This is determined by the following considerations:—It should not be longer than five times the thickness of the teeth at the major diameter. And this is too much when it exceeds one-third of the pitch cone distance. It is better to have it rather less. If it is more, the rotary cutters ordinarily supplied for bevel gears are not thin enough to pass through the tooth spaces at the inner end. When, however, a good width of face is essential, it is better to extend the larger ends of the teeth, making the pitch there in excess of that actually required. The sizes of the teeth at the small end are in proportion to those at the large end, as the inner cone distance is to the outer cone distance. Dividing the first by the second should give ⅔ or 0.666, or rather less, as just stated. This is a proportion adopted for convenience in using standard rotary cutters, *see* p. 80, and is not imperative when teeth are planed by formers or generated and planed.

GEARS WITH AXES AT OBTUSE AND ACUTE ANGLES.

RULES FOR FIGURING ANGULAR GEARS.

The Gleason Works.

To Find Tangent of	Rule.
Pitch angle of gear when angle of shafts (S) is less than 90°.	CASE I.  Fig. 59. Sine angle of shafts. $\frac{\text{No. of teeth in pinion}}{\text{No. of teeth in gear}}$ + (cos angle of shafts).

To Find Tangent of	Rule.
Pitch angle of pinion when angle of shafts is less than 90°.	Angle of shafts – pitch angle of gear.
Pitch angle of gear when angle of shafts (S) is greater than 90°. To determine whether a given example belongs to Case II. or to Case III.	**Case II.**  Fig. 60. Sine 180° – angle of shafts. $\frac{\text{No. of teeth in pinion}}{\text{No. of teeth in gear}} - \cos(180^\circ - \text{angle of shafts})$
If number of teeth in gear × sine (angle of shaft – 90°) is smaller than number of teeth in pinion, use Case II. If number of teeth in gear × sine (angle of shafts – 90°) is larger than number of teeth in pinion, use Case III.	**Case III.**  Fig. 61. Sine (180° – angle of shafts). $\cos(180^\circ - \text{angle of shafts}) - \frac{\text{No. of teeth in pinion}}{\text{No. of teeth in gear}}$
Pitch angle of pinion when angle of shafts is greater than 90°.	Angle of shafts - pitch angle of gear.

Abnormal Gears.—Reference has been previously made to the limitations of the sizes of the smaller involute wheels, due to the interferences in the roots, and to the fact that the positions of the pitch lines have to be altered in such cases in order to produce satisfactory engagements. These gears are abnormal, and correspond broadly with what the millwrights term bastard gears. That is, they are gears in which exact proportions are departed from, in order to obtain workable results, which for various reasons are sometimes desirable in practice. The term " bastard " has reference generally to pairs of abnormal bevel wheels, the pitch cones of which do not meet in a common apex, but which are rendered necessary or desirable when velocity ratios have to be transmitted greater or less than those which would correspond strictly with the correct pitch cones.

In bastard bevels the conditions to workable gear are that the teeth of the pinions, the pitch cones of which do not meet in the apex of the pitch cone of the wheel, but above or below it, shall have the addenda formed in relation to the pitch cones of the pinions. That is, the addenda will vary from greater to less than the normal from the middle plane of the teeth towards each end. There is, of course, a limit to such malformation, the extreme being that at which the pitch cone falls outside the points and roots of the teeth. But within such limits (Fig. 62) working gears are practicable, and since the pinions are generated directly from the wheel, the gear is correctly formed.

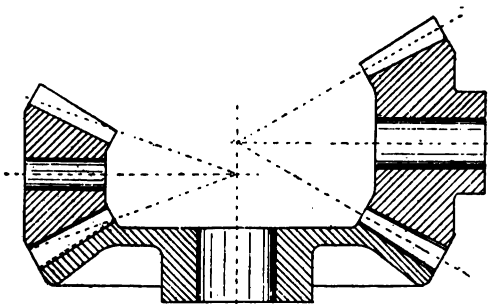


Fig. 62.

The result of varying the position of the base lines is most curious when bevel wheels are designed with pitch cones meeting at different apices. Mr Bilgram once made three bevel wheels of different diameters gear with one crown wheel correctly, though the apices were each on a different location. These wheels were generated with cutters originated from a crown wheel. The result was that the flanks developed a spiral form, as the addendum increased and diminished on the pairs in contact.

Skew Bevels.—These are related to the spiral gears when the axes of the latter cross at an angle. If these axes are continued and bevel gears are substituted meshing at a locality away from that where the spiral gears would mesh, skew bevels are produced. The action is better than that of spirals because for the sliding action of the latter the combined rolling and sliding action of bevel-gear teeth is substituted. But the gear is difficult to construct and is seldom adopted. The pitch surfaces of the skew bevels are frustra of a figure generated by the revolution of a straight line about an axis to which it is not parallel, a " hyperboloid of revolution."

SECTION II.—METHODS OF CUTTING

CHAPTER VII

FORM CUTTERS

THE principles on which the teeth of gears are formed are conveniently regarded from the methods of cutting which are practicable. They may be broadly divided into three—by rotary disc cutters, the sections of which correspond with those of the tooth spaces; by form planing; and by generation. Each has given birth to a large group of machines. The chronological order of the three methods is that of the order in which they are given.

Curves which are conjugate to others are conveniently made on a principle of moulding such as would result from rolling a gear having its teeth of a hard substance in a soft blank. A single gear, a rack, or a pinion, is the base which imparts the conjugate shapes to all gears of that pitch. The actual cutting is not done thus. It is performed in the following ways:—

By rotary cutters, the objection to which is that they are liable to error in outlines, and which, being used in relatively small sets, cut only as many wheels accurately as there are cutters in a set, the intermediate wheels being only approximately correct.

By a planing process, using a tool shaped like a tooth space, the objections to which are similar to those made to rotary cutters.

By a planing process, using a pointed tool, or one having a knife-edge, the movements of which are controlled by the edge of a form representing the tooth curves enlarged. Here accuracy depends on the truth of the form.

By a planing process, using either a tooth or a cutter shaped like a base gear which is reciprocated and rotated, the rotations synchronizing with those of the gear being cut, and of the gear represented by the cutter.

By a planing process in which synchronous movements are embodied in the design and construction of the machines themselves, and in which the cutting portion of the tool is a point, an edge, or a face-milling cutter.

Moulding by a generating process of simple rolling is not a practicable one. It must be combined with a planing action. In this the cutting point or edge comes into operation on successive portions of a tooth which is being rotated at a suitable speed corresponding with that of the pitch line. Thus the shape is generated.

Generating by a circular cutter in the form of a worm hob is utilized in spur, spiral, and worm gears. It is the most recent method, and has developed in a remarkable manner.

The Rotary Disc Cutters.—The cutters for standard gears are selected by a method of averaging, since it would be impossible commercially to make a distinct cutter for each different size of gear. Even so, an enormous stock is necessary, because a set is required for each different pitch, and usually a separate set for different machines, since the larger machines have large arbors, and use cutters of larger diameter, though the sectional shapes of the teeth for a given pitch must be the same. A small light machine is not suitable for cutting the heavier pitches, neither is it economical to put the finer gears on the heavier machines. A machine, therefore, of a certain capacity is properly intended to deal with some one range of gears, not necessarily a small range, but one which has its limits at extremes, and the machine has to be equipped with a full set of gear cutters within that range, or at least of as many different pitches as are included in a firm's requirements. To these a number of stocking cutters have to be added for the heavier pitches.

This entails a very heavy item of expenditure, and is one of the reasons why the hobbing and other generating machines are regarded with so much favour. Also, objection is taken to the practice of using gears which are produced by a small number of cutters, eight in involutes (Fig. 63), and twenty-four in cycloidal ones, because a cutter can only be absolutely correct for one gear. It was formerly said that the differences were no greater than the differences in errors of workmanship, a plea which is seldom offered now. Errors there are, of course, but these always tend to lessen as the design and construction of machines improve, and as the manufacture and the operation of cutters is also improved. Errors in workmanship and original errors do not necessarily nullify one another; they may be cumulative, and they are more obvious as gears increase in coarseness. Because of this some firms use cutters of intermediate sizes, often also cutters made exactly to the shapes required, as when large quantities of similar gears for motor-cars are wanted.

Range of Cutters.—The basis on which a system of cutters is arranged is that the same amount of difference of form should be present in each cutter in the whole range from the basis pinion of, say, 12 teeth to a rack, the extremes of which are seen in Fig. 64. Then the number of cutters to be selected will be determined by the amount of inaccuracy which is considered permissible. In the Brown & Sharpe system, which is generally adopted, eight cutters are embodied in all pitches, though the departure from accuracy will be more in teeth of coarse pitch than in those of fine pitch, not relatively, but absolutely, due to the larger dimensions. Thus, if the difference between the teeth of the smallest pinion and the rack at the point of the teeth is ¼in. in the teeth of a given pitch, it will be double that in a wheel and rack of twice the pitch, and so on.

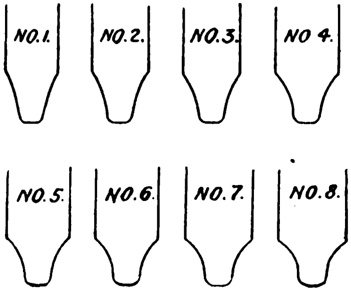


Fig. 63.

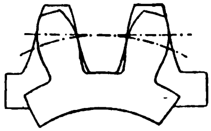


Fig. 64.

The selection of eight cutters is therefore an arbitrary one. Professor Willis, with whom the idea originated, obtained by calculation equidistant values of twenty-five cutters ranging from gears of 12 teeth to a rack, and also prepared a table showing what cutters should be selected on the supposition that a smaller or larger number of cutters were to be selected. If a single cutter were thought sufficient for an entire set of very small wheels, one for 25 teeth should be selected. And so on for numbers of cutters ranging from two to twenty-four, as given in the table on p. 79.

SELECTION OF CUTTERS PROPOSED BY PROFESSOR WILLIS.

Number of Cutters in the Set.	Numbers of Teeth to be Selected.																							
2	50	16	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
3	75	25	15	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
4	100	34	20	14	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6	150	50	30	21	16	13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8	200	67	40	29	22	18	15	13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
10	200	77	50	35	27	22	19	16	14	13	—	—	—	—	—	—	—	—	—	—	—	—	—	—
12	300	100	60	43	34	27	23	20	17	15	14	13	—	—	—	—	—	—	—	—	—	—	—	—
18	300	150	100	70	50	40	35	30	26	24	22	20	18	16	15	14	13	12	—	—	—	—	—	—
24	Rack	300	150	100	75	60	50	43	38	34	30	27	25	23	21	20	19	18	17	16	15	14	13	12

In the Brown & Sharpe cutters a difference may be made in the construction according as the pitches are finer or coarser. For gears below 10 diametral pitch the simple uncorrected involute construction can be taken. For gears of 10 diametral pitch and coarser, certain corrections are made to avoid interferences. In the first the cutter for 12 and 13 teeth is made to a diagram for 14 teeth, and the others as below:—

For	14	to	16	teeth,	diagram of	16	teeth.
,,	17	,,	20	,,	,,	18	,,
,,	21	,,	25	,,	,,	22	,,
,,	26	,,	34	,,	,,	28	,,
,,	35	,,	54	,,	,,	38	,,
,,	55	,,	134	,,	,,	55	,,
,,	135	,,	rack	,,	,,	100	,,

In the second the cutters are as follows, made to corrected diagrams:—

For	12	and	13	teeth,	diagram of	12	teeth.
,,	14	to	16	,,	,,	14	,,
,,	17	,,	20	,,	,,	17	,,
,,	21	,,	25	,,	,,	21	,,
,,	26	,,	34	,,	,,	26	,,
,,	35	,,	54	,,	,,	35	,,
,,	55	,,	134	,,	,,	55	,,
,,	135	,,	rack	,,	,,	135	,,

The cutters made by J. E. Reinecker take account of the greater interference of coarse-pitched gears than of those of the finer-pitched ones. He makes a set of eight cutters for 2¾ diametral pitches and finer, and another set of fourteen cutters for diametral pitches up to 2½ P. A comparison of these will show that for the coarser pitches the range of a single cutter is more limited than that for the finer pitches.

Half numbers of cutters are made, giving, in addition to the eight usual ones, seven more (fifteen in all), a concession to those who require accurate and silent gears. Brown & Sharpe, and others, have half numbers as follows:—

No. of Cutter.	Range.	No. of Cutter.	Range.
1½ . .	80 to 134 teeth.	5½ . .	19 to 20 teeth.
2½ . .	42 ,, 54 ,,	6½ . .	15 ,, 16 ,,
3½ . .	30 ,, 34 ,,	7½ . .	13
4½ . .	23 ,, 25 ,,		

This system is carried still further, as already stated, by the growing practice of making special cutters to exact shapes for standard gears, and it has been developed by the demands of the motor-car builders for silent change-speed gears. Many of these are cut with single disc cutters in preference to using a generating hob.

The eight cutters in the usual set are also made in high-speed steels. Prices range approximately double, while the output is about three times as great.

Each cutter in the ordinary standard sets is usually made exactly for the lowest number of teeth for which it is to be used, the exceptions occurring in two or three of the low-numbered gears. The inaccuracy in all other gears is in the right direction for easy working. Thus, when a No. 3 cutter, made right for 35 teeth, cuts 54 teeth, or any number between 54 and 35, the teeth will have more rounding off than the correct construction would entail, with a correspondingly slight decrease in bearing surface, which, however, is much to be preferred to interference.

Cutters for Cycloidal Teeth.—Cutters for cycloidal or double-curved teeth are made in larger sets than the involutes, because, excepting in the finer-pitched gears, the variations in the tooth forms are greater than in the single-curved involutes, and because they are generally reserved for the coarser pitches. Twenty-four cutters are made, designated by letters A to X instead of by numbers, to prevent confusion with the involute cutters:—

Cutter	A	cuts	12 teeth.	Cutter	M	cuts	27 to 29 teeth.
,,	B	,,	13 ,,	,,	N	,,	30 ,, 33 ,,
,,	C	,,	14 ,,	,,	O	,,	34 ,, 37 ,,
,,	D	,,	15 ,,	,,	P	,,	38 ,, 42 ,,
,,	E	,,	16 ,,	,,	Q	,,	43 ,, 49 ,,
,,	F	,,	17 ,,	,,	R	,,	50 ,, 59 ,,
,,	G	,,	18 ,,	,,	S	,,	60 ,, 74 ,,
,,	H	,,	19 ,,	,,	T	,,	75 ,, 99 ,,
,,	I	,,	20 ,,	,,	U	,,	100 ,, 149 ,,
,,	J	,,	21 to 22 teeth.	,,	V	,,	150 ,, 249 ,,
,,	K	,,	23 ,, 24 ,,	,,	W	,,	250 or more.
,,	L	,,	25 ,, 26 ,,	,,	X	,,	rack.

All the smallest numbered gears require a separate cutter, and the cutters for medium gears are very limited in their range. They are not manufactured in so large a range as the involute cutters, being stocked in a few pitches only. They are, however, made for diametral and circular pitches, while the involutes are more often made only for the first, though some firms manufacture both kinds. The cycloidal cutters have a shoulder, which the involutes have not, in order that the depth of tooth shall be controlled by the contact of the shoulder with the rim. This ensures that the pitch lines shall be in contact, and the double curves take up their exact mutual relations, a point which is of much less importance in single-curved gears.

Irreconcilable Systems.—In any attempt to work on an exact system, the diametral and circular pitches clash, since neither can be cut correctly with a set of cutters made for the other system. Yet customers sometimes want one, sometimes the other. For the best results, a larger stock of cutters is required than for average results; and sometimes gears are wanted to conform to the metric system, for which, again, other sets of cutters are necessary.

The commonest problem is that of using the standard cutters made for diametral pitch, to cut wheels of circular pitch by. The proper way is to have sets of cutters for each class of wheel. But as the demand for the latter is so small the occasional order is filled by selecting a cutter which comes nearest to the circular pitch required. Thus a 4 diametral pitch is the closest to a $\frac{3}{4}$in. circular pitch, with a variation of about $\frac{1}{32}$in.—a serious difference. A 3 diametral pitch comes very close to a 1in. circular pitch. A $2\frac{1}{2}$ diametral pitch is very near to a $1\frac{1}{4}$in. circular pitch. The blank is then sized for circular pitch, and the diametral cutter used. This is not recommended for adoption in regular practice, but it is justifiable at a pinch, and in the worst cases it is preferable to a badly-cast gear. The deviation from strict accuracy, however, is not quite so serious as appears from the difference in the pitch of the cutters. The tooth thicknesses and spaces come out above or below exact dimensions; but the involute is so elastic a type that the meshing of the teeth is practically unaffected. On large wheels, say, of over 50 teeth, the difference is much less apparent than it is on smaller wheels, say below 20 teeth, in which the deviations from accuracy are more obvious. But it is practicable to make decent gears in most wheels by using diametral cutters, and this has been done with satisfactory results. It is well to keep full-sized sections of teeth based on the diametral system for ready reference; but a close approximation to the circular pitches which correspond with diametral can always be made mentally by remembering that the relation between the two is always that of diameter to circumference, or as 1 is to $3\frac{1}{7}$, or as 7 is to 22, or, circular pitch × .3183=diametral pitch, and

$$\frac{3.14159}{\text{diametral pitch}}=\text{circular pitch}.$$

Thus, take 1in. circular pitch = 1in. × .3183 = .3183, which does not correspond with any exact diametral pitch, and so on through any circular pitches. On the other hand, take a diametral pitch, say 6, then—

$$\frac{3.14159}{6} = .524 \text{ circular pitch,}$$

which again is not an even number.

So that a 4p. (four representing a fourth of an inch or module) would give $\frac{1}{4}$in. × 3in. = $\frac{3}{4}$in. + $\frac{1}{7}$ of $\frac{1}{4}$in. = $\frac{3}{4}$in. full for the corresponding circular pitch; 2p. would give $\frac{1}{2}$in. × 3in. = $1\frac{1}{2}$in. + $\frac{1}{7}$ of $\frac{1}{2}$in., or a trifle over $1\frac{9}{16}$in. for the circular pitch. And to derive diametral pitch from circular the operation would be reversed.

Relief of Cutters.—Cutters are relieved—that is, the teeth are struck from a centre which is set eccentrically in relation to the centre of revolution. The result is that the cross-sectional shapes of the teeth remain unchanged by the repeated grindings which are done across the faces. So that a cutter may be used down to a mere skeleton without loss of accuracy or efficiency, cutting a few hundred gears during its life. It is highly essential to keep it sharp, regrinding at about every two or three hours between the cutting of gears.

Stocking Cutters.—The foregoing relates to the cutters used for finishing. But gears of about 4 diametral pitch and over are better roughed-out with stocking cutters, leaving only a small amount of metal to be removed by the finishing cutter. This tends to greater accuracy, because less distortion is likely to occur, and it prolongs the life of the finishing cutters. Stocking cutters are made for pitches smaller than 4P., but they are not employed so much for these as for the coarser pitches.

In these also improvements have been made with a view to save time and to lessen the work of the finishing cutters. The simplest form roughs-out parallel as in Fig. 65. These have been modified to take out more material from the spaces above the pitch line (Fig. 66). A closer approximation to the finished shapes has been made by Messrs Brown & Sharpe (Fig. 67). In this, plain and stepped teeth alternate. The first do the greater part of the cutting, the second break up the chips, while the tooth is roughed-out more nearly to the finished shape and dimensions than by the ordinary kinds of stocking cutters.

The radial gang cutters (Fig. 68) cut from two to ten teeth at one traverse of the cutter slide. They are useful as a means of increasing output where quantities of duplicate gears have to be cut, and where a fine degree of accuracy is not required.

Each gang will only cut one diameter of gear and one certain number of teeth. The teeth are sharpened in the same way as those of single cutters, and when the gang is assembled on the arbor the teeth are set in staggered fashion to hit and miss each other in the axial direction. These cutters are manufactured by Messrs Gould & Eberhardt.

Cutters for Spiral Gears.—Spirals, single-helical and double-helical gears, and tangent worm wheels are all cut by means of the disc cutters, the relationships of which have been established in a previous chapter.

Single-helical Gears.—These are usually cut in the universal milling machine with one of the standard disc gear cutters, provided of course that the gears are not of large dimensions. In this machine the dividing head, and the feed screw of the table on which the head is mounted, take charge of all the movements of the blank, the table providing its angular setting, while the cutter is simply rotated by the machine spindle. The table screw and the change gears in unison impart the helical movement to the blank, and the indexing is done by the index plate and peg. The table of the machine is set to the angle of the spiral on its pitch line. But if the angle exceeds 45° attachments are provided for swivelling the cutter. Several firms provide these attachments for the machines which they manufacture.

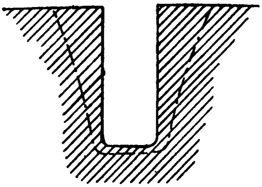


Fig. 65.

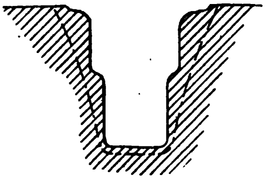
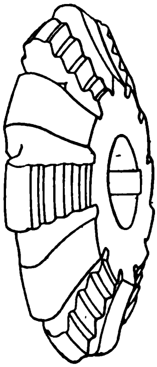


Fig. 66.

In the wide range of angles other than 45° all cases are met with from 10° or 12°, in which the tooth section varies but little from that of a spur gear, to high angles of 70° or 80°, where the difference is extreme. In the first there would be only about one tooth difference in the selection of cutters and the spur-gear cutter of the same diametral pitch. In the second a rack cutter would be used of a very fine pitch. In the first case a single tool, like that used for cutting worm threads in a lathe, would frequently be used.

Messrs Brown & Sharpe advocate fitting a templet to the curve of the flank along the line of the normal pitch, which will, of course, be flatter than the curve along the plane of the circular pitch. The diameter of a circle is found to coincide with the curve of that templet, and taken as the addendum circle of a gear for which a cutter has to be selected. With increase of angle of spiral there will be greater flattening of the curve, and the larger will be the number of spur-gear teeth from which the cutter will be selected.



Double-helical Gears.—Spiral gears of large diameter are usually termed helical gears simply. But the prefix " single " helical should then be employed to distinguish them from the true or double-helical gears, to which the term properly applies. Yet, as we have seen, these differ from spirals only in the double arrangement of teeth, and not in any essential. Hence we must study them through the spiral gears with which they must be classified. The duplication of the teeth on opposite hands entails difficulties in cutting, which do not arise in the single-helical gears. The successful cutting of large double-helical gears in the solid has been accomplished only in recent years. This difficulty here arises from the meeting of the teeth at the apex, hence none of the cutters which are ordinarily used for spiral gears can be employed for double-helical teeth because the meeting of the right and left hand teeth at the apex arrests the movements of the cutters. These can only be employed in the Wüst design, in which the teeth hit and miss, thus breaking their continuity.

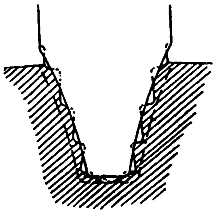


Fig. 67.

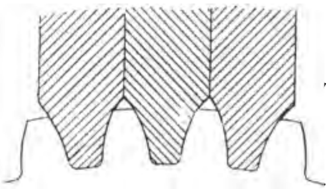


Fig. 68.

Since rotary disc cutters cannot be used when right and left hand teeth meet in an apex, only one practicable method is available then, that of an end cutter having its axis set perpendicularly to the rim of the gear which is being cut, and rotating and traversing along the spiral path.

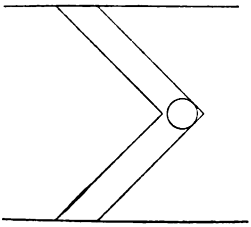


Fig. 69.

In the end-milling process the mill is shaped to the section of the normal tooth space, or that taken at right angles to the direction of the spiral. It is the same as the section of a rotary cutter. But, unlike the latter, the cutting teeth are so few in number, and the tool is so small, that it wears rapidly, so that the method is ill-suited to wheels of fine pitches, and having a large number of teeth. Also, as a radius is left at the inner angle of meeting teeth (Fig. 69), either this must be cut away subsequently, or, which is easier, the apex of the outer angle of the mating wheel must be removed and rounded by hand-work.

In these respects it compares unfavourably even with the disc rotary cutters, and very much more so with the hobbing cutters. But it has no rival, because no other shape will answer. And the cutters are easily renewed. Their teeth are coarsely pitched to prolong their lives. Fig. 70 shows a group of cutters used by Messrs David Brown & Sons, Ltd., of Huddersfield, in cutting their solid double-helical spur gears.

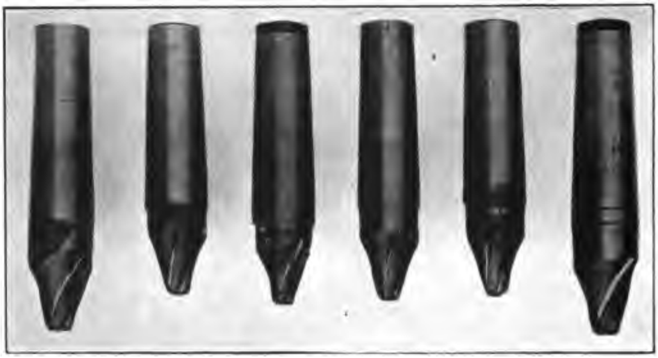


Fig. 70.—End Mill Cutters used for Double-helical Gears.

In using a rotary gear cutter for single helical gears, or for the hit-and-miss system of double-helical gears, the problem is rendered difficult by reason of the difference between the normal and circular pitches, and tooth shapes. The cutter is selected for a gear which corresponds with the normal pitch of the spiral. But such a disc does not cut a correct tooth because the disc is flat, and the tooth space is a spiral, which results in interferences, increasing when the cutters are large and the spirals of short lead. Such cutters are set with their axes normally to the direction of spiral.

The difficulties and objections involved explain why the manufacture of solidly cut double-helical gears has been so long delayed by comparison with that of other kinds of gears. Hence a practice of long standing has been, and is still, adopted: that of cutting two distinct single-helical gears separately, and bolting them together. Though many gears have been made in this way, an objection to them is the weakening of the teeth at the apex, due to the discontinuity of the metal there, and generally also the dislike to the union of two pieces in a gear, though in many high-class dividing worm gears an identical practice is followed, but with the object of making fine connections. Another method of cutting is to separate the two sets of teeth, right and left hand, by an annular space, sufficiently wide to allow the disc cutters used to clear out from the ends of the teeth.

Cutters for Bevel Gears.—The formation of the teeth of these gears with rotary cutters is far from being a simple matter. The case is not at all like that of cutting spur teeth, in which the cutter section is exactly the same as the tooth section. The bevel-gear cutter has a section much thinner, in order to permit it to pass through the tooth spaces at the small ends. And whether the cutter is selected to have the profile of the teeth at the large end, or a little way inward from that end, obviously its profile is only quite correct for one sectional position. This is why the cutter leaves the tooth points too wide at the small end, as indicated by the dotted lines in Fig. 71, which excess has to be filed away.

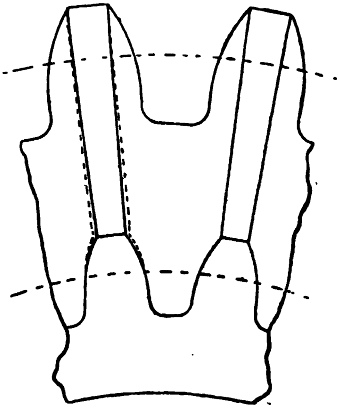


Fig. 71.

Moreover, at least two adjustments or " rolls " have to be made, independently of the stocking or roughing-out cut, being one adjustment for each side of a tooth space (Figs. 72 and 73). The effect is to alter the proper shape of the tooth, so that a pinion of small cone angle, which would have to be rolled more than a wheel of large cone angle to get the correct tooth thickness at the pitch line, would have its faces thinned somewhat, and its roots thickened at the large end. Less, however, in that case has to be filed off the faces at the small end of the tooth.

A way to lessen these slight inaccuracies at the large end in small pinions is to select a cutter one or two numbers removed from that suitable for larger gears—as, for instance, a number 6 or 7 instead of 9. These cutters, being thinner at the roots, do not round off the faces of the teeth quite so much. In either case the cutter selected can only be an average to cut to the nearest approximation right through the tooth, and thus lessen the amount of corrective filling or subsequent corrective cutting.

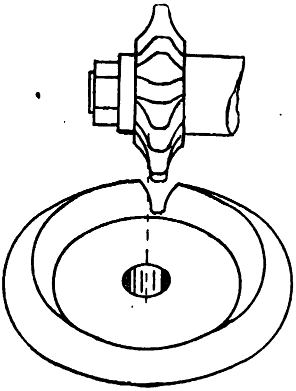


Fig. 72.

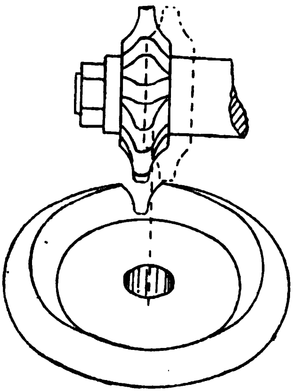


Fig. 73.

The difficulty of producing bevel-gear teeth with rotary cutters is enhanced by the approximate and corrective character of the settings which are necessary before the exact amount of roll can be ascertained which is required to give the right thickness to the teeth at the larger and the smaller ends. The following explains the nature of the operations:—

In all but the smallest gears three cuts are taken through a tooth space, a central parallel stocking cut, and two flank cuts, one on each side. In small gears the central cut may be omitted. A bevel-gear cutting machine is illustrated at p. 170, showing, in Fig. 153, the provisions for setting the blank to the cutting angle, or sectional angle corresponding with the angle of cut, which may be that of the working depth, or the cone angle of the roots. In Fig. 151 provision is shown for setting the cutter out of centre for cutting the large ends of the teeth. It includes vernier measurement on a graduated dial for reading to thousandths of an inch. The cutter being set away from zero, the blank is rolled up into contact first with one side, then with the other. In this rolling of the blank the tooth flank is swivelled about the apex of the cone of the gear, so bringing the large end of the tooth radially into line with the small end. The cutter is set out of centre to bring the side that is cut centrally with the blank. Hence, when these adjustments are made, considerably more material will be removed from the large end than from the small. The exact amount necessary is in the first place tested by trial cuts, but when quantities of similar gears are being cut these trials need not be repeated. The amount of trial distance may be one-sixth of the tooth thickness at the large end in a 12-toothed pinion, and ranges from that to one-seventh in larger numbers of teeth. After a trial cut has been taken off each side a tooth gauge is used, one at the large and another at the small end. Any further corrections which may be found necessary are made before the remainder of the teeth are cut.

CHAPTER VIII

FORM PLANING, AND GENERATING METHODS

Form Planing.—Bevel-gear planers using forms, or formers, which are enlarged profiles of the teeth, are popular. They are made in several designs, which differ in the methods of carrying and rolling the blank, in those adopted for operating the reciprocating cutter, in the location of the form and in the practice of cutting one side only, or both sides of a tooth simultaneously.

The Form.—The principle of action of these machines is simple enough. A form, which is a piece of steel having an edge cut to an enlarged profile of the teeth to be planed, becomes a guide which controls the lateral movements of the tool between its reciprocations. The feed thus takes place along the curved edge of the form. The gear blank is held stationary during the actual cutting, and is indexed or pitched when a tooth flank is finished. Obviously the accuracy of the teeth that are cut depends primarily on that of the form, and therefore the latter is made on the same principle as a dividing wheel usually is, namely, larger than the tooth being cut, for which it is the guide. As a rule the curves of the form are from three to four times the dimensions of the gear tooth at the major diameter so that errors in the form are reduced in the gear which is cut. Fig. 74 shows the form in relation to the gear. A B is the cone distance of the gear, and A C that of the form—not, however, set out to its real distance in the diagram. The line B D represents the back cone distance of the gear, and the line C E that of the form. With C E as radius, describe the arc A in Fig. 75. On this arc as a pitch line, construct the tooth curve *a b c*, remembering that all the tooth elements are enlarged in the exact proportions which exist between B D and C E in the previous Fig. 74. If C E is three times the length of B D, all the tooth curves, the pitch, addendum, dedendum, and root clearance, will be three times larger than those in the actual gear. The enlarged tooth curve *a b c* is now used as a centre line for a series of circles shown, the diameter of which is exactly that of the diameter of the form roller used in the machine. The curves drawn tangentially to the series of circles are the curves for the upper and lower formers. The outer edges of the formers are cut so as to bring the arm of the machine level when the centre of the roller is at the point *b*. The holes shown are stud holes for attachment. The tooth curves of forms are now produced in a machine which generates the curve, so that the marking out of the teeth is discarded.

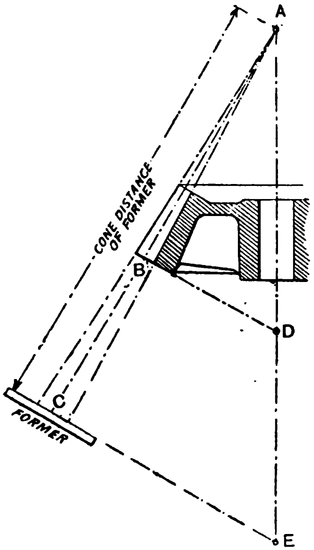


Fig. 74.

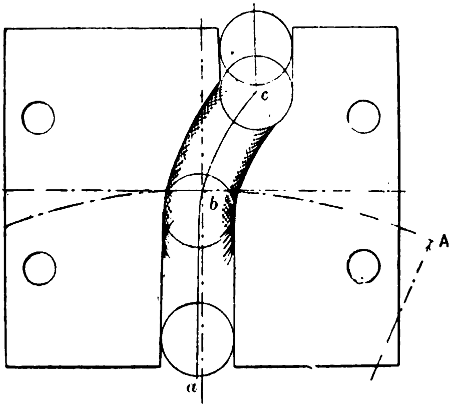


Fig. 75.

The results of form planing are more satisfactory than when rotary cutters are used. If the forms are correctly shaped, and the machine is accurate in its action, and the blank and tool are set up properly, the teeth thus planed sufficiently approximate to perfection for most practical purposes. The cost of cutting is usually rather high, and sometimes the results have not been satisfactory, due to faults in the form, in the machine, or in the setting up. The planer types of machine are of especial value in the coarser gears, or for those of over 4 or 5 diametral pitch. The rotary cutters are satisfactory enough for fine-pitched gears for ordinary duties.

A slight difference in using rotary cutters and planing is that in the first case the root line is cut to the cone centre, in the second it is set to the angle of the working depth of tooth and produces a root which lies parallel with the point of the engaging gear. This, however, has the advantage of imparting slightly more rounding to the small ends than if the correct angle were set by.

Objections.—A fact in favour of planing is that the rotary cutter must have a longer travel than a planer tool in order to clear the teeth at each end. An objection to planing is the large proportion which the lost return strokes bear to the cutting ones, and that fine feeds are necessary to lessen the effects of spring of the tool, which is more liable to occur because the thickness of the chip is greater at the larger diameter than at the smaller. The cost of the forms is considerable, since there should be at least as many of them as there are cutters to a pitch, usually there are more. To secure the full advantage of the planing methods there should be a form for nearly every different gear in the smaller and medium numbers of teeth, though not in the larger numbers. These reasons, coupled with the growing desire for absolute accuracy, explain the rapid advances which the bevel-gear generating machines are now making.

Principles of Generation.—Not so many machines are made for generating spur gears by planing as there are for bevels. The reason is that spur gears can be cut much more accurately than bevel gears can be with rotary disc cutters. To understand the action of the generating gear cutters it is advisable to approach the subject from the point of view of the moulding generating process (*see* p. 77), though that is hardly yet a practicable method. In this a hard gear is imagined to be rolled in a plastic blank, such as wax or clay, gears and blank being mounted on their axes and rotated at the velocities corresponding with their diameters. The teeth thus moulded and generated would mesh correctly with those of the hard gear.

The Fellows Cutter.—Put into a practicable shape this is the basis of the Fellows spur-gear cutting machine. The hard gear here is an actual planing pinion-shaped cutter (Fig. 76), which makes a part rotation between each cut taken along a tooth, the cutter being fed round continuously with the gear blank, between each reciprocation. Though the rack tooth does not appear here it nevertheless lies at the basis of the method, because the pinion cutter is ground by a rack-like generator (Fig. 77), and any gear that is cut with a rack will mesh with any other gear cut by the gear, or by the same rack, and, consequently, will generate any such gear.

The gear-shaped cutter is finished by grinding to the exact form, after roughing-out and hardening, using a wheel having a flat face set at the angle of the rack tooth which is selected as the basis (*see* Fig. 77). This being rotated, the cutter is rotated also, and rolled past it in the relations which subsist between a rack and a gear.

The motion of the cutter is produced by steel tapes which are wrapped upon a disc that represents the pitch diameter of the cutter. One side of each of the cutter teeth is ground first and afterwards all the other sides. The cutter is resharpened by grinding the ends only of its teeth.

Clearance of Cutter.—The teeth are not backed off as are the milling or rotary disc cutters, but an equivalent of that is imparted during the grinding. To effect this the face of the grinding wheel is set at a slight angle with the axis of the spindle which carries the cutter, with the result that the teeth are sloped slightly, one side of the teeth being portions of a right-hand spiral gear, the other side that of a left-hand spiral. The outside of the cutter is also backed off sufficiently to preserve the correct relation between the pitch thickness and the addendum. As the cutter is sharpened on its face only, it is reduced in diameter to a very minute extent. And as the involute curves for the entire length of the cutter are generated upon the same base circle, the teeth generated by the cutter as it wears are always alike. This results from the fact that the base circle is the generating surface in involutes, and not the pitch circle as in cycloidal teeth.

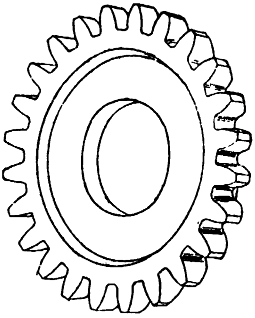


Fig. 76.

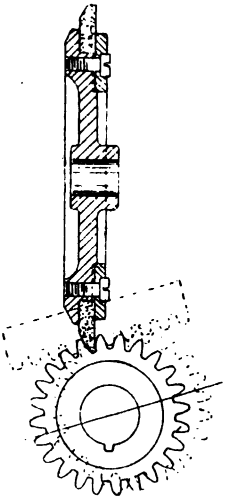


Fig. 77.

Action of Cutter.—When producing interchangeable gears from this pinion which stands for one of the set, the principle is, as previously stated, that which is termed moulding-generating, though the cutting action is strictly that of planing or shaping. During the intervals of cutting, the pinion cutter and the blank rotate or feed very slightly in unison. At the commencement, the cutter is fed in to the proper depth of tooth, after which the rotation commences, the cutter finishing as it and the work rotate (Fig. 78). All these movements are automatic. The successive action is shown in Fig. 79, in which A is the cutter and B the blank. The lines drawn indicate the shapes of the successive chips, which run out to a thin edge at the finished surface, but are thick and coarse in the middle of the tooth space. The bulk of the metal is thus removed there, leaving only a fine shaving to be taken off the sides of the tooth, so combining a roughing and finishing cut in one. Also, as indicated at the upper left hand of Fig. 79, the cut is a draw-cut, taking place towards the cutter slide, which fact is favourable to rigid and steady action.

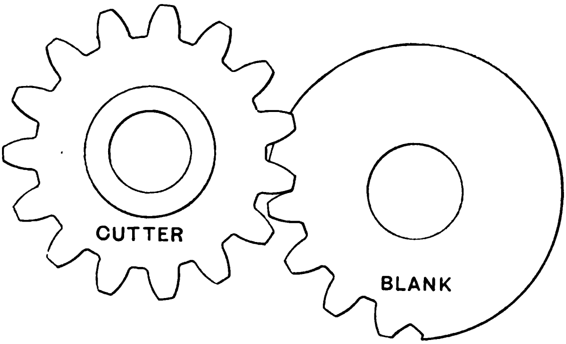


Fig. 78.

The Rack-shaped Cutter.—But for convenience the rack tooth itself or its equivalent is almost invariably incorporated in a generating gear-cutting machine. Such being the case, it is obvious that a piece of rack may be formed into an actual cutter and reciprocated precisely as the pinion just mentioned is, and the rack and wheel blank may be fed in the intervals of reciprocation. This is the principle of the design of the Sunderland spur-gear generator (made by Messrs Parkinson & Son), in which an actual length of rack comprising several teeth is used (Fig. 80), having a sufficient number of teeth (six) to make contact with the maximum number of teeth which can engage with it at one time. The movements of the rack cutter and the blank are identical with those which occur in actual engagement. All gears of a given pitch that are cut thus will engage with the rack and with each other. No corrections are required, because the teeth of every gear are generated. The bottom teeth are always roughing, and the upper teeth always finishing. Attention is called to this fact on the ground that the top teeth, being less subject to wear, can be relied on to finish accurately. Cutters can be made cheaply to suit any pressure angles. They are manufactured by milling, and finished by grinding the sides; but these are never touched afterwards, all the resharpening being done on the ends or faces. A small grinding wheel is used to produce hollow faces. The cutter is inclined to an angle to cause the grinding wheel to cut deeper at the base than at the point (*see* the section in the upper part of Fig. 80). Thus keen edges are produced. The cutters are of high-speed steel.

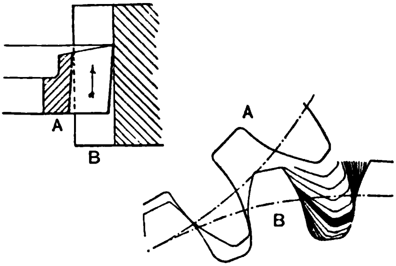


Fig. 79.

The Single Rack-tooth Cutter.—A more common method adopted is that of using a planing cutter shaped like a single rack tooth. The cutter reciprocates and the blank is rotated in the intervals. Different methods are adopted. In one the tool has the complete section of a rack tooth and both sides cut, so planing opposite sides of the same tooth space. In another only one edge corresponds with a rack tooth, and the tooth shapes of one side only are pro-

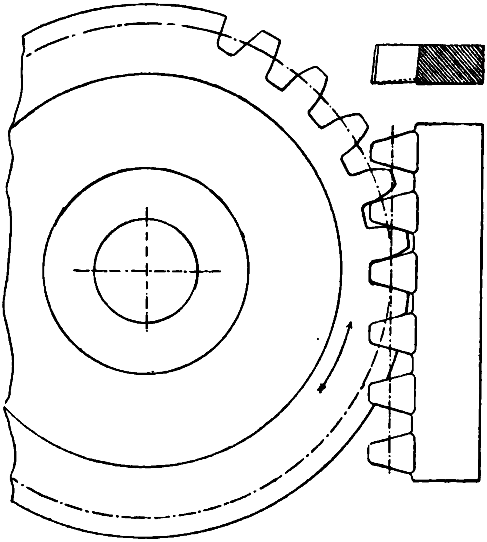


Fig. 80.

duced during one series of cuts, another similar tool following on the opposite sides.

Bilgram's bevel-gear shaper was the first planing generating machine made. The system employs a tool which corresponds with one tooth of a rack for an interchangeable set of gears of a given pitch. The rack tooth, in the case of bevel gears, must be that of a perfect crown wheel—that is, a disc having teeth converging to the disc centre as an apex (Fig. 81). This is clear from a consideration of the relations of the base line and the involute curve.

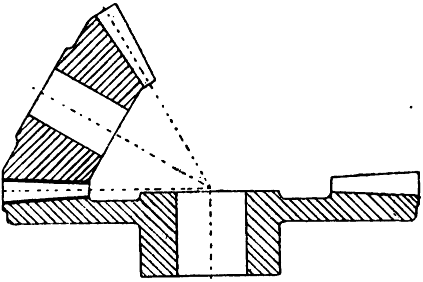


Fig. 81.

The curve is obtained by the rolling of a point in a straight line—the line of action—upon the base. As the base in a crown wheel is a plane, the tooth profile cannot be a curve, but must be a plane normal to the angle of obliquity of action of the gear. Therefore the tooth profiles of a crown wheel must be like those of a rack. From those plane profiles, set at the angles determined on as standard, the teeth of all other gears are generated; with the result that any wheel which gears with the rack teeth will gear with any other wheel of the set made on the same basis. Also, suppose such a rack tooth to be used as a reciprocating cutter, and a wheel blank to be rotated against it at a suitable rate, the tooth itself will cut perfect teeth in the wheel blank (Fig. 82).

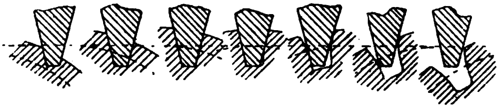


Fig. 82.

For such a system the cycloidal tooth is unavailable, because its shape cannot be developed by the rotation of the wheel blank. That system only can be utilized which generates a single curve. So that while in the planing system, in which the use of a form is embodied, teeth of either kind are planed, the generating machines are limited to the cutting of single curved teeth.

The De Fries Cutter.—In the spur-gear slotting machine by De Fries & Co., Akt. Ges., the wheel blank is fixed to the work-table, with the tool set to the required depth of tooth. The action is seen in Fig. 83, A showing the tool outside the body of the blank, and its front edge tangential to the periphery. The path of the wheel is then as indicated by the arrows, combining a lateral and a rotational movement on its own axis. One side of a tooth is thus generated, the tool cutting down to the full depth, and the opposite side of the tooth is then dealt with as the tool works up from the root to the point, until the tool leaves the finished tooth as at B in Fig. 83.

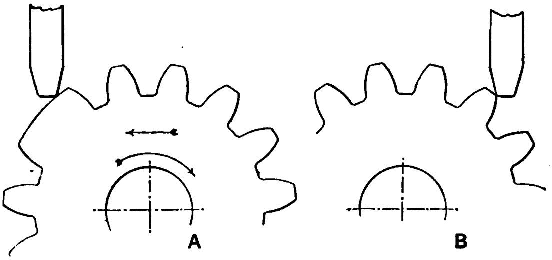


Fig. 83.

The successive positions of the blank in relation to the tool are clearly seen in Fig. 84, A. To save the work of slotting out the mass of metal in a tooth space when dealing with coarse pitches, the practice is to use a parting tool making a narrow groove as shown in B, Fig. 84, on opposite sides of the central area of metal, leaving a triangle of nearly the size of the tooth. This piece falls out, and the tooth curves are finished with fine cuts.

Either single or double cutting tools are used, the first cutting on the downward stroke, and having a relief motion for the return. The tool can be turned over when the edge has become blunt and be used right up. Two tools are employed in a rocking holder for the double-cutting system (similar to double-cutting planers), the holder being rocked automatically at each reversal, and the tool that is out of use being swung clear of the cut.

The Perrot Gear Cutter.—In this design spur gears are generated of any angle of obliquity and any desired thickness without employing intermediate change gears. The principle is that of a cam in contact with a rack bar, and the obliquity of this can be varied, so that through gear and screw connections the sliding movements of the cutting tools can be controlled. The arm of the cam rocks about the axis of the blank to be cut, the cam being located at the opposite end to the pivot. The inclined rack bar descends during cutting, and is coupled by transmission gear to the tool-holder, so that the descent of the bar corresponds with the rotation of the blank being cut. The cam corresponds with an enlarged master involute curve, and the oblique bar is equivalent to the flank of a rack tooth. Altering the angle of the latter therefore alters that of the rack tooth and consequently the pressure angle. Two cutters are ground and set in a holder, at the angles of the tooth (Fig. 85), and provided with screw adjustment for varying the distance between them, so as to increase the thickness of tooth on pinions and the smaller gears. This machine is the invention of Henri Perrot, of Argylls, Limited, Alexandria (Dumbartonshire).

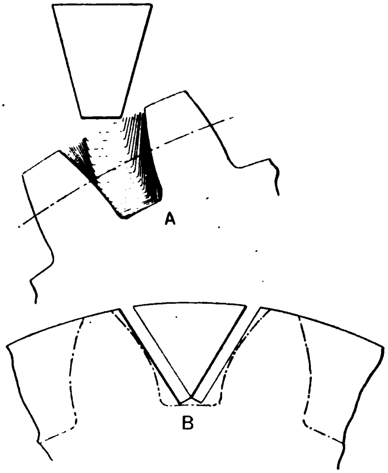


Fig. 84.

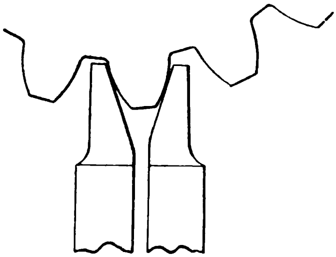


Fig. 85.

But it is not necessary that the cutter should bear any resemblance to the rack tooth. It may be only a point, or an edge, in which case another system of producing the shape of the teeth has to be substituted for the moulding-generating method. The tool reciprocates, and the gear blank is controlled by mechanism in such a way that the involute curve is generated. By means of various ingenious devices, the equivalent of the base circle from which the involute curve is generated, is embodied. The tool, being fixed and reciprocating only, shapes the tooth curves as the blank is fed round under the particular method of coercion embodied in the design of the machine.

Hobbing Cutters.—In the hobbing process a worm-like cutter is used. As the section of the teeth of a worm is that of a rack, the effect is that the blank being rotated at the speed suitable, and the worm set at the proper angle and revolved and fed across the face of the blank, it generates teeth of the correct section. The angle at which the worm is set is that of the angle of its helix, in order to present the normal thread sections for cutting (Fig. 86).

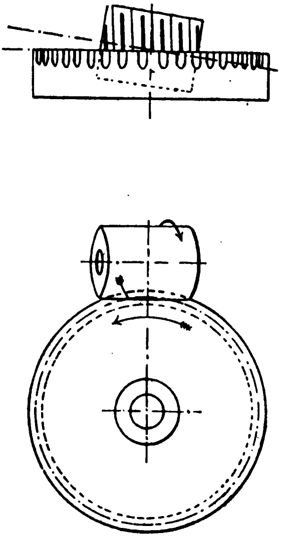


Fig. 86.

The process of hobbing is one which covers a field much larger than that of the production of spur gears, since it includes that of racks, worm-wheel teeth, and helical and spiral gears, all of which have a common basis in regard to tooth shapes, and are essentially a group apart from the bevel gears. The study of the machines in which these operations are performed is one of much present interest. Their work is but slightly invaded as yet by the practice of planing, which occupies so large a field in bevel-gear cutting; but the choice mainly lies between the old single-disc cutters and the later hobs. The utilities of each are counterbalanced by corresponding practical disadvantages.

Each of the gears just named—spurs, racks, spiral gears, both worm and helical—that can be cut either with single-disc cutters or rack-like cutters can be cut with hobs. But the worm wheels, the helical and the spiral gears, are not done satisfactorily with single cutters, because of the twisted form of the teeth, which causes interferences between them and the cutter. The hob cuts them all correctly by a process of generation.

The process of finishing worm-wheel teeth by hobbing has long been practised. But the first machine for generating spur-gear teeth by a similar process appears to have been made by Messrs Juengst & Sons, of Croton Falls, New York, in 1893. In 1894 Mr Reinecker had a hobbing machine for cutting spur, worm, and spiral gears, of which large numbers have been built. The gear blanks are carried on a horizontal mandrel. But most machines are now made with the mandrel disposed vertically, of which some dozens of designs are now built. We may consider the principle of action embodied in the hobs.

Forms of Hobs.—The hobs employed are of two types, parallel (Fig. 86) and tapered (Fig. 87); the first named being used for spurs, spirals and worms, and the second for worm wheels only, and those chiefly of high-angled spirals. The essential difference in their action is that the parallel hob is fed inwards gradually (Fig. 86), until the depth of tooth is attained, as there is no lateral motion of the hob; but the tapered hob is set at the commencement to the full depth and is fed tangentially across the blank as it rotates (Fig. 87), in which the commencing and finishing positions are shown at A and B. The tapered hob is preferable for cutting wheels to mesh with worms of high angles, as the teeth are generated more correctly, and it cuts lightly at entrance (like a taper tap), gradually deepening. The teeth of hobs are fluted spirally in order to make the cutting angle equal on both sides of the teeth.

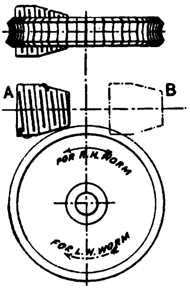


Fig. 87.

Action of Hob.—In this process the basis is a rack tooth, and the hob is merely a convenient device for the embodiment of the rack-tooth section in a practical form. This is illustrated in the diagram (Fig. 86), in which the spiral thread of the hob is so set as to cause it to take the place of a succession of rack teeth—that is, the section of the hob thread taken normally to its spiral is identical with that of a standard rack, and the axis of its revolution has to be set at an angle with that of the gear being cut, to bring the thread sections into the same identical positions which would be occupied by the rack teeth. The hob and the blank are then rotated at the ratio which corresponds with the number of threads in the hob and of teeth in the gear, just as the imaginary rack would operate. This is simple in principle, and apparently so in operation. But the working out of the details and the results are not uniformly satisfactory, for which there are several reasons.

In theory the action of a hob in generating the teeth of spur gears is perfect; in practice it is not. The worm-shaped hob containing several threads, rotating continuously, cuts the teeth of the spur gear, the blank of which also rotates continuously as the hob is fed inwards in its rotation. Cutting, therefore, starts from the top edge of the blank (Fig. 86), all the way round to the full depth of tooth, and progresses downwards with the feed of the cutter slide from the top to the bottom edge of the blank, during which period both blank and cutter will have made many revolutions under the control of the change gears set up. This method is in favourable contrast with the employment of single-disc involute gear cutters, of which eight form a standard set, and which are nearly doubled with half numbers where very good results are demanded for high rates of revolution. And these, moreover, are each absolutely accurate for one size of gear only, being approximate for those immediately adjacent. In theory every gear cut with a hob is absolutely accurate, provided the sectional shapes of the hob threads in the *normal* pitch conform to the rack section, which is the basis of the system of cutting, and with those corrections which are embodied in single cutters to avoid interferences in low-numbered pinions, as explained on. p. 17.

The hobbing machine is perfect for pitching, due to the regular rotation of the blank and the correctional character of the hob. The first named avoids local heating; the second is a consequence of the spiral form of cutter, which provides a large number of teeth acting in succession, and with very little wear, the teeth retaining their keenness of edge for a very long time. The hobbing process is much more rapid than cutting with single cutters.

Supposing these conditions are all fulfilled, there still remain some possible sources of inaccuracy—not imaginary, but so real that when very accurate gears are required for motor-cars and other service, some firms reserve the hobbing machine for roughing only, performing the finishing correctional cuts with single-disc cutters.

Objections to the Hob.—The first objection to the hob is that its action is not quite perfect. The teeth produced by it are not so entirely accurate as those which are made by the same hob when cutting worm wheels. The cause of this appears to lie first in the slight difference of the section taken in the axis of the worm and of that in the axis at which it is set, a matter of from about 3° to 5° (Fig. 86). But the principal one is due to the successive nature of the action of the hob in slipping from tooth to tooth while the blank is continuously revolving, which produces extremely minute facets on the teeth. Actually, when a very high degree of accuracy is insisted on, other methods of cutting have preference over hobbing, but for ordinary or average gears it is a sufficiently accurate method.

Though the difference of from 3° to 5° of angle is not much, it is enough when the tooth section is imparted to the hob parallel with its axis to leave the teeth a little too thick on the points. To avoid this some hobs have the tooth flanks made slightly hollow; in others the points are well rounded. In some instances the tooth shape is taken normal to the direction of the thread. If corrections are made in this way the action of the hob should be practically perfect. Machines, however, which use hobs must be built stiffly, since the stresses to which they are subjected are of a severe character, and are detrimental to the efficiency of the hobs.

The irregular surfaces, caused by the distance between the cutting edges of the hob operating on a wheel blank while revolving continually, may be minimized as stated. But when the best possible results are desired some means of correction seems desirable. A " reamer " design of hob has been patented by Messrs David Brown & Sons, Ltd. It is a parallel hob corresponding with the worm of the gear and with continuous teeth of rack section, but the sides of the teeth (or threads) are formed with a number of fine cutting teeth or serrations, arranged in different fashions similarly to the teeth of single, or double-cut files, and the hob is hardened and ground to eliminate distortion. The amount of metal removed is very small—a mere film correction—which is all that is required. It may follow the ordinary tapered hob in correcting high-angled worm gears, or it may be used for correcting spurs.

The Grant hob (Fig. 88) is designed to avoid or to reduce the formation of flats on the teeth arising from the combination of the revolution of the blank and the successive cutting action of hob teeth, which are spaced widely by the ordinary mode of fluting. In the hob shown, finely pitched teeth are cut, and their sectional shape is as illustrated in the sectional view (Fig. 89), providing clearance when cutting, and also clearance for the grinding wheel which is used to sharpen them. The Richards Machine Tool Co., of London, is the representative of the firm in this country.

Making the Hobs.—Three methods of producing hobs are adopted, two of which are commonly followed. There are turning and milling, and less frequently hobbing. The method of turning is familiar to the metal turner, using a screw-cutting lathe, set up with change gears. The profile of the flat tool employed corresponds with that of the thread space, and being set at the angle of the worm thread, it reproduces the thread of rack section. In the milling method the procedure is similar, excepting that in place of the single-edged tool a rotating milling cutter is used. Neither of these methods produces theoretically correct results, because the degree of accuracy attained depends on the truth of the tool or cutter, and on the control exercised over it through the change gears. These sources of error are minute and are distributed over several threads, which become mutually corrective in the action of a hob when cutting a gear. But high-angle worms are used to an increasing extent, and the hobbing method affords an accurate and rapid mode of production, alike for hobs as for use as running worms. The difficulties encountered in cutting multiple-threaded worms with a lathe or thread miller, on account of the interference of the cutter, are avoided by hobbing. A special section of tooth, having more or less curvature on the sides of the teeth, is necessary, because an ordinary rack-tooth hob would give involute teeth to the worm. Messrs Henry Wallwork & Co., Ltd., mark out the shape of hob tooth on the drawing-board to produce correct worm teeth, and make the hob from the profile thus obtained. Extreme accuracy is not necessary, because the worms are corrected by grinding after they have been case-hardened. This firm recommends a spiral

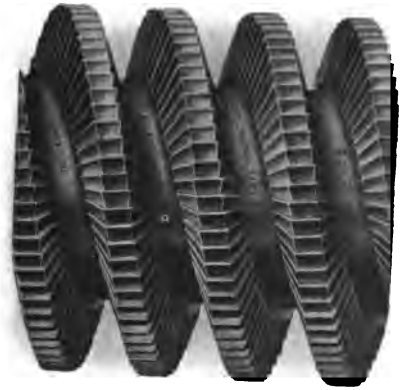


Fig. 88.—The Grant Corrective Hob.

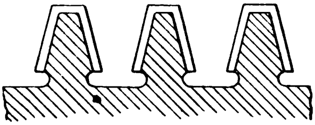


Fig. 89.

angle of between 30° and 50° to obtain high efficiency, and a pressure angle of $22\frac{1}{2}°$, which gives an included angle of thread of 45°.

The Humpage Hob.—This (Fig. 90) has inserted teeth, the object being to facilitate hardening and truing of the tooth profiles, and advantage is also taken in the construction to embody an ingenious way of producing the backing-off without the trouble of doing it as in solid hobs. The grooves in the body are tapered, with their centre axes not radial, but tangential to a circle concentric with the hob. The blades containing the teeth are set in reverse fashion to that in the sectional view, during turning and cutting of the threads, so that when they are reversed, their position in the grooves imparts the necessary relief. The teeth are ground accurately after being hardened, and are drawn down into their seatings by keys, the nuts at the end preventing lateral movement. Messrs E. G. Wrigley & Co., Ltd., are the makers.

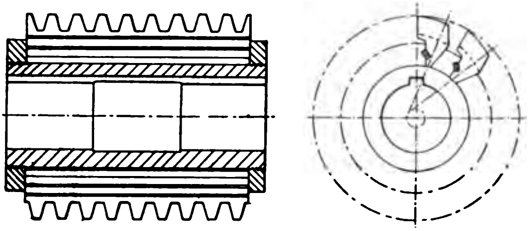


Fig. 90.

The Wüst Process.—The Wüst helical spur gears are hobbed, the teeth being discontinuous and staggered. Wüst gears cut with hobs, are remarkable for the very small size of the pinions which are practicable. This is one of the advantages which the helical design possesses over the ordinary spurs.

The axes of the hobs used in cutting the Wüst teeth are not inclined, but are set at right angles to the axis of the wheel blank. The axial sections of their teeth are those of a rack. A pressure angle of 20° is adopted, and the teeth are shorter than the Brown & Sharpe standard, the height of addendum being:—

$$\frac{0.8\,p}{\pi}$$

The practice is adopted of enlarging the pitch diameters of pinions under 20 teeth to avoid undercutting, the formula being:—

$$d = \{(0.95 \times \text{number of teeth}) + 1\}\frac{\text{Pitch}}{\pi}$$

for 20° pressure angle, and 0.89 addendum. The teeth are cut at an angle of 23°.

Another method embodied is that of a milling cutter, of which one face produces the tooth curves on the blank, the cutter being set at the angle corresponding with that of the rack teeth; the same result is obtained if a grinding wheel be substituted for the milling cutter.

The Chambon Hob.—This is designed to cut bevel gears by means of a hob (Fig. 91), which on first thoughts seems impracticable. The appearance of the hob itself does not help one to an understanding of the method until the principle of the design is grasped. It represents the tooth of a crown gear. But since the tooth tapers, the hob combines the tooth sections at major and minor diameters and at all intermediate positions. It is conceivable that separate hobs should be used, as one to shape the teeth on the major diameter, another on the minor diameter, and another on others intermediate. How to combine them in one hob of the same diameter is the difficult problem, since not only the dimensions but the angles of thread vary with the difference in pitches.

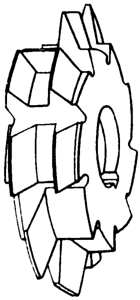


Fig. 91.

To do this a blank is turned and cut to the section of a thread corresponding with the section of the tooth of a crown gear on the major diameter and to the angle of thread corresponding with the pitch on that diameter. A second thread is cut on the same blank corresponding with the section of a tooth of a crown gear on the minor diameter and to the angle of thread which corresponds with the pitch on the diameter taken. Since the crown teeth have straight sloping flanks the angle of these will be alike in both threads. But the angle corresponding with the pitch will be different, so that in cutting the second thread a portion of the first thread will have been cut away, and a portion of the second will be wanting. The smaller thread is to have the same depth of tooth as the larger. But the metal that is left has at two points a section common to both threads, and if it is fluted at those points these will form cutting edges, and the one hob will take on itself the functions of the imaginary hobs for the major and minor diameters and for all intermediate positions. The hob is fluted at other positions all round. The hob now takes the place of the crown gear in generating bevel gears of the same pitch.

To hob bevel gears with this representative of a crown gear three movements are necessary. One is that of revolution in relation to the blank, similar to that of the spur-gear hobbing machines, the hob making as many revolutions to one revolution of the gear blank as there are teeth in the latter. The hob must also traverse across the width of the teeth with a constant change of angle suitable to the change of angle corresponding with the pitches. And the hob must be rocked while being fed across the teeth. These movements complicate the construction of the machine very much. The table for the work is carried on the horizontal face of the main framing of the machine with horizontal adjustment for cone angle and having the dividing wheel actuated from the head of the machine. The cutter is mounted on a spindle disposed vertically in a swinging carrier sliding on ways on the face of the head, the movements being driven through a telescopic shaft. The head oscillates to change the angle of the cutter, which change is regulated by a templet.

Generation by Milling: The Grant Machine.—A curious machine of the generating type was designed by Mr G. B. Grant, of Lexington, for producing the toothed profiles of a particular form of cycloidal teeth (Fig. 92) with a circular milling cutter. As the tooth flanks in this system must always be straight, the term "planoid" is applied to this *epi*-cycloid plane-flank system. The flanks are shaped with a thin slitting type of cutter (Fig. 93), the blank being set at such an angle that one face of the cutter forms one radial flank correctly, both in relation to the centre of the wheel and to the apex of the cone. Afterwards the cutter is set up in the centre of the slot and revolved, while the wheel blank is turned through an arc of a circle, causing the tooth points to receive the *epi*-cycloidal curve (Fig. 94). This therefore differs from the gears which are produced by plano-generating processes. The surface is finished in one rolling cut; but, more than that, the actual contact of teeth takes place only at and close to the pitch line, because the *hypo*-cycloidal curves, which would be struck by rolling circles in contact, are obliterated by the necessity of using a flat cutter. One advantage of this is that there is no oblique pressure as in involute teeth, and as in a lesser degree in cycloidal teeth. Moreover, it is a system which

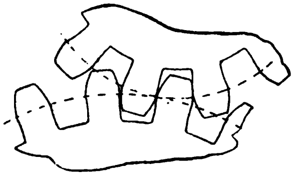


Fig. 92.

has been employed to a great extent in cutting the teeth of change wheels and of mortice wheels.

In the formation of these teeth it is not necessary that the flanks shall be cut on the generating machine. They can be produced on an ordinary gear cutter or milling machine, so setting free the generating machine for the formation of the curved faces. Another point claimed in its favour is that, as the teeth are of epicycloidal form, the system is better applicable to small gears than the involute, which, when of small size, require to have the angle of obliquity or the position of the pitch line altered, and the relative lengths of tooth above and below the pitch line. This is an advantage in regard to the turning of blanks, which can then be done by the standard rule for addendum allowance. In this system the same cutter serves for all sizes and shapes of teeth, because it has no special relation to any particular base.

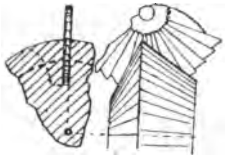


Fig. 93.

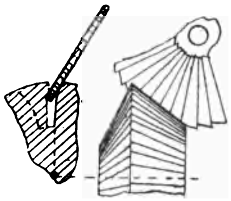


Fig. 94.

The Warren Bevel Gear Generating Machine.—This was an early one of very special type, which the Pratt & Whitney Company made originally for cutting the gears of the chainless cycles which were boomed some years ago. It was constructed for cutting one size of gear only, and produced the same kind of tooth as the Bilgram, but in a different way. Instead of using a rack tooth as a generating cutter, milling cutters were employed on the Warren machine, having straight teeth inclined at the same angle as the teeth of the crown bevel wheel which lies at the base of the system. Two milling cutters were used to operate on the outer sides of adjacent teeth, acting with reciprocating strokes, while both the gear blank and the cutters are rolled past each other. The cutters are rotated in slides which are fed along at the bevel required, the loose connection required to permit of the double motion being effected through telescopic shafts with gimbal joints, driving spiral gears on the spindles.

Though one cutter cannot be made to cut both sides of a tooth space at once, yet two cutters in this machine effect the same economy by being made to act on opposite sides of adjacent teeth at one time (Fig. 95). This is effected by the arrangement of the slides, the angles of which in relation to each other correspond with the angles of the tooth slides, so that each cutter runs on an axis perpendicular to the surface being cut, and their faces if prolonged would cut the apex of the pitch cones.

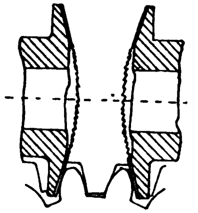


Fig. 95.

The cutters are fed along the teeth by the slides, which, as the machine is made to turn out one size of gear only, are carried on fixed bases. The cutters are rotated at the same time that they are fed forward lengthwise to the tooth, and the gear blank is also oscillated through a distance equal to the length of the arc of contact of the teeth. The machine, too, is entirely automatic from start to finish. The cutters have quick return, and when a gear is finished the machine rings a bell and stops.

The Beale Machine.—In this, shown by Figs. 307-310, p. 301, two cutters A, A, are milling cutters, having inserted radial teeth, of which *a*, Fig. 311, is the operating face. The spaces between the teeth leave room for the teeth of the other cutter to interlock, so that the faces of the teeth cut are those which lie opposite to the cutter body. The angles of the cutters in plan view (Fig. 309) can be varied to suit different pressure angles of the teeth. There is also another adjustment—that in an angle from the vertical—to correspond with the bevels of the teeth in section or to the apex of the pitch cone. The cutters represent the rack or crown gear of the involute system of bevel gears. In the rolling of the blanks past the cutters the tooth curves are generated. The bottoms of the teeth are left very slightly concave.

SECTION III.—MACHINES

CHAPTER IX

MACHINES USING FORM CUTTERS

Automatic Machines.—Figs. 96 to 103 illustrate a spur-gear cutting machine of automatic type by the Brown & Sharpe Manufacturing Company of Providence, R.I., U.S.A., in which rotary disc cutters are used, and of that design in which the cutter is traversed in a horizontal slide beneath the wheel blank, carried on a horizontal arbor. Fig. 96 is a side elevation of the machine; Fig. 97 is a plan view; Fig. 98 is a front elevation, or right hand as the attendant stands; and Fig. 99 is a rear-end or left-hand elevation. The general design has been embodied in machines by different makers.

General Design.—The principal parts of the machine are as follows:—The bed A supporting the cutter slide B, and at one end the upright C. On the face of C the work spindle slide D is carried. The overhang of the spindle is taken by the supporting bearing E which is carried in the upright bracket F, and this is adjustable along the edge of the bed A in tee-grooves. The work spindle is actuated from the pulley G belted from the countershaft, which latter makes 230 revs. per minute, and the feed and speed motions of the cutter carriage and cutter spindle are derived from the same pulley. The shaft H conveys the motion to the work spindle through the group of indexing gears at J, and the locking disc and its adjuncts in the box K. The movements are transmitted through the vertical shaft L, through equal spur wheels to the worm which drives the indexing worm wheel M. The spur wheels, *aa*, which have 30 teeth each, are indicated in dotted outline in Fig. 96 above the worm, also dotted, in front of the wheel casing M. The pulley G drives to the worm wheel N on the feed screw O, and also to the worm P on the end of the cutter spindle. Here it will be noted that the driven member is the worm and not the worm wheel, an interesting and unusual mechanism. The rates of both these movements are varied by the feed and speed change gears contained at Q (Fig. 98). R is the pulley by which

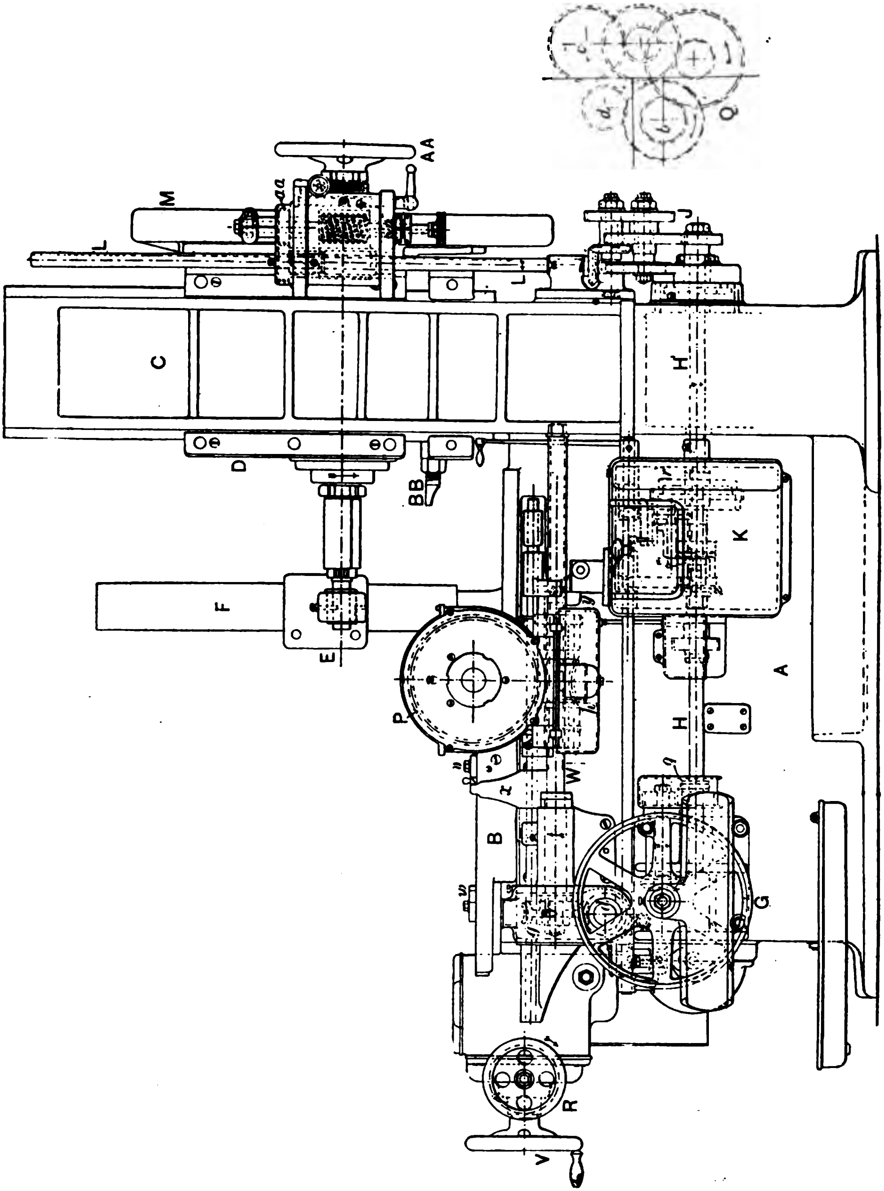
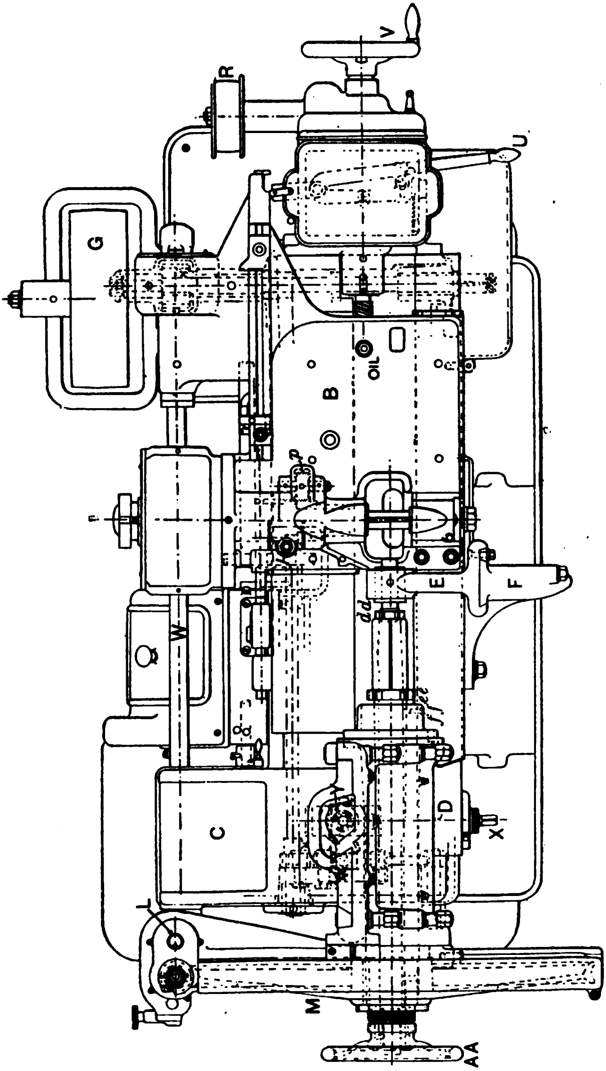


Fig. 96.



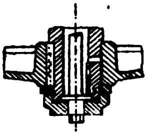


Fig. 97.

the movement of the cutter slide is reversed. These are the arrangements in outline. We will next trace out the details.

Details.—Beginning with the bed A of the machine, its section is seen in Fig. 100, which shows the sloping opening down which the cuttings and waste lubricant fall, the latter to be removed from the trays at the sides, seen in the general views, and returned by the pump. The cutter carriage B is gibbed to one shear only of the bed —the right-hand one in Fig. 100—and it has a slight clearance next the square edge on the side opposite. The overhang of the right-hand side of the carriage is counteracted by a bearing and gib on the under

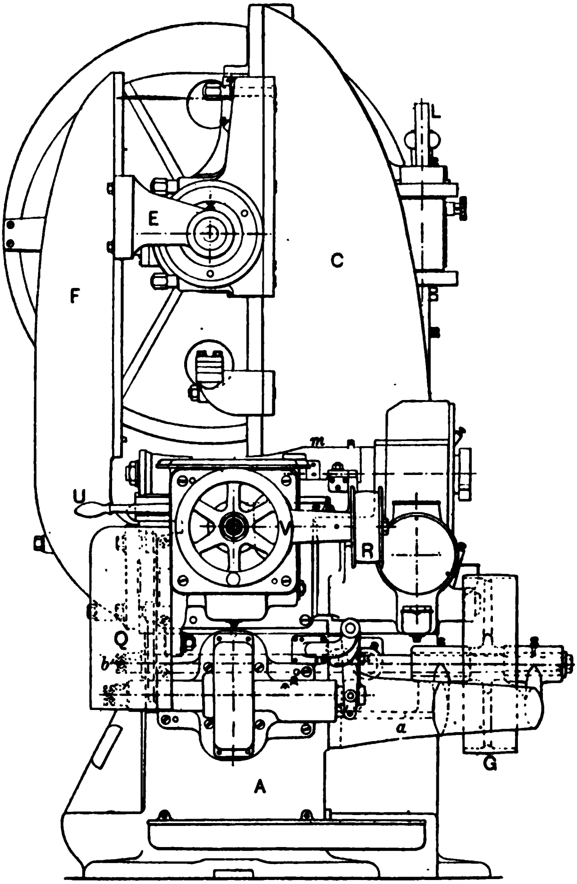


Fig. 98.

side of the narrower shear at the left hand. The upright C, which is cast in one with the bed, has spaces for tools and index change gears, and a portion of the bed is recessed for feed change gears.

The pulley G drives both to the cutter carriage and to the cutter

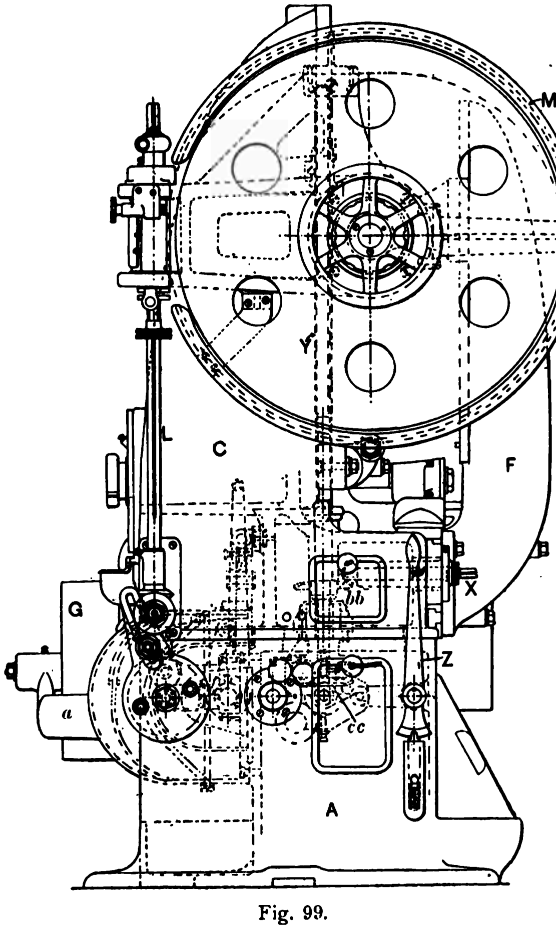


Fig. 99.

spindle. It is keyed on the driving shaft, the bearings of which are bushed (Fig. 98), and carried in a bracket *a* which is bolted to the side of the bed. The driving shaft *b* passes through the bed. This shaft is the first in operating the trains of change gears at Q. These transmit the motion for feeding the cutter slide, as indicated in the change gear diagram to the right of Fig. 96, terminating in the worm *c* (Fig. 101), which drives the worm wheel N on the feed screw shaft O. The speed worm shaft *d* is driven by pairs of gears only, one on *b*, and one on *d* (Fig. 98), the latter carrying the wheel which drives the worm gear P (Fig. 100) on the end of the cutter spindle. In this particular machine there are twelve rates of slide feed, and six rates of revolution of the spindle. The slide travels at $\frac{5}{8}$in., $\frac{13}{16}$in., $1\frac{1}{16}$in., $1\frac{3}{8}$in., $1\frac{13}{16}$in., $2\frac{3}{8}$in., $3\frac{1}{4}$in., $4\frac{1}{4}$in., $5\frac{1}{2}$in., 7in., 9in., and 12in. per minute. The spindle makes 20, 26.7, 35, 45·7, 60, and 80 revs. per minute. The gears for these changes are given in tables sent by the firm.

Whatever the rate of revolution of the cutter spindle, the return

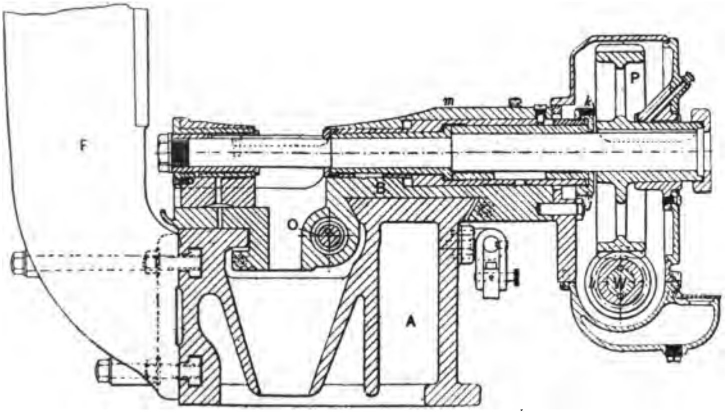


Fig. 100.

is at a constant speed, driven from the pulley R, making 498 revs. per minute. The mechanism is shown in enlarged detail in Fig. 101 (compare with Fig. 97). The pulley R drives the bevel gears S, the larger gear running loosely on a sleeve which is keyed on the feed screw. When the feed is on, the feed screw O is driven by the worm wheel by the clutch T, feather-keyed on the extension of the feed screw and moved to the left, the worm *c* becoming operative. The large bevel gear S then runs loosely with its sleeve, driven by its pulley R. When the clutch is moved to the right it engages with an intermediate clutch plate at the back of the bevel gear, and makes connection without shock through the medium of the friction washers, which are fitted in annular recesses in the gear. The clutch is operated by the lever U, sectioned in Fig. 101, and seen in plan in Fig. 97.

Provision is made for end-thrust and take-up on the feed screw. A collar on the screw runs between friction washers and end play is taken up by the washer *e*, which is pinched by the set screw *f*. The feed screw, the end of which is seen at the left-hand end of the drawing, is quadruple-threaded, 1⅛in. lead, left hand. The gears in the case (Fig. 101) are covered in, the cover being removable for inspection, and a screwed plug in the bottom of the case can be removed for drawing away waste lubricant. V is the hand wheel by which the adjustments of the cutter-spindle carriage are made when setting up the work.

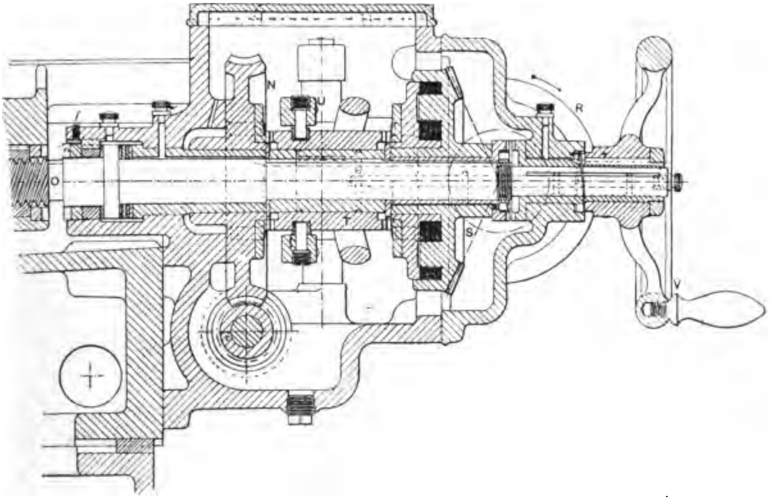


Fig. 101.

The rates of revolution of the cutter spindle are derived, as already stated, from the gears at Q, Fig. 98, by six alternative combinations of gears driving from the pulley shaft *b* to the speed worm shaft *d*. In Fig. 96 the worm on the shaft *d* is indicated in end view, actuating a worm wheel *g* enclosed in a box. The double-splined shaft W, on the end of which *g* is keyed, runs in a long bearing adjacent to the wheel, and is supported at the opposite end in the upright C. It carries the worm wheel *h*, sliding with the feed movements, and rotating in balance by the double keys on opposite sides, and driving the worm P on the end of the cutter spindle (Figs. 96 and 100). The worm wheel *h* in this case is smaller than the worm, and is the driver. The worm wheel runs in an oil box, the section of which is seen in Fig. 100, with an oil cover at one side. The entire box is covered in with a top, which is removable for inspection and when end play of the cutter spindle has to be taken up. This is done by adjusting the nut *j* and locking it by the screw *k*.

Setting the Cutter.—Provision is made for setting the cutter centrally, as follows:—A nut *l* (Fig. 97) is loosened, and the dirt guard *m* (Fig. 98) removed. Then the indicator (Fig. 102) is clamped, holding between the dotted portion and the screw *n*, and the screw *o* is adjusted against the pitch line of the cutter, first on one side and then on the other, until the zero line on the arm is equidistant from zero on the plate. Or the screw *o* can be adjusted so that the zero lines on the arm and plate will coincide. The worm *p* (Fig. 97) is used to adjust the cutter spindle, and when set the nut *l* is again clamped.

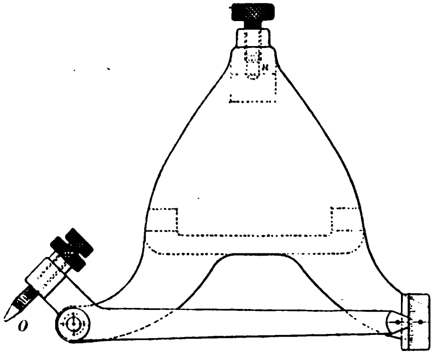


Fig. 102.

Indexing.—The drive to the indexing gear J starts from the pulley G, and is transmitted through the shaft H. It is commenced by the spur gears *q* (Fig. 96), the first of the gears being driven by mitre gears from the shaft of the pulley G, and the speed is reduced from 230 to 202 revs. per minute. The motion is transmitted to another shaft in the same plane, H^1, and reversed by gears *r*. The shaft H^1 carries the indexing mechanism K for the change gears J. This is shown enlarged in Fig. 103. *s* is the locking disc, notched at the edge to be turned one, two, or four times with different arrangements of change gears, particulars of which are supplied in tables. To set these, the knob *t* (Fig. 103) is lifted, and the sliding block *u* is set at the recess which corresponds with that given in the tables supplied by the firm. The indexing is effected automatically by the movements of the cutter slide from the dogs *v* and *w* (Fig. 96): *v* is clamped on the slide in contact with the reversing dog *x*, when the cutter slide is moved forward sufficiently far to leave space between the cutter and the work for indexing. *w* is clamped at the termination of the cut while resting against the reversing dog *x*. By the movement of the stop rod which carries *x*, a trip at *y* moves a clutch at *z* and renders the indexing mechanism operative.

The change gears at J transmit motion to the vertical splined shaft L (Figs. 96 and 99), which drives through equal spur gears *aa* the worm which operates the dividing worm wheel M. The shaft L being splined permits of the vertical adjustments of the work spindle.

Work Spindle.—The work spindle slide D is adjusted vertically either by hand or power. The first is effected at the crank handle X (Figs. 97 and 99), operating the bevel gears *bb* on a shaft Y. The second is controlled by the lever Z (Fig. 99), operating the bevel gears *cc* (Fig. 99) on the shaft Y.

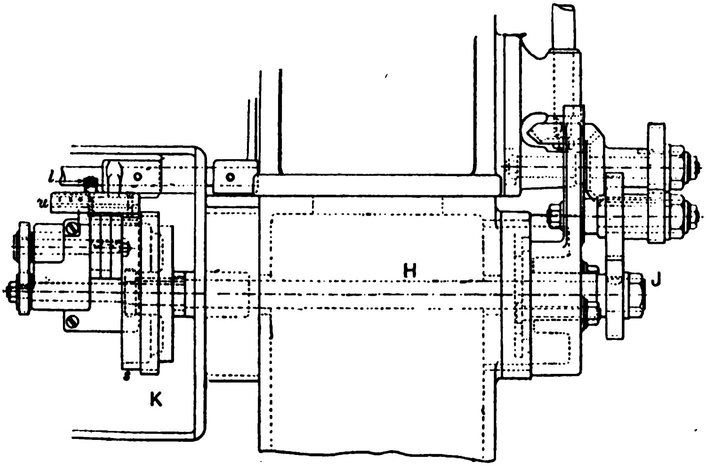


Fig. 103.

The section through the work spindle and its slide is outlined in the plan (Fig. 97). The slide is gibbed, and six bolts clamp it to the upright C. The blank is fitted by its bore over the split bushing at the end of the arbor in the spindle, confined by the nut *dd*. On the removal of this nut the bushing is forced off the arbor by the nut *ee*. The arbor is released by screwing the nut *ee* against the end of the spindle, taking hold of the hand wheel A A and then drawing the arbor back through the spindle. The bushing is then inserted in the bore of the blank, and being held in position, the arbor is pushed through it and the nut *dd* is screwed against it, expanding it in the bore of the blank. The nut *ee* is then loosened a little, and the arbor can be drawn into the tapered hole in the spindle by the hand wheel. The arbor can be taken out to be changed for one of another size by removing the hand wheel A A and the screw *f f*. The arbor is then drawn out at the front.

Setting the Work.—Work is set as follows:—The clamping bolts on the work spindle slide are loosened, and also those on the outer support E. The crank or hand wheel at X is used to lower the slide until the cutter just touches the blank. The cutter slide is then moved back until the cutter clears the blank. There is a dial at X, which is now set to zero. The work spindle slide is lowered a little more than is required for setting to the depth of tooth, and then raised to the exact depth. This is done in order to let the weight of the parts rest on the screw. The work spindle slide and the outer support are then clamped. The rim rest B B is set in position.

The Loewe Machines.—The automatic spur-gear cutting machines (Figs. 104 to 106), constructed by Messrs Ludw. Loewe & Co., Akt. Ges., of Berlin, are of the horizontal work spindle type. The largest machine deals with a maximum diameter of 78¾in.

The speed changes are produced through a stepped cone in each case. The feeds are obtained through a feed box, with eight changes, these being driven on the smallest type from a separate belt from the countershaft; but a safety device is included to connect to the cutter spindle, which prevents the feed from continuing if the cutter spindle should stop, or its revolutions fall below a certain minimum through belt slip. The cutter spindle is then driven by the feed belt until the main belt works properly, or until the feed is arrested. As soon, however, as the main belt pulls effectively the device is automatically disconnected. The two larger types derive the feed from the main drive. The quick return of the cutter slide takes place at about 4in. per second.

The indexing is driven independently of the cutter spindle and of the cutter slide feed, and provision is incorporated for preventing the slide from advancing until the indexing is completed. This is done by the use of an index wheel, which must make one, two, or four complete revolutions before the slide can advance. This wheel has a divided rim for adjustment for wear, and is bolted to the work spindle by means of a flange connection. The adjustment of the work slide upon the upright is effected by means of a hand wheel and screw, with ball-thrust collars, and a vernier permits of reading to .002in. A tailstock rest is provided for supporting the outer end of the work arbor, and there is also a rim rest to receive the thrust close to the tooth space being milled.

One of the machines is arranged for hobbing as an alternative to the use of a disc cutter, this being advantageous for milling narrow spur gears with many teeth—where the reversals would be so frequent—and also for roughing-out purposes. The cutter slide is modified then as shown in Fig. 107, to swivel to 5° in either direction, being adjusted by means of a scale and vernier. The rates of feed when hobbing vary from 0.013in. to 0.079in. per revolution of work.

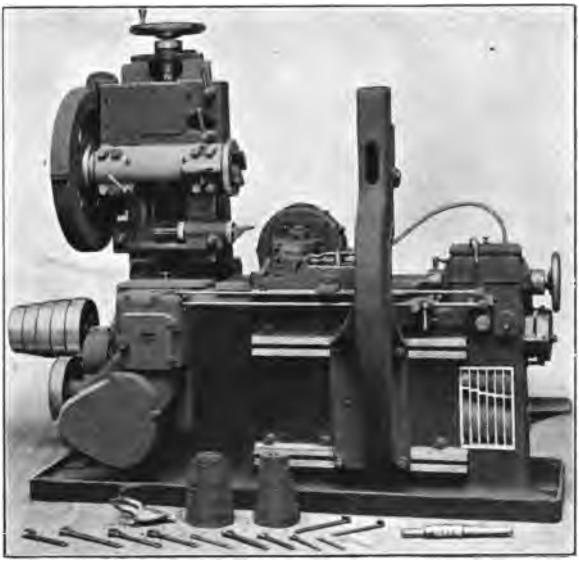


Fig. 104.—Loewe Automatic Spur-gear Cutting Machine (Front View).

In ordinary cutting, the feed of the cutter slide may be varied from ¾in. to 4¾in. per minute.

Automatic Machines by Messrs Darling & Sellers, Limited, of Keighley.—This firm makes a series of automatic spur-gear cutting machines in which the work spindles are set vertically, differing thus from the usual horizontal construction. The advantages claimed for this arrangement are, that the cuttings fall away from the cutter instead of falling on and clogging it; the work is always in balance, so that the risk of straining the arbor is lessened; and that it is easier to set and to remove the work by a simple vertical movement. The objection that in this design the dividing wheel is more liable to receive cuttings on it is met by completely encasing the wheel. These machines are made in various sizes up to 6ft. 6in. capacity. A 48in. by 12in. machine is selected for illustration, with general views and details.

Fig. 108 gives a side, and Fig. 109 an end elevation. The work arbor A is set in the spindle B, which passes down through the saddle C, fitting the bed by square edges, and carrying at the bottom the divided casing that encircles the worm wheel. E is the cutter slide, balanced, which moves downwards for feed and upwards for quick return.

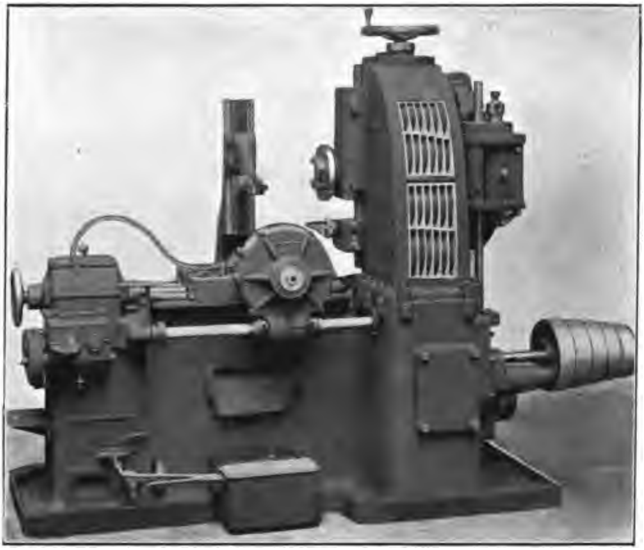


Fig. 105.—Loewe Automatic Spur-gear Cutting Machine (Rear View).

Driving.—The drive is taken first from the pulley F, belted from the countershaft. F drives through spiral gears a vertical splined shaft G, which passes through a bracket on the cutter slide E. This bracket is shown enlarged in Fig. 110. Here G turns a gear *a*, fitting by two long keys, and *a* engages constantly with another gear *b*. On the upper end of the shaft *b* is a stud with a spring plunger, enabling different change gears to be put on to engage with a gear on the top end of the shaft carrying the spiral pinion *c*, with 9 teeth.

By altering these change gears, six different speeds can be imparted to the cutter spindle. *c* engages with the 36-toothed spiral wheel *d* on the end of the cutter spindle, and the boss of *d* fits in a gland-like bearing *e*, in which it may slide endwise. This is to allow of the slight lateral adjustment of the spindle and its bearing, which is provided for adjusting the cutter exactly central with the work. *f* is a cover for the hand hole in the casing surrounding *d*, and *g* is a hinged guard covering the change gears just mentioned. Instead of putting the cutter direct on the spindle, separate arbors are employed, so that any size of cutter can be put on without regard to the size of its hole. The arbor is lettered H in Fig. 109, and its outward bearing, J. The latter is adjustable longitudinally, and is secured by bolts fitting in tee-slots on the side. A pointer K is pivoted above the arbor, and is brought down when required for setting the cutter true by its centre line, the end of the pointer being central with the axis of the spindle B.

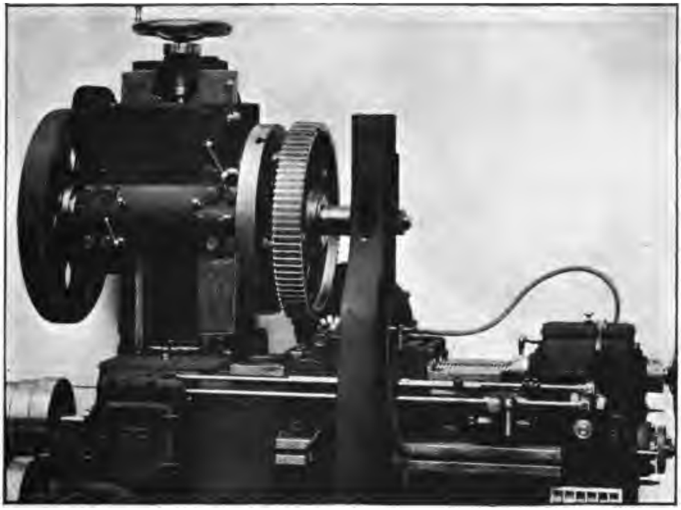


Fig. 106.—Loewe Machine Illustrating Use of Steady.

The Cutter Slide.—Square lips, with a narrow guiding ledge, are employed for mounting the slide E. It is fed by a spiral rack and pinion driven primarily from the pulley F, which operates spur gears (not shown), and thence a wheel *j* driving a pinion *h* (Fig. 108). The latter operates change gears in the box L, which has a hinged door so that different gears may be put on, a range of nine speeds being obtainable. From the box L a worm gear just above it transmits the motion at right angles, and it is again changed by another set of worm gear to the rack pinion on shaft M. A hand wheel N, the rim of which is seen dotted in end elevation, actuates worm gear for turning M, when adjusting the cutter slide. A trip lever N permits the hand feed to be disengaged when not required. The tripping of the automatic down feed is effected by the dog O, bolted on to a stepped surface at the side of the slide to prevent risk of the dog slipping, adjustments being finally made by the screw locked with a nut. This screw strikes the lever P, which trips a clutch, so throwing out the drive to shaft M. Q is a lever coupled to P for throwing out the clutch by hand. When the feed has been tripped the slide E is free to rise, and it does so rapidly under the pull of the counterweight attached to the chain which is seen passing around the pulley at the top of the frame. In the buffer bracket at the top of the standard is a spring which takes the shock on the return of the cutter slide. On the stop screw R is an adjustable nut which

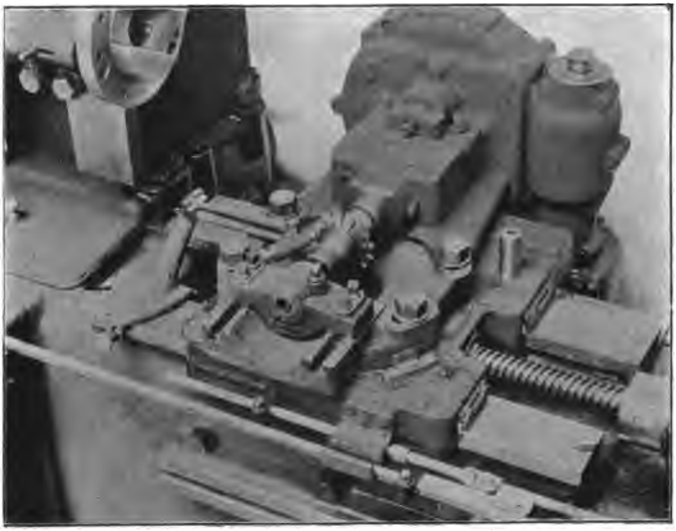


Fig. 107.—Cutter Slide Modified for Hobbing.

can be locked in any desired position, and which comes into contact with the hoop which compresses the spring. The air cylinder (not shown) is regulated by a slide valve on the front side of the machine. It will be noted that this method of returning the slide obviates the necessity for a good many quick-return gears and other details.

Indexing.—At the rise of the slide the indexing motion is put into action through a trip bracket S at the terminal portion of the ascent. This operates a lever, releasing a catch U, which holds a disc V that is attached to a top shaft passing through the machine. This shaft is thus allowed to make one revolution, a block on a wheel *t* stopping it after one turn, and at the completion a dog or piece attached to V strikes the lever U, lifts the ball lever T, and re-engages the feed clutch.

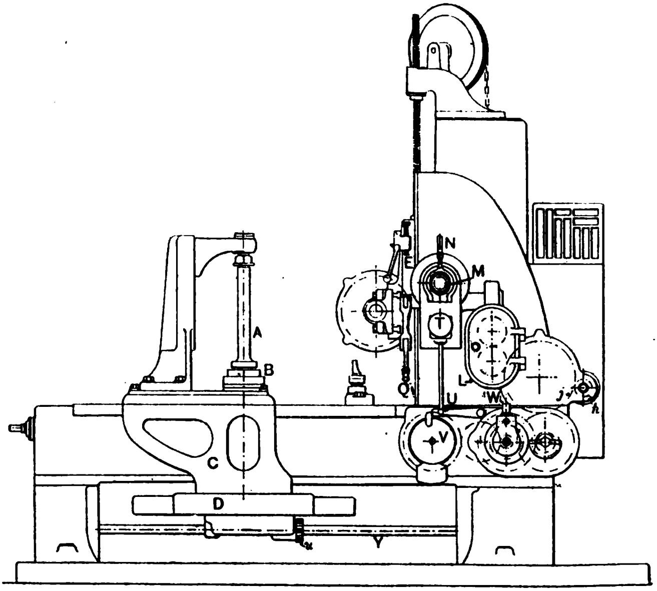


Fig. 108.

The stop shaft is rotated by a series of spur gears actuated from *j*, at either of two rates. Figs. 111 and 112 give an enlarged view of the details, the latter figure showing the outer bearings for the shaft and studs. The spur gear *k* is driven by *j*, and it rotates gears *l* and *m* through friction washers tightened up by the two adjusting nuts. Gears *l* and *m* remain stationary until the disc V is freed, when the friction washers commence to drive. *l* is in mesh with a gear *o*, and *m* with a larger one *p*. *o* and *p* run loosely on a bush, but a sliding key moved by the collar *q*, operated by the handle W (Fig. 108), causes either of them to turn the bush, on the end of which there is a pinion engaging with a wheel *s* (Fig. 111), driving *t* on the top shaft. As previously mentioned, the latter passes through the frame, and then transmits by mitre gears to a box X (Fig. 109), in which suitable change gears are arranged for the indexing required. From X a shaft Y passes alongside the machine to actuate the worm of the dividing wheel in the case D. A splined gear *u* encircling Y engages with a gear *v* (*see* Fig. 113) held on the shaft of the worm *w* by a split boss. Alongside *v* is a hand wheel for turning the worm by hand for adjusting purposes, and on the other side of *v* a micrometer dial with 200 divisions is keyed.

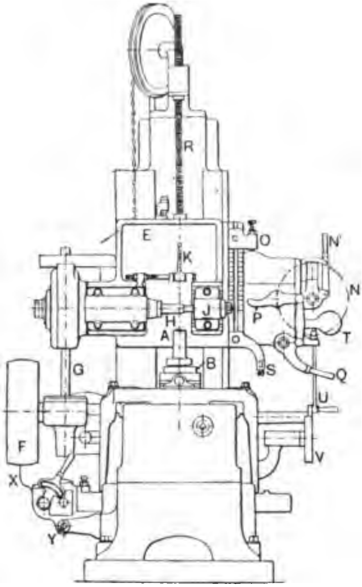


Fig. 109.

The worm may be withdrawn rapidly from mesh with the wheel by means of a lever Z, fixed on a spindle fitted with eccentric bushes. The spindle passes through slotted lugs in the casing which forms the bearing for the shaft of *w*, and the effect of moving Z is to draw the casing and worm backwards, thus leaving the worm wheel free for spinning the blank.

It will be seen from Fig. 113 that the worm wheel has a divided rim bolted together, so that adjustment can be made for wear. In this figure is also shown the provision for steadying the wheel by a set screw above, and a sliding block *x* below. *y*, *y* are fibre shields which go on the ends of the worm bracket and maintain a dust-proof joint.

Minor Details.—Referring again to the general view (Fig. 108), three points may be noted. The saddle is moved along the bed by a screw, turned by a squared end which is seen projecting, with a micrometer dial attached. A steady bracket supports the top of the arbor, and is applicable for pinions and wheels of moderate diameter. Another stay is provided for wheels of larger diameter. This steady is seen in the side elevation, as is also a rim support (on the bed close under the cutter slide) adjustable to bear under the rim of a wheel, close to the teeth, to counteract the downward pressure when cutting. Provision is made for lubrication by a pump (not seen), the machine standing in a tray. Change gears are stored in the pigeon-holes in the top of the cupboard at the back of the column.

The machine will cut up to 2 diametral pitch and 1½in. circular pitch. There are 42 change wheels, which will divide all numbers up to 100, and all, excepting prime numbers from 100 to 240.

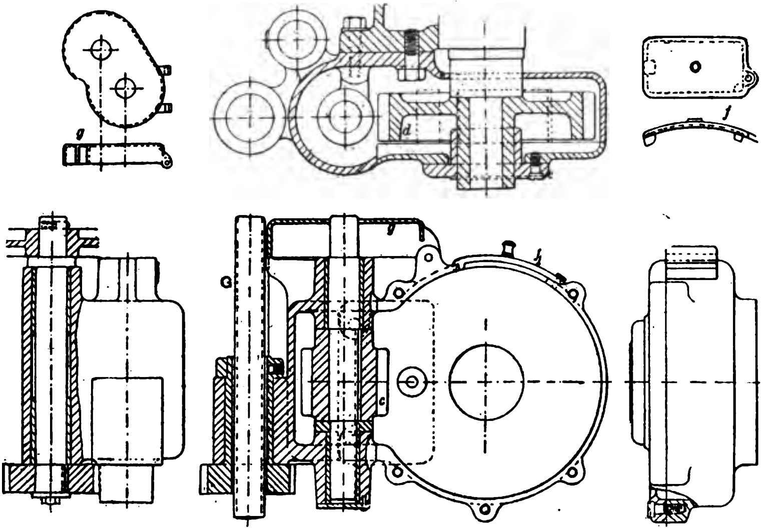


Fig. 110.

Operation.—In operating the machine, the following points have to be observed, the sequence in setting up for cutting being as follows:—

First, the worm is withdrawn from the dividing wheel by moving the lever Z, so that no strain may be brought on the teeth of the dividing wheel when tightening the work arbor. Then the socket B carrying the work arbor A is locked by a plunger pin, seen in the end elevation (Fig. 109). Care must be taken to see that the arbor and its hole are both perfectly clean, and the arbor is then inserted and the nut underneath tightened sufficiently to hold it from turning.

The gear blank is now placed in position, noting that the collar and packings for the arbor are free from burrs, and the top nut of the arbor is tightened down.

The cutter is set centrally by the pointer, similar precautions regarding the packings being observed, so that it shall run perfectly true on the arbor.

When the blank has been properly secured on its arbor the rim rest is brought up into contact lightly, using the fingers only for adjusting its nut, and it is then locked. Revolve the blank by hand to test its truth, both edgewise, by bearing on the rest all round, and over the periphery. If it runs out, the cause may be burrs, dirt, defective bushings and collars, bent or untrue arbors, etc. The worm is now re-engaged with the dividing wheel, care being taken

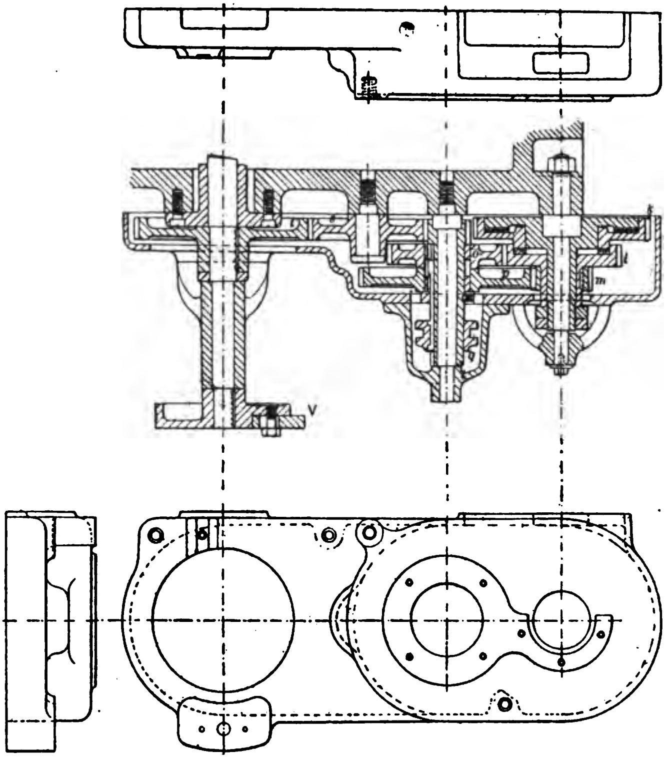


Fig. 111.

to operate the handle gently until the wheel teeth have entered the threads of the worm, the latter being then lightly pressed home and locked. The stop is made adjustable in order that the exact degree of engagement may always be ensured between worm and wheel, there being no play between the teeth, and, on the other hand, no wedging.

To set the cutter slide, the saddle C is moved forward until the cutter just clears the blank, and the slide is brought down until the centre of the cutter arbor is just below the under side of the work. The stop bracket O is set and locked in the groove of the cutter slide above the rocking lever P. The screw in bracket O is now adjusted to bear on P when the latter is in its lowest position. Next the cutter slide is run up clear of the blank, and the saddle is set for depth of cut (as explained in the next paragraph) and the cutter slide is adjusted until the cutter nicely clears the work. The stop nut on screw R may now be put into contact with the top buffer, and locked, and the small bracket S carrying the side tappet is set, say, ½in. above the plunger lever, which it trips during the terminal portion of its ascent.

The setting for depth of tooth is performed as follows:—Bring the saddle forward until the blank is just grazed by the revolving cutter, release the micrometer dial on the end of the saddle screw, set it to zero, and tighten again. Run the cutter slide up to clear, and set in the depth required in revolutions and fractions of a revolution of saddle screw, the micrometer indicating thousandths.

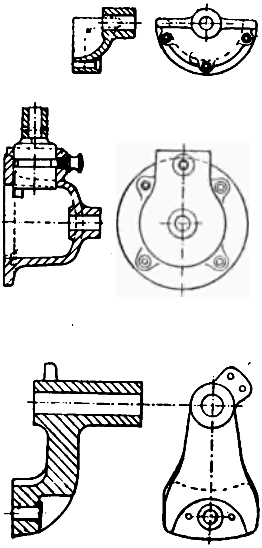


Fig. 112.

Adjustment of the air valve is made by means of a small sliding bar projecting from the bed underneath the spacing gear, and on this being moved in and out it regulates the elasticity of the air cushion to which the balance weight for the cutter slide descends. Should the slide appear to hesitate during its ascent, open the valve very slightly; or if, on the other hand, the return of the slide is so rapid as to end with too great a shock, close the valve until the slide is brought gently to a standstill.

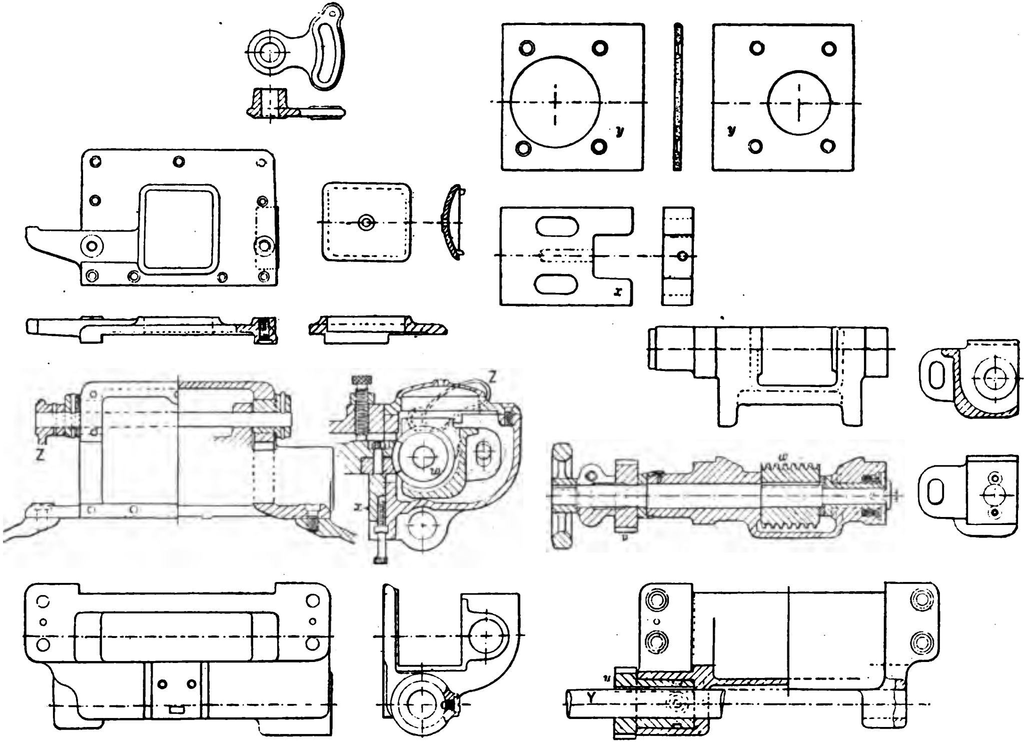


Fig. 113.

Change Gears.—The change wheels for the dividing motion are selected from the blueprint which is supplied with the machine, and are placed on their respective studs, care being taken not to engage them too deeply. The spindle-drive change wheels are also obtained from a table, which gives the number of spindle revolutions per minute for each combination of wheels. The feed-drive change wheels are given with their respective rates in terms of revolutions of the spindle per inch, on a brass plate attached to the cover of the feed-gear box.

When cutting large gears it is advisable to arrange a face plate drive for the work rather than rely solely upon the friction of the collars and work-arbor nut, besides which the face plate steadies the rim of the wheel. A face plate of the usual lathe type mounted on and secured to the collar of the arbor socket is all that is necessary, fitted with loose dogs or other drivers of the form best suited to the work. An opening is left in the boss of the face plate to permit the locking plunger on the top of the saddle to engage with the socket.

Until the operator has become thoroughly conversant with the working it is advisable, after all adjustments have been made, to allow the machine to go through its operations for two or three teeth without touching the work, in order to make sure that the stroke, clearances, etc., have been properly allowed for. Also, in cases where the blank to be operated upon is of a costly nature, it is as well to guard against mistakes in mounting the change wheels by letting the cutter just mark the rim of the blank through a complete revolution if it is to have a few teeth, or for a sufficient number of teeth to determine the accuracy of the pitch in the case of larger wheels. For this operation the hand motion to the cutter slide will be used, first bringing down the slide until the cutter marks the rim, and then returning it to the top for the tappet to strike the plunger for the spacing motion.

The object of the two-speed dividing motion (Fig. 111) is to save time in dividing, and reduce the strain on the change wheels. The handle by which the change is effected is set to readings "40 or under," or to "41 or over," according to the number of teeth to be cut.

The friction drive for the spacing motion is so adjusted that very little or anything should be required in the way of adjustment for some time; but if, on starting, it appears to slip at all, first examine the shaft bearings and meshing of dividing wheels to see that all is perfectly easy and free to revolve; and then, if still inclined to slip, tighten very slightly the two check nuts provided and oil the friction faces through the hole provided for the purpose. The friction plate should feel warm to the touch after running for a short time.

The micrometer adjustment on the shaft of the worm *w* (Fig. 113) is only used for a special purpose—that of touching round the teeth of a wheel on one side only. This is rarely called for; but if it should be, the set screw which clamps the partly split spur gear *v* (thereby disconnecting it from the spindle) and the worm is rotated by the hand wheel alongside *v*, until the work is found to be offset sufficiently. Then re-tighten the set screw and proceed with the spacing as usual. On no account must the mechanism be restarted before the clamping screw has been tightened properly.

The machine as stated is intended to cut gears up to 1½in. circular pitch or 2 diametral pitch. For 2½ diametral pitch and coarser in cast iron, and 4 diametral pitch and coarser in steel, a stocking cutter should be used to remove the bulk of the material. Heavier pitches than those first named can, of course, be dealt with by taking a series of cuts.

The Gould & Eberhardt Machine.—A 72in. by 18in. automatic spur-gear cutting machine (Fig. 114) is the latest representative of a type which has been built for many years past. It is seen in rear view to illustrate the motor drive. It will be noticed that the central type of construction is followed, the work spindle passing through the hollow upright, instead of lying to one side of it in an overhanging position. The cutter-slide feed screw is also located directly behind the cutter in a central position. The drive is furnished by the constant-speed motor, or alternatively by a constant-speed pulley, and connects through gears to the cutter spindle, finally driving by worm and bronze worm wheel. The range of speeds proceeds in geometrical progression. The cutter slide has also a geometrical range of feeds, and a uniform rate of quick return independent of these. The various automatic movements are connected so that nothing can start working to the detriment of the machine, cutter, or work until the prescribed motion has been completed; thus the cutter slide cannot commence its travel until the dividing is fully accomplished, nor can it commence to return before its allotted stroke is finished, and it must be back in position ready for the next cut before the gear indexes for another tooth space.

The index wheel is of the split rim type, with hobbed teeth, and the final finishing is done with the wheel in position on the machine. After it has been hobbed to depth, the split ring is unbolted and shifted half-way round on the wheel, and the teeth are matched up to locate any error. The error is then divided and the wheel again hobbed. This operation of splitting the error and rehobbing is repeated until the error is eliminated.

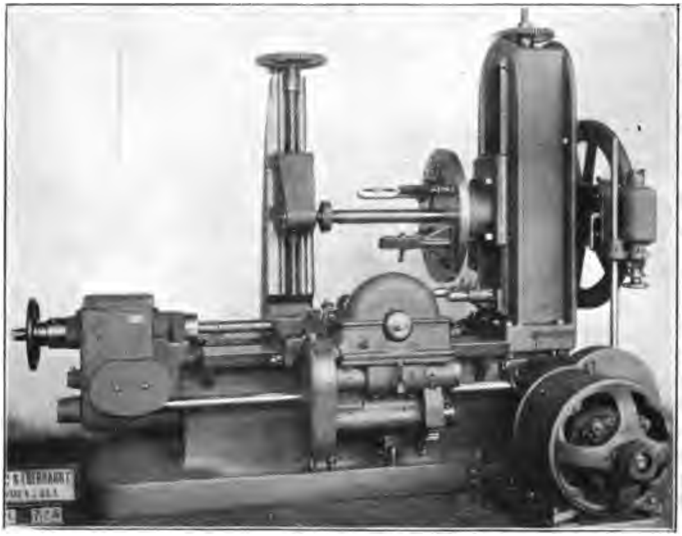


Fig. 114.—Gould & Eberhardt Automatic Spur-gear Cutting Machine.

A slight tension is constantly maintained between the stop cam and worm, in the direction of rotation, which prevents all danger of inaccuracy through backlash. The dividing mechanism is so arranged that the gear blank can be spun for testing, or turned round for other purposes, and it is also possible to index for two or more tooth spaces, if the Eberhardt multiple cutters (*see* Fig. 68, p. 85), finishing more than one tooth space, should be in use.

A face plate with supporting jacks, and two arms or dogs with bolts, are fixed on the spindle nose, and a rim rest is attached to the upright to take the thrust of the cutter. The firm supply a special rim clamp which holds the rims of large gear blanks as in a vice, to prevent them from creeping under the cut.

The outer arbor support is held upon a tee-slotted bracket by bolts, and is adjusted readily for different lengths of mandrels or for temporary shifting to remove or replace work. Verniers are provided to accurately adjust the bearing head in alignment with the main spindle.

Quill Gears.—All ordinary spur gears or pinions can be readily cut upon the regular machines without trouble, but in the case of quill gears, where there is a wheel and a pinion, in one with a sleeve it is not always practicable to cut the pinion teeth, on account of the wheel fouling the slide when brought down sufficiently low for the cutter to reach the pinion. An attachment is consequently used with a supplementary cutter spindle raised above the main one on the Brown & Sharpe machines (Fig. 115), and driven by a train of gears.

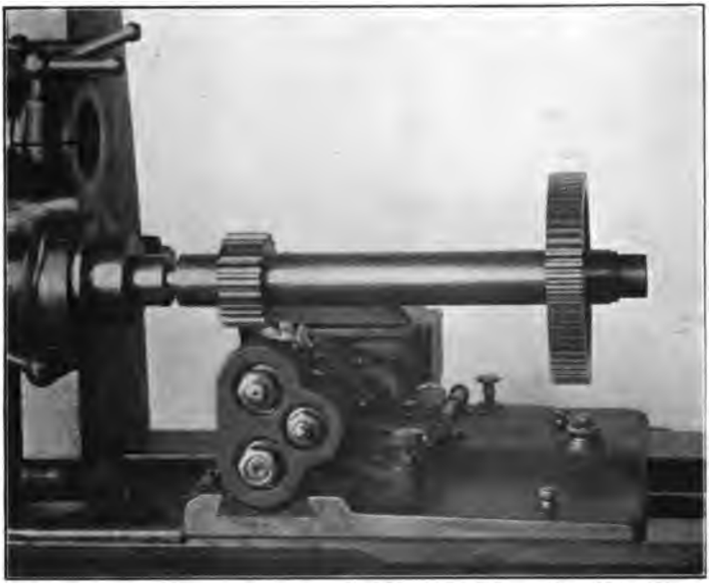


Fig. 115.—Quill Gear being cut on a Brown & Sharpe Machine.

Cutting on Milling Machine.—The employment of a large attachment on a Brown & Sharpe universal milling machine is seen in

Fig. 116 representing a magnified type of index centres, with a slotted extension above the headstock bearing carrying a screw with adjusting nuts to take the pressure of the rim during cutting. The spur gear being cut is a fairly large one for the size of machine it is done upon. An alternative method sometimes adopted when large gears have to be cut on a milling machine is to let the cutter work underneath, and raise the index centres up on blockings so that the cutter spindle passes across just above the table and the gear stands above, the capacity depending upon the range of extension afforded by the raising blocks.

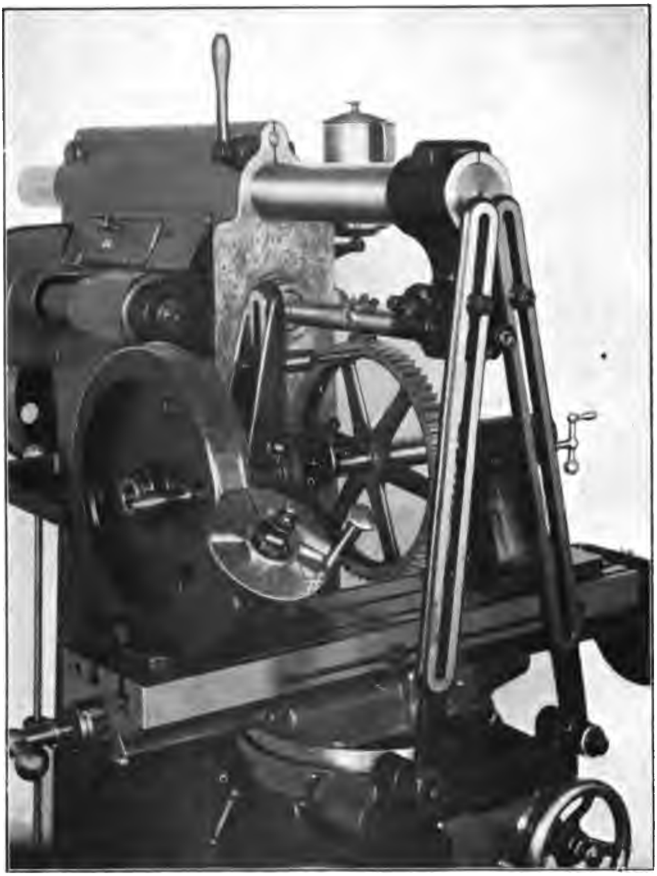


Fig. 116.—Cutting a Large Gear on a Brown & Sharpe Milling Machine.

Rack-cutting Machines.—These include some of large and complicated types. Racks are frequently cut on milling machines and on universal gear cutters. Rotary disc cutters, single or multiple, are used. But such machines are not fully automatic, nor are they designed for heavy duty, or suitable for dealing with quantities. For these reasons special machines are built, fully automatic and of heavy design, in two broad types. One may be termed of the pillar-and-knee milling-machine type, the other the planer type. The first deals with the shorter, lighter racks; the second with the heavier ones. One of each class, by the Walcott & Wood Machine Tool Company, of Jackson, Mich., U.S.A., is shown by the accompanying drawings.

Figs. 117 and 118 show the first type of machine in side and front elevations respectively. The framing comprises a column and top housing. The knee A is carried on the front of the first, and the bearings for the cutter arbor B in the second.

Cutter Arbor.—The cutter arbor is driven from both ends, as seen in Fig. 118. A three-stepped back-geared cone C, belted from the cone D on the countershaft, drives the arbor through gears at both ends, one set *a* being of double-helical form, the other set *b* comprising common spur gears, the object being to eliminate torsion and vibration. The three cutter speeds are changed at the countershaft to suit work done in ordinary, or in high-speed steels. In the No. 2 machine shown they are, for ordinary steel, 14, 20, and 30 revs. per min., and for high-speed, 19, 28 and 41 revs.

The Work.—The rack to be cut is bolted on the table E, the range of movement of which in this particular machine suffices for racks up to 36in. long and 8in. width of face. This, of course, permits of cutting narrower racks laid side by side up to that total width. The table E has a traverse motion (for pitching) on the cross slide F, and the latter has a transverse movement on the knee A for feed of cut, both being automatically produced. The knee has the usual elevating screw, operated by a handle placed at *c*. The indexing for pitch is independent of the speed of the cutter arbor, the drive being obtained directly from a small pulley on the countershaft to the large pulley (G, Fig. 118). To accommodate the tension of the belt to the varying height of the apron, the idler pulleys *d, d* are provided. The indexing drive from G to the table E takes place through spur gears, change gears, bevels, and spurs to the end of the lead or index

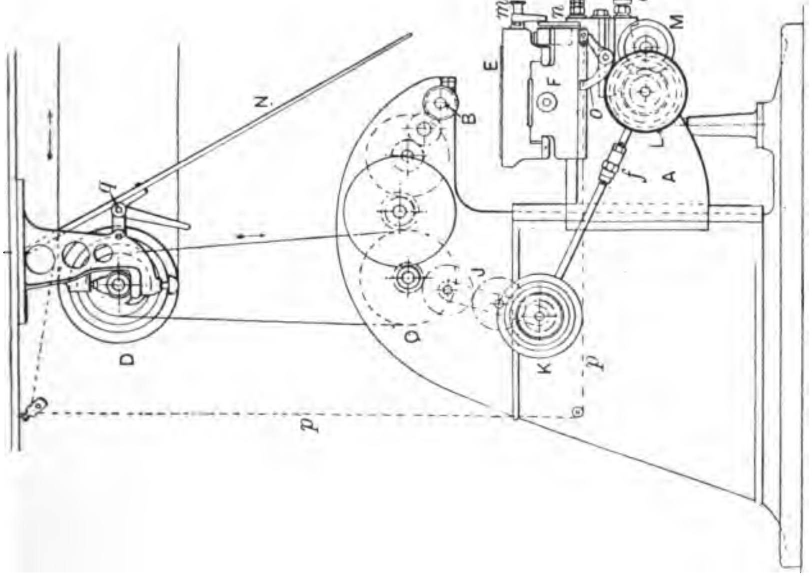


Fig. 117.

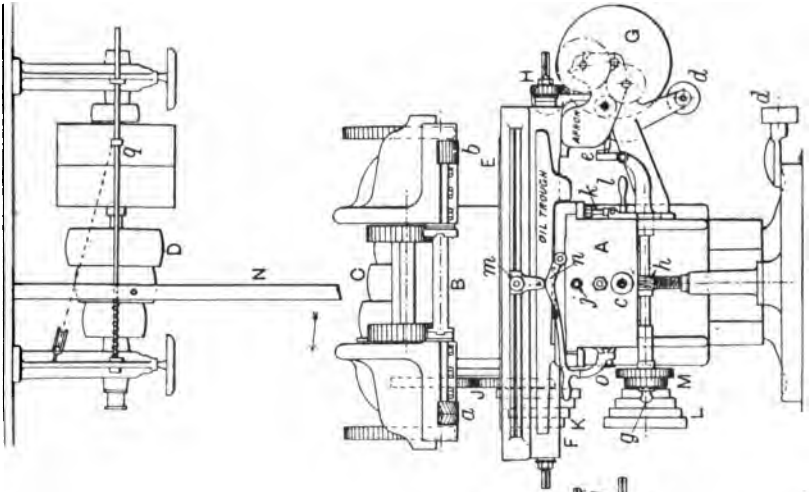


Fig. 118.

screw at H, on which the last change gear is seen. The collar of the screw is indexed to read to $\frac{1}{1000}$in. Positive clutches take charge of the indexing. The trip-dog for indexing is seen at *e*. Multiple indexing can be done through change gears. Only the outlines of these features are indicated in the drawings. The feed for cutting is actuated from cone pulleys K and L, and gears at the left hand of the machine, eight feeds being provided for each speed of the cone which drives the cutter arbor. They range in this machine from 0.037 to 0.43in. per rev. of cutter. For racks of 1 diametral pitch, which is the limit on this machine, a roughing and a finishing cutter are used. For finer pitches, gang cutters are used.

Feed Drive.—The drive to the stepped feed cones takes place from the shaft of the cones C, through spur gears J, to the upper feed cone K belted to the lower cone L. As the centres of these cones vary with the movements of the knee, a belt-tightener *f* accommodates K thereto, which swings on a link connection. The drive is continued through pairs of spur gears M, which are put in and out of action by means of a sliding key, the knob of which is seen at *g*. The shaft which they drive, at eight distinct speeds, as stated, carries the feed worm *h*, whence, through gears not shown, the feed screw *j* is driven, imparting the transverse movement to the cross slide F. The feed is tripped by the dog *k*. There is a quick return, the lever of which is seen at *l*.

When a rack has been cut there is a provision made for automatically stopping the machine. A dog *m* is clamped in a suitable position at the edge of the table. When it strikes the lever *n* this trips the dog *o*, which pulls the chain *p* coming from the belt-shipper bar *q* on the countershaft, so pulling the belt on to the loose pulley. N is the lever for operating the shipper bar by hand. Machines of this type are made in five separate capacities.

The Planer Type of Machine.—This is illustrated by Figs. 119 to 123. The details are not fully shown, but enough are given to permit the design and method of operation to be understood. The cutter arbor is driven from the cone pulley A at the base of the machine, but the indexing is done directly from the countershaft to the pulley B at a constant speed which is independent of the speed of the cutter arbor. It is also wholly automatic. There are

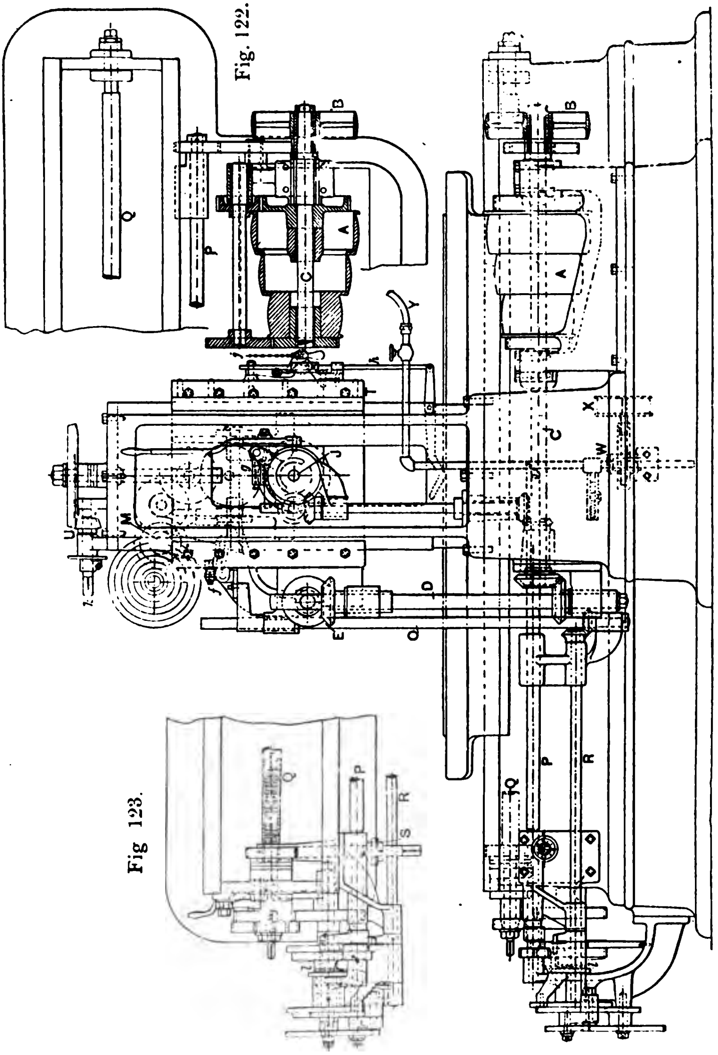


Fig. 122.

Fig. 119.

Fig 123.

eight rates of feed, each of which is available for each speed of the cutter arbor.

Cutter Drive.—Taking the cutter drive first—it is transmitted from the cone pulley A (Fig. 119), through the shaft C, through bevel gears, to the main driving shaft D; thence through mitres E to the cross shaft F. Shaft F carries a splined bevel gear *a* (Fig. 121) which drives the larger gear *b*, on the shaft of which is a small spur gear driving through a train to the cutter arbor G (compare Figs. 120 and 121). The arbor is carried in the bracket seen attached to the saddle H. It is rotated at the three speeds corresponding with the steps of the cone A, and countershaft drive. For ordinary steel they are 13, 19 and 30 revs. per min. For high-speed steel they are 17.5, 27 and 41 revs. The eight different feeds to the saddle H carrying the cutter arbor are imparted to the screw J from a pair of cone pulleys, and a rather formidable-looking nest of gears on the upper cone-feed shaft, thus:—

Feed Drive.—On the other end of shaft F which is driven by the mitres E, is a four-stepped cone pulley K which drives to the pulley L above. The four feed changes thus obtained are doubled, and reversed, with a quick return by the nests of gears seen in Figs. 119, 120 and 121, which need not all be separately lettered, as their functions can be studied in the views. They drive down to the feed screw J through spurs and mitres. The four speeds are doubled at the gears *c* (Fig. 121) by means of a sliding knob. The nest of transmission and quick-return gears, comprising spurs and mitres, are actuated from the shaft M. The clutches are seen at *d*, *e*, and the quick-return shaft at N, having the bevel gears C′ at its opposite ends driving back to the shaft C. The clutches are operated by a trip arrangement, the shaft of which is seen at O (Fig. 119); and the latches, studs, dogs, pin and yoke are indicated in outlines at *f* and *g*, for producing the rotation of the feed screw J (Fig. 119).

The indexing is commenced from the pulley B. The method of its transmission to the shaft P is seen in Figs. 119 and 122, being taken through gears. From P, motion is carried to the feed screw Q through the nest of change gears seen to the left of Fig. 123. The feed screw has an acme thread. The feed is tripped by the shaft R, which is actuated by the trip of shaft O, just now noticed. It is thrown out of action by the clutch *l*. The amount of the indexing is indicated on the ring S, which is provided with a friction sleeve.

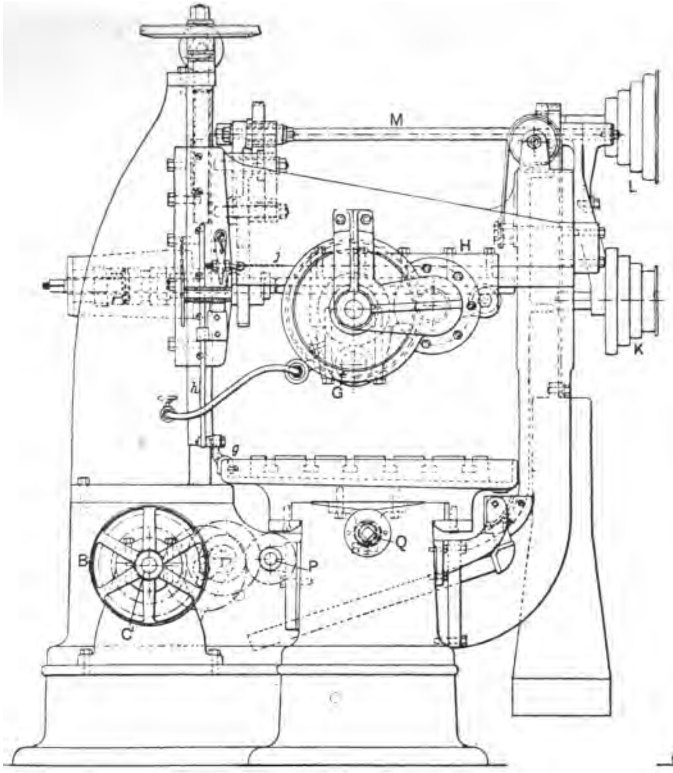


Fig. 120.

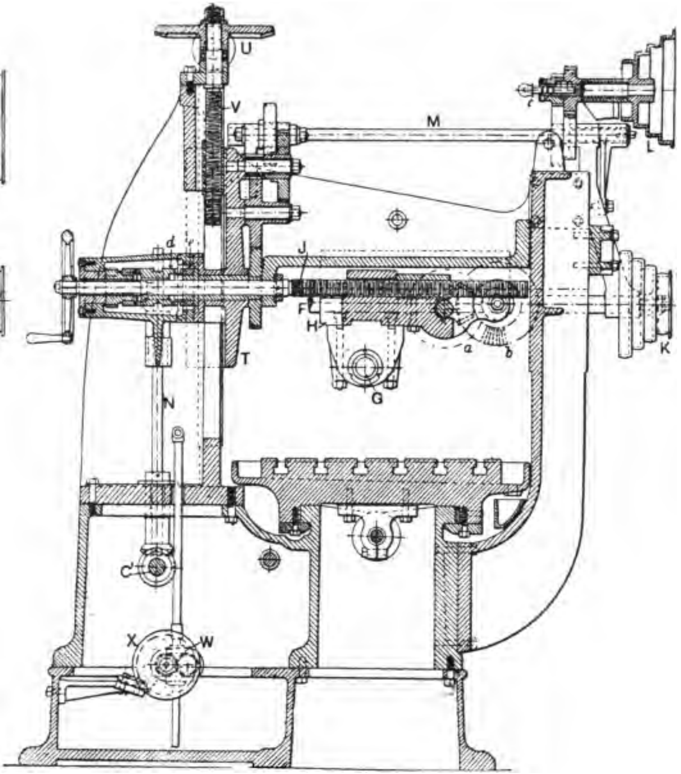


Fig. 121.

When a rack is completed the machine is stopped automatically by a dog *g* (Fig. 120) striking a lever which actuates a rod *h* (Figs. 119 and 120), the movement of this tripping the chain *j*, which pulls over the belt-shipper rod on the counter, so pulling the belt on to the loose pulley.

The elevation of the knee or slide T which carries the cross rail along which the saddle H is fed, is effected by gears U and a screw V. These are hand actuated from the handle *k*.

Provision is made for forced lubrication. A pump W is driven by the pulley X from the countershaft, and a flexible pipe Y conveys the lubricant to the cutter, whence it returns to a tank in the base.

The capacity of the machine shown is for racks up to 96in. long, and 10in. width of face, and to 1 diametral pitch. The table is 108in. long by 24in. wide, and the machine weighs about 5½ tons. It is therefore rather a massive example of a specialized machine.

Rack-cutting Attachments.—Short racks may be cut in quantity by mounting a series of cutters upon an arbor in the milling machine, and holding a number of racks upon the table, the traverse being then in the long direction of the table. For the ordinary cutting, however, it is necessary to have a special attachment to drive the cutter upon an arbor at right angles to the main spindle. There is then an opportunity to get in a much longer rack, and it is only a question of indexing the table along between each cut. Either a single cutter is used, or two or three are mounted side by side, or a roughing cutter precedes a finishing one, so that a space is roughed out alongside one that is being finished. This is seen in the view of the attachment by J. Parkinson & Son (Fig. 124). There is space on the spindle to permit of several cutters being clamped together, to a total width of 3in., and as seen, the arbor has an outer bearing, a construction much preferable to the method of allowing the end to overhang. A pair of spiral gears transmits the drive from the main spindle, and the gears are hardened, with ball-thrust collars provided for each. The outer end of the attachment is supported by the bracket of the overhanging arm. A special long vice is employed to hold the blanks, and another special provision is the indexing device (Fig. 125), which saves time, and ensures accurate pitchings, obviating the bother of setting by the use of the micrometer dial of the table screw. The device consists of

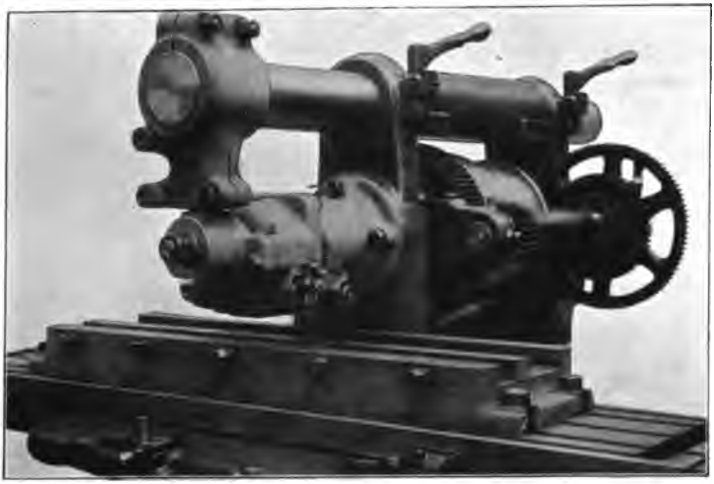


Fig. 124.—Parkinson Rack-cutting Attachment for Milling Machine.

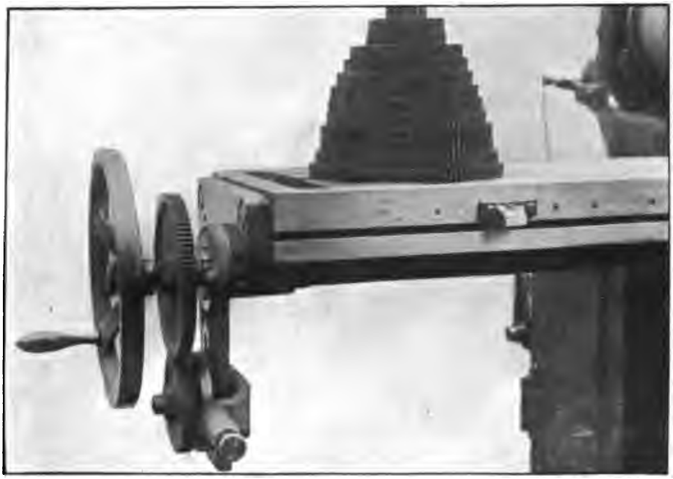


Fig. 125.—Indexing Device for Parkinson Attachment.

an index plate with a plunger, and sixteen change gears. The divisions are obtained by combinations of these gears, and the index plate is given a half turn, or a complete turn, before the locking plunger is allowed to slip into the notch. If a complete revolution of the plate is required, the notch which is not in use is closed, to prevent possibility of error resulting in false pitching. In a plain milling machine with rack feed the device is mounted at the front, and is connected to the shaft which traverses the table.

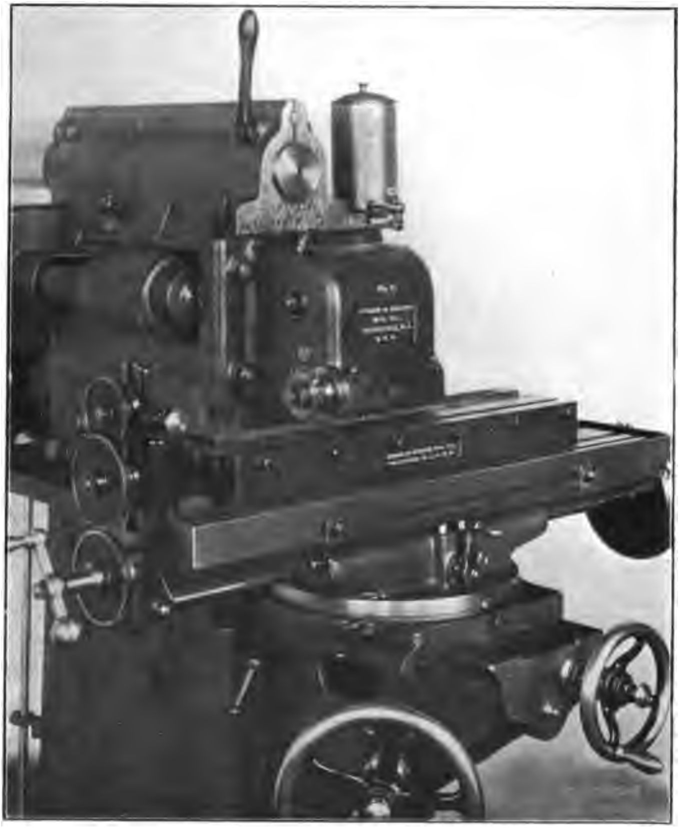


Fig. 126.—Rack-cutting Attachment to Brown & Sharpe Milling Machine.

Fig. 126 represents a Brown & Sharpe milling machine with the setting up of attachments and vice for rack cutting. The bracket is clamped to the column face, and contains bevel and spur gears to transmit the drive from the main spindle to that of the attachment. The indexing is performed with the change gears and slotted disc on the end of the table, the gears being provided either for English or for metric pitches as required.

Internal Gears.—It does not pay to use a large machine for regularly cutting small internal gears; a very small machine is quite suitable, such as is used for change-speed gears and the ratchets for cycle free wheels. Messrs A. A. Jones & Shipman, Limited, of Leicester, manufacture a special machine for this service (Figs. 127 to 130), which occupies a small compass. The operation is automatic, and several gear rings can be cut simultaneously. The cutting is performed by a slotting tool which is formed to suit the shape of the tooth required—spur, or ratchet, or otherwise. A cutting rate of 50ft. to 60ft. per min. is employed. In the drawings the tool is marked A, and is clamped in a horizontal slide B, which is reciprocated by a belt pulley C, acting through a pinion and a gear D, the latter being slotted to drive a connecting rod E, pivoted to the slide B.

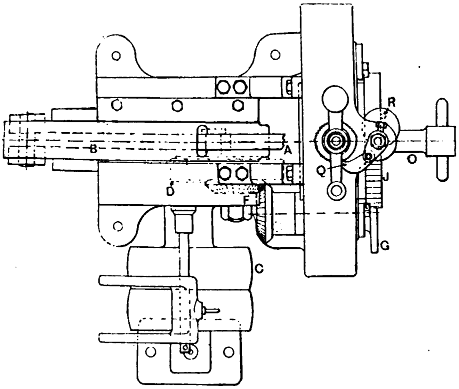


Fig. 127.

At each stroke of the tool, the gear or gears being cut are given a part rotation, equal to the pitch, and after one complete revolution a depth feed is imparted, this sequence being continued until the teeth are finished. The arrangements for pitching and for feeding are as follows :—The mitre gears F, driven from the shaft of gear D, rotate a cam G, and this operates a roller lever H, which is rocked over and pushes the ratchet wheel J. According to the number of teeth in J, which wheel can be changed to suit the work, the part rotation is given to suit the desired pitch. J is secured to a socket K, which turns within a slide L movable up and down within the upright frame. The gear blanks, of which three are seen (M, Fig. 130), are clamped against the inside of a bush

N by means of a screw and pad O, the screw fitting in a block P which is inserted and locked after the insertion of the blanks.

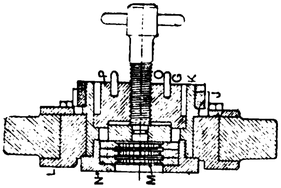


Fig. 130.

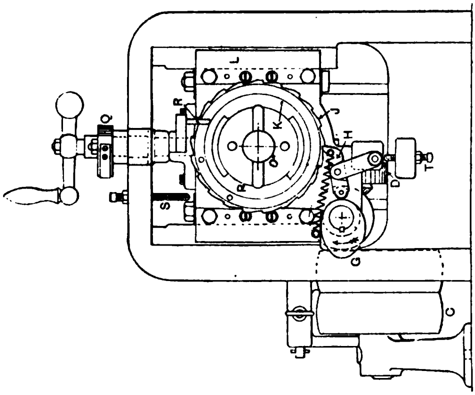


Fig. 129.

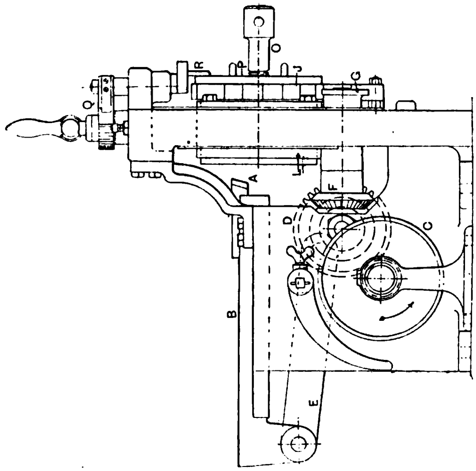


Fig. 128.

Downward feed for the depth of teeth is obtained through the ratchet Q, which feeds the vertical screw. The ratchet is rocked at each complete revolution of the wheel J by a pin standing out from its face, this pin pushing a vertical pin R outwards and so moving the ratchet pawl. A stop screw S (Fig. 129) controls the highest setting of the slide L, and another, T, the lowest setting. The latter serves to stop the machine when the full depth of cut has been reached, because the ratchet wheel at Q is fitted between fibre friction washers, and when the slide L abuts against the screw T these washers slip, and the machine ceases to operate. The capacity of the machine, which weighs 190lb., is for diameters up to 3¾in.

The internal attachment for the Loewe gear-cutting machines is seen in Fig. 131, comprising means for carrying the cutter at the end of a right-angled bracket which is bolted down on the cutter slide. A train of gears connects up within the bracket from the main cutter spindle to the spindle of the cutter on the nose of the bracket. In other respects there is no difference in the mode of operation. If the internal gear is not of such a form that the inside shoulder of the ring of teeth stands away sufficiently from the face plate for the nose of the attachment to have a clear run, it is necessary to pack out the gear.

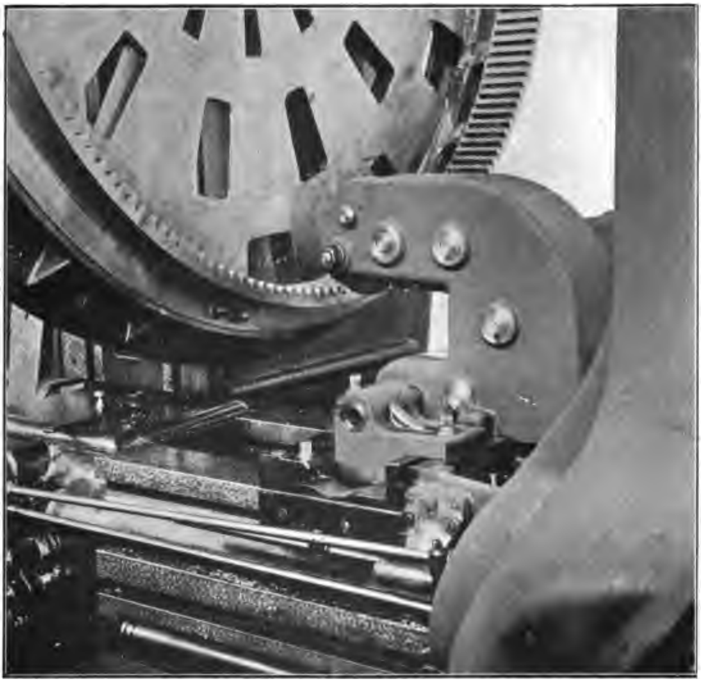


Fig. 131.—Internal Cutting Attachment to a Loewe Machine.

CHAPTER X

MACHINES USING FORM CUTTERS (*continued*)

Milling Worm Threads.—The employment of a milling cutter in place of a lathe tool for producing the threads of worms has extended with great rapidity during recent years, and many firms mill all their worms now. The chief advantage is of course the increased rate of cutting possessed by the multiple-tooth mill, while another feature is that the whole depth can usually be cut at one pass, while a lathe tool requires a number of traverses.

Fig. 132 shows a simple method of laying out the angle of the sides of a tool for planing out spaces between rack teeth. Upon a line A B draw a circle, and from B lay off B C and B D, each equal to one-quarter of the diameter of the circle. Then lines A C and A D, drawn as shown, are correct for the inclination of the sides of the rack tool, 29°. The end of the tool is made .31 of circular pitch, and then the corners are rounded to leave fillets at the bottom of the teeth. Thus, if the circular pitch of a rack is 1½in., and we multiply by .31, the product, .465in., will be the width of the tool at the end for a rack of this pitch before the corners are taken off. This width is shown at *x y*. The foregoing rule is also correct for a worm-thread tool, but sometimes the tool is not rounded for fillet. In cutting worms, the width of top of thread should be left .335 of the circular pitch, which will bring the depth of thread right.

Worm milling is done either in a plain or a universal milling machine, in a lathe with a rig-up on the slide rest, or in a regular worm miller, the latter being the most economical and suitable, particularly for large work. An attachment for a Garvin milling machine is seen in Fig. 133, from which the essentials for worm milling may be gathered. The cutter is necessarily carried by a swivelling head, so that it may be angled to the inclination of the threads. Suitable change gears are set up to connect the table screw to the end of the work spindle, so that the worm is rotated at a correct rate as the table carries it along past the cutter. The reason for the separation of the work spindle into two sections, each carried in its own bearing, is to provide for the cutting of multiple threads which is effected by a flange clutch situated between the bearings. The teeth of one part of this may be disengaged from those in the other, and re-engaged at another position, so as to turn the blank half, or quarter round, or to other settings as may be desired. The ordinary index head and tail stock of a milling machine may be used in place of these attachments shown, but the machine in this instance happens to be a plain and not a universal one, hence the provision of the special heads illustrated.

Worm-milling Machines.—In the regular worm-milling machines the work may have no longitudinal motion, while the cutter traverses, or the opposite may be the case, both designs being popular. Both types are illustrated in Figs. 134 and 135, a Holroyd, and a Pratt & Whitney machine respectively. The first machine has a sliding headstock which is fed by screw and change gears as its spindle is rotated, the work being supported in a bush in the fixed tailstock. The cutter head, which is sufficiently powerful to drive a cutter enabling a 1½in. pitch worm to be milled at one cut, is provided with the required adjustments for depth of cut and angular setting.

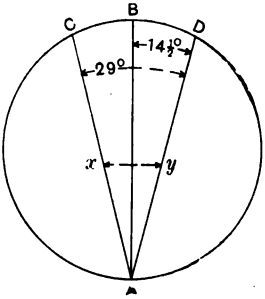


Fig 132.

In the Pratt & Whitney machine (Fig. 135), in which is shown a shaft having two sections of worm thread, the general model is that of a lathe, with saddle carrying the swivelling cutter head. The spindle of the latter receives its motion through the stepped cone and splined shaft running along the bed, connection being thence made through bevel gears and telescopic shaft up to the saddle, this provision being necessary on account of the changes in position of the cross slide according to the distance of the cutter from the centre. The headstock spindle also derives its drive from the cone pulley through a feed-gear box, and change gears are connected up to the saddle lead screw. The shape of cutter used for worm threads is shown by Fig. 136. A staggered type of cutter is, however, employed in some cases on the Pratt & Whitney machines, one side of each alternate tooth being absent, so as to leave ample clearance for the cuttings to wash out.

Spiral or Single-helical Gears.—Three regular methods of producing these are employed. (*a*) By planing, employing a tool having the same section as the tooth space; (*b*) by using the same single rotary cutters that are employed for spur gears, which in the smallest spiral gears is not an absolutely correct method; (*c*) by employing a hob like that used for generating spur gears, also not wholly accurate though much better than (*b*). In these last two, inaccuracy is due to the interference of the cutter with the sides of the teeth, resulting from the twisted form of the helix. This error can be minimized by reducing the diameter of the cutter, or the hob, which, however, lessens its wearing qualities. As, however, the interference cuts away more of the faces towards the points, this may be accepted as rather an advantage, tending to easier running, being similar to the slight corrective rounding which is given to gear teeth in general with the same object in view.

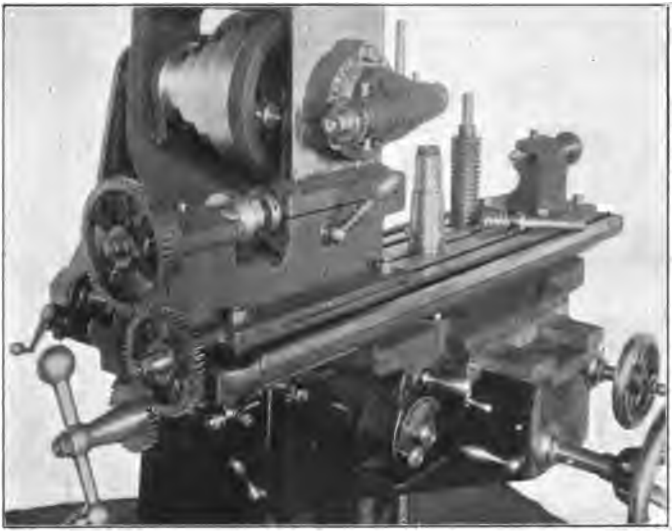


Fig. 133.—Worm Thread Milling Attachment to a Garvin Milling Machine.

The difference between a forming method and a generating method is: in the first the cutter must be of the same section as the tooth space which it forms; in the second it has the rack section, and the movements of the tool and the rotation of the blank must synchronize in order to produce the development of the tooth shapes.

Methods Adopted.—The differences in the methods of cutting single-helical gears which are open to selection entail the employment of machines of different kinds. When planing tools are employed the shaping class of machine is the most convenient to use, or a lathe geared to drive directly to the lead screw. Rotary disc cutters are adopted on universal milling machines, and on many gear-cutting machines of suitable design, and also on the thread-milling machines. Hobs are utilized on any of the numerous spur-gear hobbing machines, which are fitted also with gear and mechanism for producing the spiral movements, and also on the special Wüst machines. End axial cutters, like those in Fig. 70, p. 86, are occasionally used for small gears on universal milling machines. But they are employed almost exclusively on the special machines which are built by the firms who make a speciality of this work. These designs are constructed broadly on the pattern of the lathe, or on that of

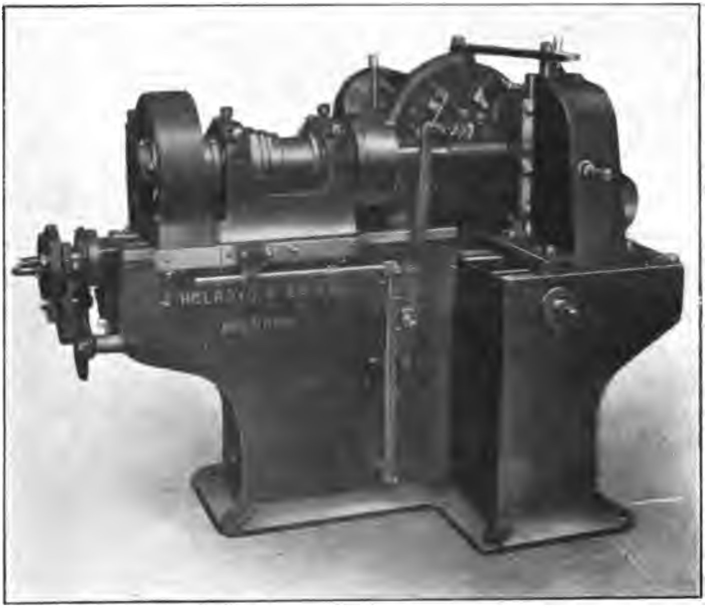


Fig. 134.—Holroyd Worm Thread Milling Machine.

horizontal drilling and boring machines, the resemblance to these being of course barely superficial.

Tools Used.—As in worm gears—which are also a particular form of true spiral gears—so in spirals, the rack might be supposed to be of a hard substance, which would mould correctly the teeth of all wheels in a softer substance to gear with it, the rack being moved over the cylindrical surface of the wheel blank. In practice the method of planing with a fixed cutter, or of milling with a rotary cutter, is adopted. Planing in this case is the equivalent of turning a worm in the lathe with a tool shaped to the section of the tooth space, and milling is done with a rotary cutter in a milling machine.

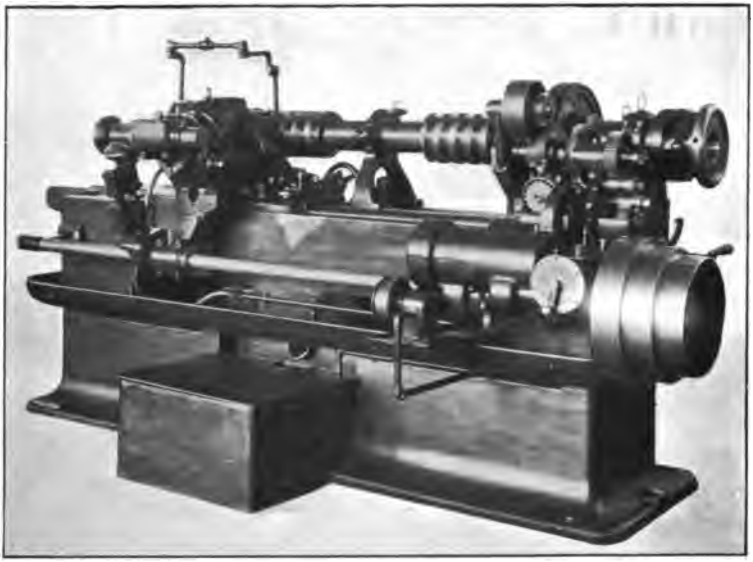


Fig. 135.—Pratt & Whitney Worm Thread Milling Machine.

The tool used for shaping the teeth by the planing operation must be of the section taken in the line of the normal pitch, and must be set at the angle corresponding with the normal. In strictness, the tool used for cutting a common single-threaded worm, made by the diagram in Fig. 132, should be so shaped and presented, but the difference in the normal and actual pitch is so slight that it is a matter of little importance in the case of the worm. In the spiral gears of high angle the distinction is of the greatest importance, affecting as it does the entire system of cutting.

The Essential Movements.—The advance of the blank for spiral gears corresponding with the lead of the primary helix is produced by change gears, the rotation of the blank by other gears, and these must bear exact relations to each other. As they vary in different combinations they have to be calculated and set up afresh, as in screw cutting. The indexing may be imparted by hand, or be set automatically. During the period in which the tool has cut through a tooth space, the blank will have been rotated through a fraction of a circle which bears the same proportion to a complete rotation that the length of spiral tooth bears to the complete lead. It is convenient to look at the subject from the familiar point of view of the screw-cutting lathe, cutting worms, which are spirals of very short lead. In fact, many spirals are cut in lathes and some machines built for cutting double-helical gears are designed after the lathe model. Here the lathe spindle imparts rotation, and from it change wheels impart traverse to the cutting tool, the combination of the two producing a screw or spiral outline on the blank. The movements are synchronized by the change gears, and remain unaffected by any changes in the speed of rotation, such as that which is caused by changing the belt for roughing or finishing, or by slip of the belt. If the worm is a single-threaded one no further arrangement is necessary. If it is double, or multiple threaded, corresponding with a helical gear, then some device is made use of for pitching the different threads. The old way, still suitable, is that of chalking the change gears in divisions of two or three, and starting the saddle from a definite point. Some lathes are instead fitted with thread-catching devices. But if instead of a worm, a spiral gear has to be cut, then a proper indexing device is employed. It may be the regular dividing head with index peg, which, however, is not a part of the equipment of the regular engineer's lathe. It may be a face plate slotted to receive a lathe dog, the number of slots corresponding with the number of teeth in the gear. But these devices are suggested rather as analogies than offered as workable commercial methods,

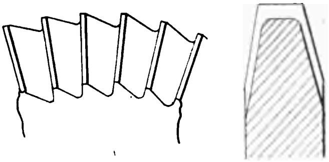


Fig. 136.

because spirals of long lead are much better done elsewhere than in the lathe, although the principles are identical. They can, however, be cut in those lathes which are provided with a direct drive to the lead screw, of which there are a goodly number built. The difference is that of driving back from the lead screw to the spindle, being a case of speeding-down instead of speeding-up. These principles of operation appear in any type of machine tool used for cutting spirals, some in a rather makeshift fashion, in others in a legitimate and quite satisfactory manner.

The setting-up for cutting a spiral gear on a Brown & Sharpe universal milling machine is illustrated by Fig. 137, showing the swivel of the table to suit the angle of helix, and the setting up of the change gears to connect to the table screw. As each tooth space is milled through, the next one is pitched by the index plate and so on.

The use of a special attachment on a Cincinnati machine is seen in Fig. 138, the cutter head being arranged with a swivel fitting. Gears of any angle, from a spur gear to a rack, may be dealt with thus. In the illustration a 3-pitch steel gear, 3½in. diameter, is being cut at a rate of feed past the cutter of 1in. per minute.

Planing Spiral Gears.—These may be planed in a shaper. In a Gould & Eberhardt design additional mechanism is fitted, consisting of a long table bolted to the knee, and carrying the blank between centres, driven by a headstock bolted to the table, that contains the mechanism through which the movements of the blank are controlled, responsive to the reciprocating movements of the ram which holds the cutting tool. These movements are those of a helix corresponding with that of the gear to be cut. They are derived from a long rack attached to the shaper ram, which, being reciprocated by it, controls the movements of a spiral gear and change wheels on the head. The spiral gear is the first or leading element, the variations in lead are produced by the change gears. On the return stroke of the tool the rack returns the blank to its original position. The down-feed of the tool is effected by hand. The pitching is done by a notched index plate into which in succession a locking bolt on the head is dropped. This is a perfectly mechanical method of cutting, although done by the aid of a rig-up to a machine not primarily intended for it. It is much better than the lathe method.

Double-helical Gears.—The patent machine of S. Stückgold, for cutting double-helical spur gears, is designed to set the axis of the blank horizontally. The increasing demand for double-helical gears with continuous teeth has given rise to several patented devices for effecting the necessary reversal of the gear at the apex of the teeth for right and left hand cutting. The one illustrated will give an idea of the kind of mechanism which can be employed.

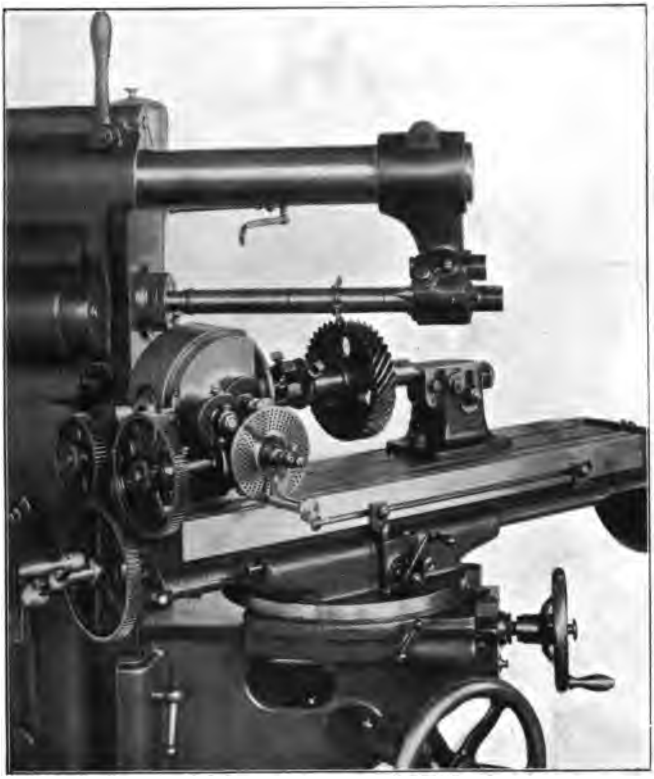


Fig. 137.—Cutting a Spiral Gear on a Brown & Sharpe Universal Milling Machine.

Fig. 139 is a side view, Fig. 140 a section at right angles to Fig. 139. The motions are all derived from the stepped cones A on the shaft B from which the cutter carriage C is travelled along the bed D, carrying the cutter *a*, which is rotated by the spiral gears *b*, *c*.

The Work-head.—The movements of the work-head, Fig. 139, are actuated from the shaft B (Fig. 140) through various gears and mechanisms which provide for the rotary movement of the blank, in forward and backward directions, corresponding with the cutting of each half of the width of the teeth. These movements are transmitted from the worm gears *d* and *e* seen at the right-hand end of both figures to the worm gears *f*, *g*, at the left hand of Fig. 139. The various degrees of angle of the different wheels to be cut are obtained by the change gears, *n*, *j*, at the left of Fig. 140, and the reversing mechanism is comprised in the rack and pinion, etc., arrangements seen in Fig. 139, and in section in Fig. 140. The details are as follows:—

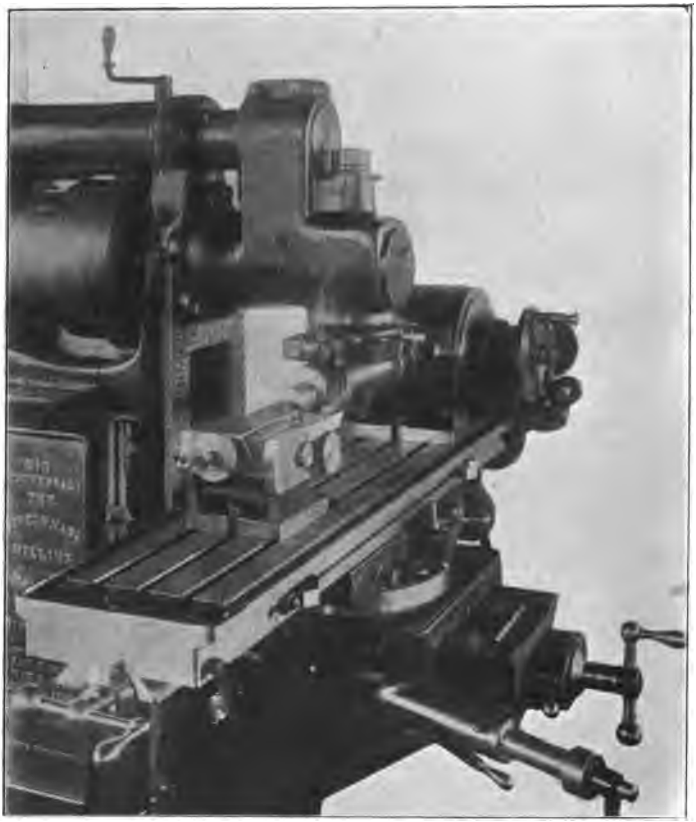


Fig. 138.—Cutting a Spiral Gear on a Cincinnati Universal Milling Machine.

Method of Driving.—In Fig. 140 the shaft E is rotated from the worm gears *d* and *e*, through a reversing clutch for the cutter head.

From the shaft E movement is transmitted through gears *m* and *n* to the shaft F, thence through bevel gears *o* to the shaft G, on one end of which the first change gear *h* is placed. Through these gears, *h, j, k,* the short shaft *p* is driven which carries the pinion *q* at its inner end. This pinion operates the pin rack H, engaging alternately above and below it, and thus changing its direction of movement, and through it the movements of the gears *r, s, t,* and of the shaft *u,* and thence the spur gears *v, w,* Fig. 139, and shaft J, and the bevels *x, y,* which actuate the vertical shaft K, which through spur gears L, M,—the latter carrying a dividing plate with index peg above,—drives the worm shaft N, and the worm *f*, and worm wheel *g* which rotates the blank.

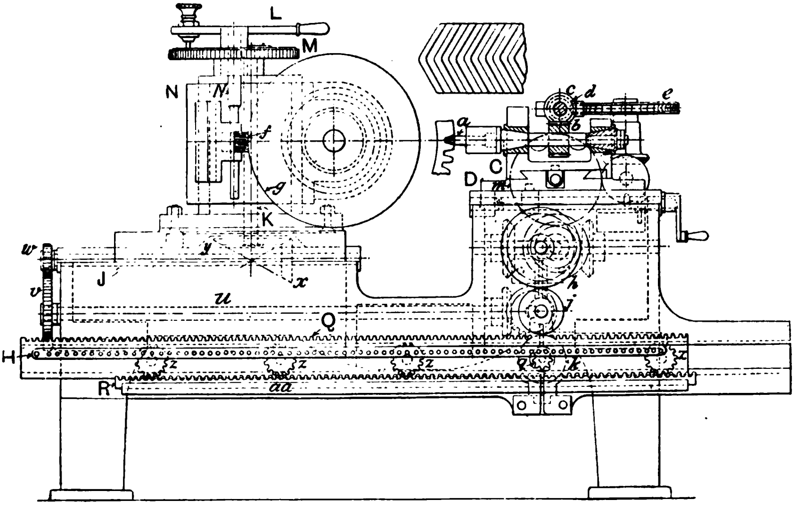


Fig. 139.

Reversals.—The details of the reversing rack motion are these: Several pin teeth of the rack H stand out beyond the width of the pinion *q* (*see* Fig. 140) to engage eccentrically in toothed discs *z* seen in both figures, which fulfil the function of crank discs having the same radius as the pinion *q*, and which rotate on pivots on a hinder rack Q of ordinary type. These eccentric cranks fulfil also the function of pinions by virtue of teeth formed on their peripheries to engage in the teeth of a lower loose rack R, which slides longitudinally in a guide *a a* formed at the lower edge of the rack Q. The object of the latter is only to ensure regular and constant movements of the gears *z*. The rack Q, with R, is guided by the dovetails seen in Fig. 140, and in its movements actuates the gear *r*, and thence the train to the work-head as already stated.

The reciprocations are produced by the relations of the pinion *q* to the pin rack H. The pinion *q* is located above the rack H until the latter has completed the range of its travel. Then, since the connection of the rack H to the rack bar Q is produced through the toothed crank disc *z*, the radius of which is equal to the radius of the pinion *q*, these cranks are turned vertically upwards or downwards according to the direction of motion of the two racks H and Q. The rack H, as soon as it is pushed along by the pinion *q*, will, in consequence of the tendency of the crank disc *z* to rotate, be pressed firmly down into the wheel *q*, and thereby any further rotation of the crank disc *z* will be prevented. This continues until the pinion *q* falls out of engagement with the rack H at the end of its travel, and for so long the rack Q will actuate the pinion *r* and its train. But when the pinion *q* gets out of mesh with the pin rack H, the latter loses its support in the pinion. Then, following the pressure exerted by the pinion *q* on its last tooth, it moves from one position into the other, a lower to a higher, or higher to lower, being under the coercion of the crank discs *z* which no longer rotate. Then the pinion engages

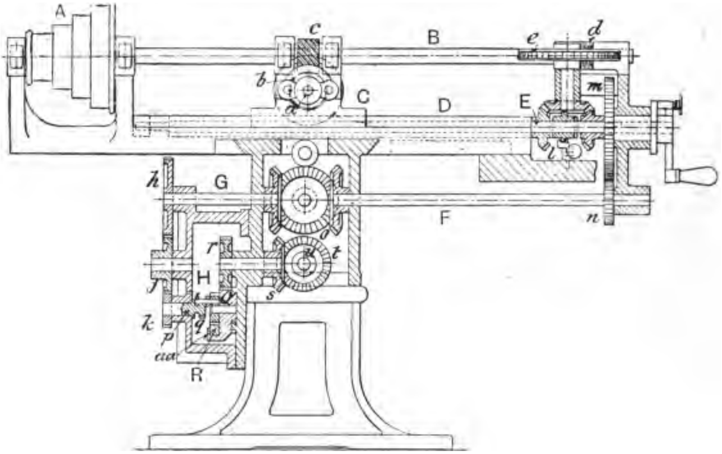


Fig. 140.

in the other side of the rack, reversing its direction and that of the train of gears which it operates.

The Lorenz Machine.—An extensive practice is done on the machines built by the Maschinenfabrik Lorenz, of Ettlingen, Baden, for whom Messrs C. W. Burton, Griffiths & Co., of London, are the British agents. These machines are built in two types, one on the lathe model, the other on the horizontal boring and drilling machine type. The first is made in two sizes for the smaller diameters of gears, the second in three sizes for large gears ranging up to six metres diameter. In the lathe design the blanks are held between centres, the axis lying horizontally; in the other the axis is vertical.

The Lathe Design.—In the lathe design (Figs. 141 to 143) the blanks are held between a dividing head at the headstock end, and the loose poppet at the other, and are further supported by steadies. The drive takes place from the single pulley seen at the left-hand end (Fig. 142) directly from the line shaft. The pulley is thrown in and out of action by a friction clutch. From this the dividing head is driven through worm gear of the globoid type enclosed behind the face plate, the wheel being of steel and the worm of bronze, with ball bearings for end thrust. The drive to the dividing head takes place through change gears to impart the angle or lead to the gear. The cutter slide is of the front or face design, being carried on vertical ways to leave the entire space above the bed available for gear blanks.

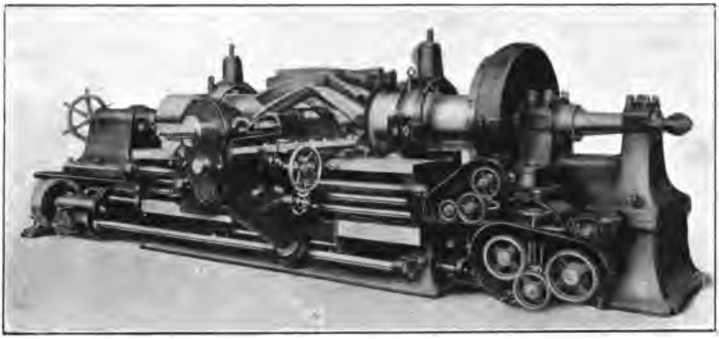


Fig. 141.—Cutting a Double-helical Pinion on a Lorenz Machine.

Method of Action.—The action of the machine is as follows:—The cutter is set to work to the depth of the tooth. After reaching this

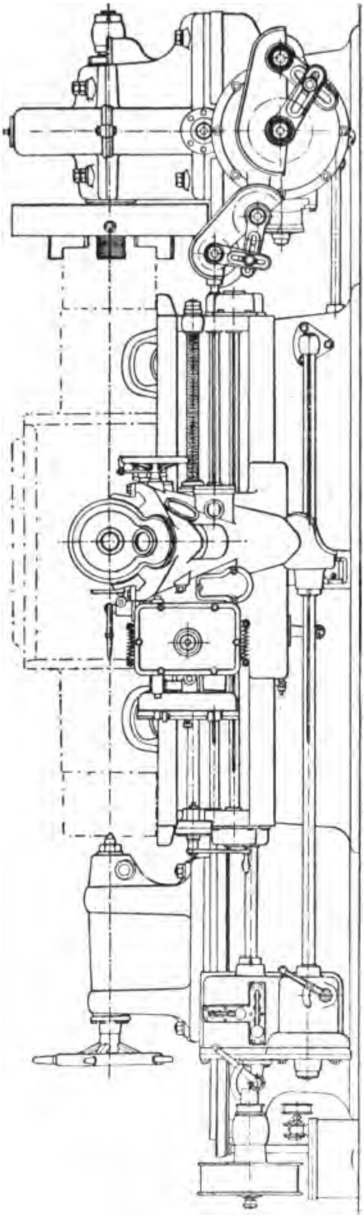


Fig. 142.

depth, the longitudinal feed comes automatically into play, carrying the saddle and cutter along the bed, while the change gears impart the necessary circular motion through the dividing head to the blank, the two combining to produce the tooth angle. The tooth space is finished out at one traverse. When this is complete, the cutter makes a quick withdrawal, and the saddle returns to its original position, the blank at the same time being turned through a distance equal to the pitch, completing a cycle. All these movements are automatic. The withdrawal of the cutter and the quick return of the saddle occur independently of the depthing and feed motions.

The Boring Machine Type.—Figs. 144 to 146 show a heavy Lorenz machine, which combines the functions of hobbing on the right-hand head, and cutting helical gears on the left-hand head, with end mills. For the automatic cutting of spur and worm wheels by the hobbing process, the ordinary hobs are employed, with straight rack teeth. Disc cutters removing a tooth space at a time can also be utilized in the case of large wheels which would require very expensive hobs. All the non-cutting movements are effected in-

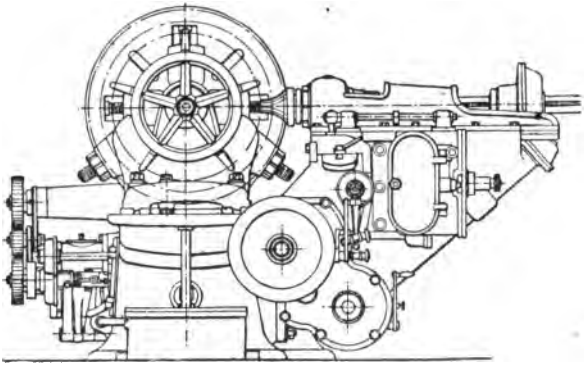


Fig. 143.

dependently of the working speeds, with great saving in time. As soon as a tooth space is cut through, an auxiliary motor is automatically thrown in, and the cutter slide runs upwards to its highest

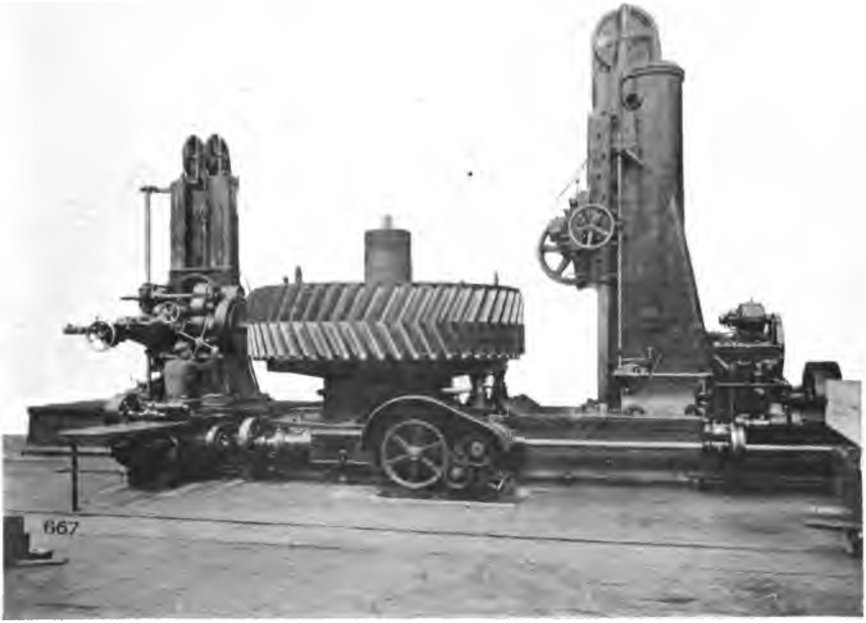


Fig. 144.—Cutting a Double-helical Gear on a Lorenz Machine.

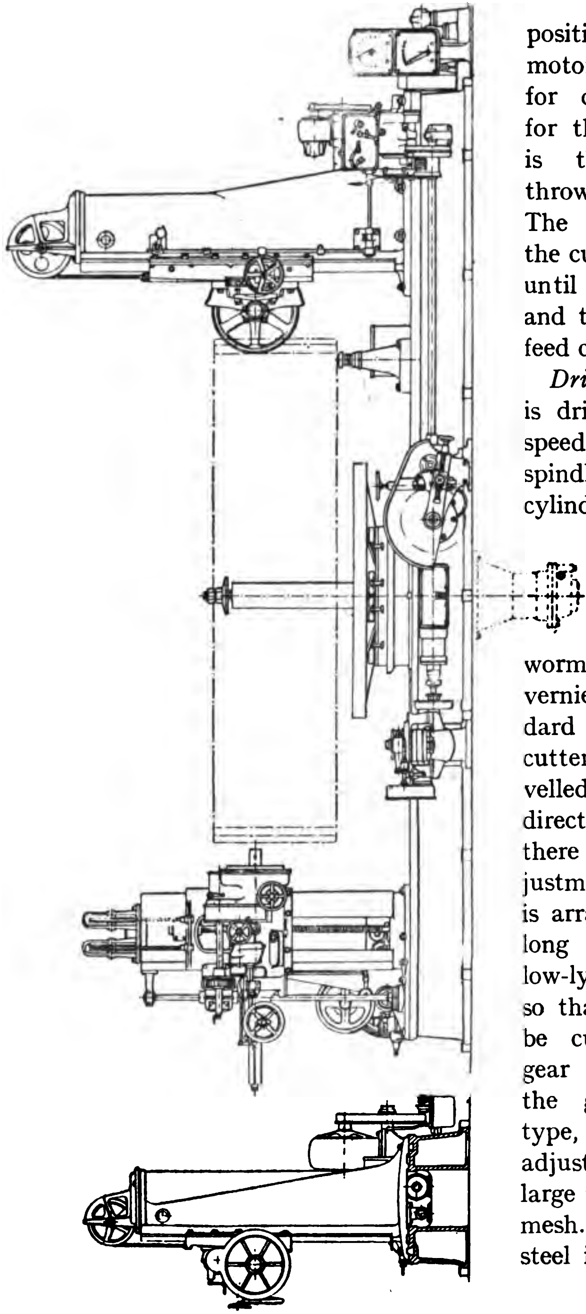


Fig. 145.

position. The auxiliary motor supplies the power for dividing the wheel for the next tooth, and is then automatically thrown out of action. The self-acting travel of the cutter slide is locked until the pitching is done, and then the cutter-slide feed commences again.

Driving.—The machine is driven by a variable speed motor. The cutter spindle runs in long cylindrical bearings, and its head is arranged to swivel, through a worm and worm quadrant, by a vernier setting. The standard with its balanced cutter slide can be travelled rapidly in either direction by power, and there is a fine hand adjustment. The standard is arranged to slide with long guides over the low-lying dividing wheel, so that small wheels can be cut. The dividing gear in the bed is of the globoid worm-gear type, the worm being adjustable and having a large number of teeth in mesh. Siemens - Martin steel is employed for the ring of the dividing wheel, and the worm is of phosphor bronze. A ball thrust receives the axial pressure of the worm, and the whole drive runs in oil.

Feeds.—The feeds for all the cutting operations are obtained through change gears within very wide limits, and each self-acting movement is adjustable through stops, which automatically trip. Fixed stops arrest the further travel in a positive manner.

The left-hand side of the machine is arranged for helical wheels and pinions, either with solid teeth, or staggered if necessary. End mills are used for the solidly-cut teeth. This side of the machine is driven by a motor with constant speed, and the cutter-spindle revolutions are varied by nests of gearing, producing eight changes.

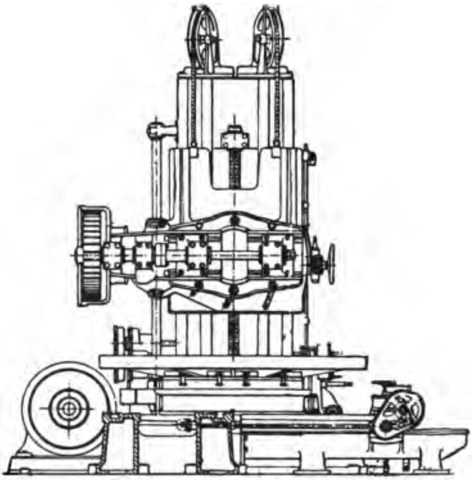


Fig. 146.

Operation.—The method of working when cutting double-helical teeth is as follows:—The cutter is set first of all into the intended depth, and is then started automatically in its longitudinal movement. The motion of the dividing wheel is synchronized through change gears suitably to produce the angle of helix. During the travel of the cutter over the whole tooth width, the wheel is turned by a reversing arrangement, so that the double angle is imparted to the teeth in one passage. When the tooth space has been cut through, the cutter is quickly withdrawn, and the cutter slide returns to the commencing position. During this quick return the wheel is pitched for the next tooth space, and the operation goes on until the wheel is finished, being entirely automatic. The feeds for depth and longitudinal travel are variable in a large range through change wheels. The quick return of the cutter out of the space, also the rapid return of the cutter slide, are effected at a constant speed, independently of the rates of cutting.

Spiral gears can also be cut with the help of an attachment

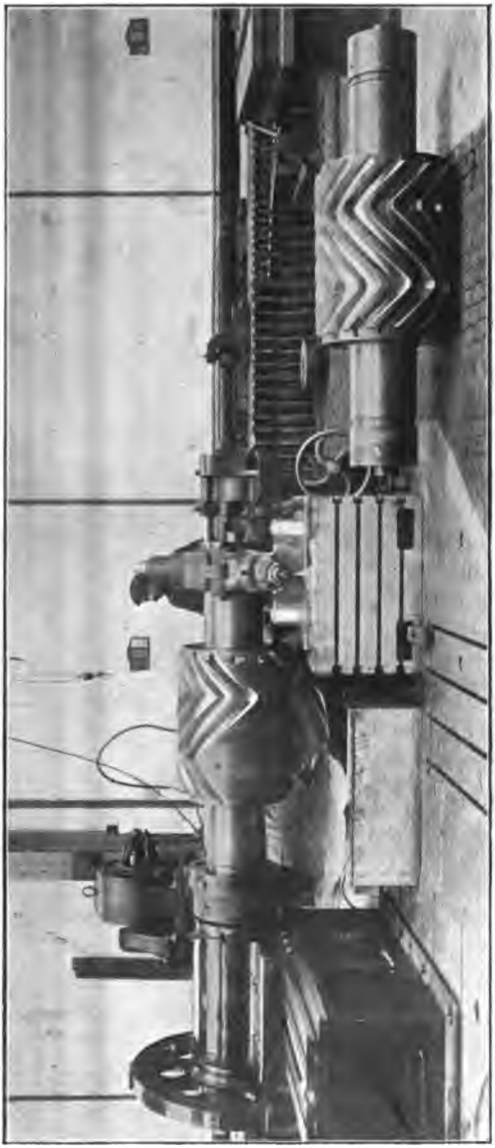


Fig. 147 —Cutting Citroën Double-helical Pinions.

fastened to the cutter-spindle frame, using disc cutters. With the combination of the two standards it is possible to cut spur wheels (the tooth numbers of which are divisible by 2) by milling with single disc cutters and dividing, and finishing by the hobbing process. This relieves the wear on the costly hobbing cutters.

The Citroën Gear.—The illustrations, Figs. 147, 148, show examples of spur cutting in the shops of Messrs Citroën et Cie, done with end formed cutters. Further reference to the practice is given in the following chapter, *see* p. 185.

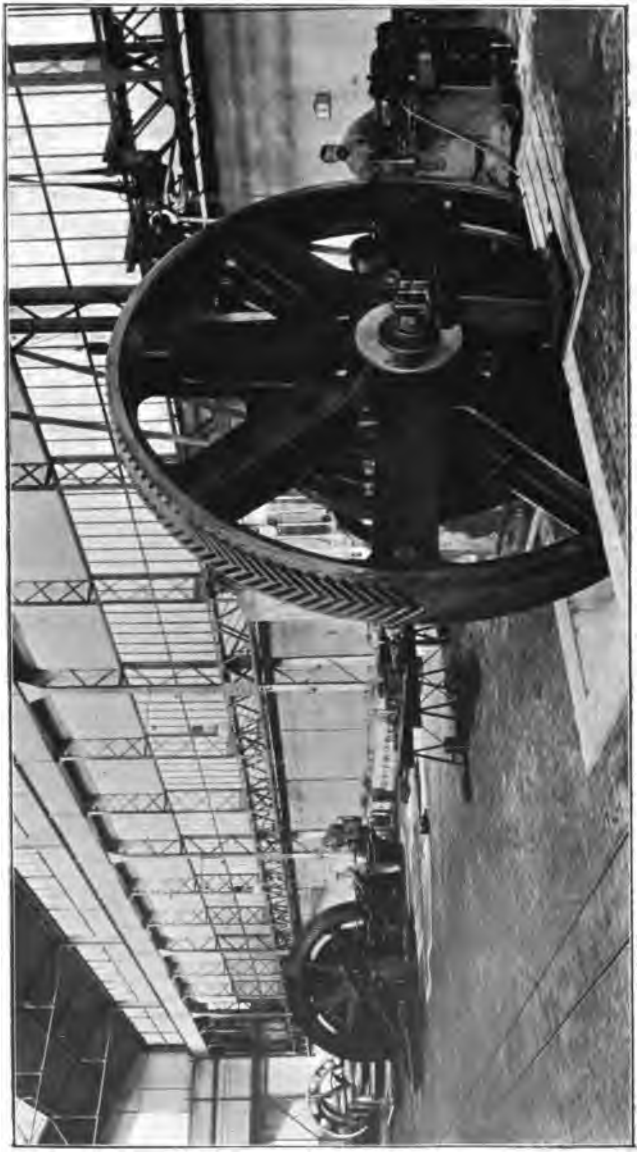


Fig. 148.—Cutting Citroën Double-helical Gears.

CHAPTER XI

MACHINES USING FORM CUTTERS (*continued*)

THE Brown & Sharpe combined spur and bevel gear cutting machine (Figs. 149 to 153) illustrate the spur-gear cutting machine described on pp. 109 to 118 with a bevel-gear cutter slide substituted for that which is used for spur gears alone. The capacity which this has for setting to any angle up to 90° entails certain alterations in the method of driving, which are shown in the accompanying drawings. Where the parts of this machine are identical with those of the spur-gear cutter the same reference letters are employed, but the parts special to this machine are indicated by other lettering. Fig. 149 is a right-hand elevation, Fig. 150 a left-hand ditto, and Fig. 151 a plan view.

Movements.—The driving pulley G is retained, and the pulley R and the feed-reversing mechanism in Fig. 101, on p. 115, are identical in both machines. The letter A indicates the bed, C is the upright, and *a* is the bracket bolted to the side of the bed which carries the bearings for the driving shaft, as previously explained. But the method of driving from the pulley G to the slide is different. Bevel gears at CC, Fig. 151, driven by the pulley G, drive the shaft DD to the bevels EE to the cutter spindle FF in the cutter slide GG (compare with Fig. 152). The box which carries the gears CC swivels on its shaft to be accommodated to the setting of the shaft DD to angle, which has to vary with the angular position of the parts HH, Fig. 151, details of which are shown in Fig. 153. The motion to the cutter slide is given by the screw-splined shaft KK, taking the place of the screw in the previous machine, and operated from the same feed and reversing mechanism as in Fig. 101, p. 115. But the hand-wheel adjustment in that figure is not used, a rack and pinion being employed instead (*gg* and *hh*, Figs. 152 and 153).

Cutter Slide.—The main body HH on which the cutter slide GG travels is hinged at *jj*, Fig. 153, and adjusted for angle round a quadrant slot *kk* with a locking nut *ll*, adjustment being made with a pinion *mm* and a quadrant rack seen in Figs. 152 and 153. The rack is graduated, and a vernier is fitted just above the pinion. The body and quadrant, locking bolt and pinion, are carried in the bed slide JJ, which is moved along by the pinion *hh* and rack *gg*. In this machine a horizontal movement of 12½in. is provided for. The cutter slide GG has a travel of 5½in. The operation of the cutter slide is shown in Fig. 153, taking place from the splined shaft KK, driven, as before stated, from the feed and reversing mechanism at the end of the bed. A sliding bevel gear on the shaft KK drives another, *nn*, both being confined in bearings secured in the bed slide. Thence gears OO in the pivot transmit motion to spur gears *pp*, which operate a screw *qq* of quick pitch in the cutter slide GG, so imparting a traverse. As the splined shaft KK is driven and reversed by the same mechanism as the feed screw in the spur-gear cutting machine, the return non-cutting stroke of the slide GG is effected at a rapid and constant speed.

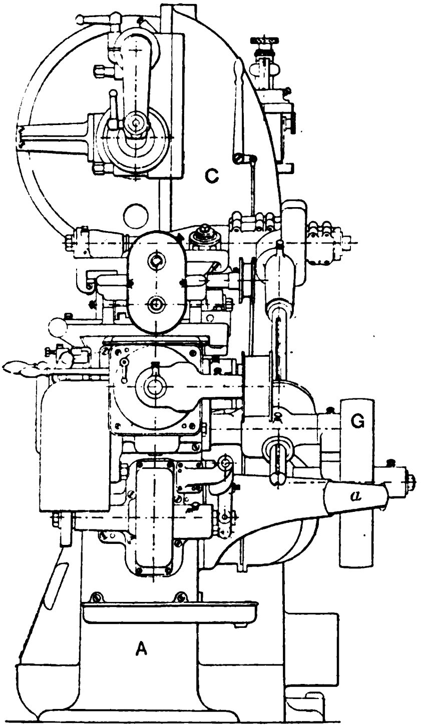


Fig. 149.

In order to take up any backlash present in the train of gears, and to prevent shock in the movements of the cutter slide, the weight is taken up by a cord passing round a pulley LL, Fig. 152, thence going over guide pulleys above, and held in tension by weights. On the spindle of this pulley is a pinion *rr* which engages with a rack SS under the slide GG. The movement between the pinion and rack maintains the cord vertically in all positions of the cutter slide.

The Carriage.—To adjust the bed carriage JJ, swing the hinged stop *tt*, Fig. 153, out of the way, loosen the gibs MM of the carriage, Fig. 152, and move the carriage out of the way by means of the rack and pinion *gg* and *hh*, Fig. 153. Set the work, and having thus determined the position of the carriage, clamp the block *uu*, Fig. 153, to the bed carriage in such a position that the screw *vv* may be adjusted until the end just touches the fixed top *ww*. Then tighten the check nut on the screw. The block and screw being left in this position, the carriage can be moved on the bed and brought back to the same position. Before beginning to cut, tighten the gibs, swing up the stop *tt*, and adjust the screw *xx* until the end bears against the top screw *vv*.

Setting the Cutter.—The cutter is set centrally on the cutter spindle by the indicator, as for spur gears mentioned on p. 116. But as a cutter cannot be thicker than the width of the space at the small end of the teeth, the cutter must be set out of centre for the large end, and the blank rotated to right and left to give the correct width and bevel. To indicate the amount of movement of the cutter out of centre a graduated dial is provided reading by a vernier to thousandths of an inch. It is shown at NN, Fig. 151, and the enlarged section above is the pinion and rack on the spindle sleeve by which the spindle is adjusted endwise. The two set screws retain the teeth and this spindle in position. They must be slackened before making adjustments, and after tightening they are secured with lock nuts.

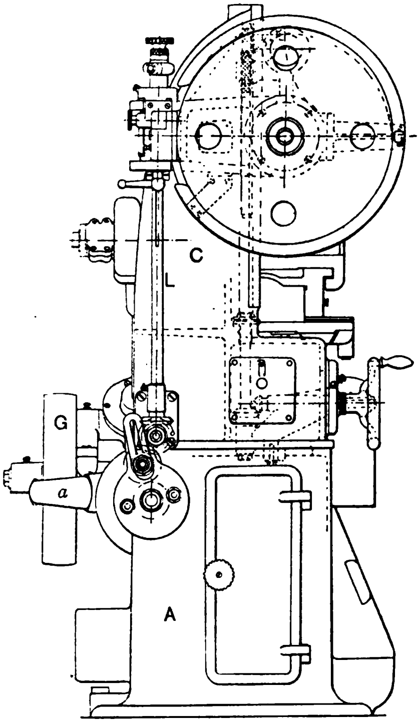


Fig. 150.

The Birch Automatic Machine.—The next gear-cutting machine is of a type that has long been employed in English shops, comprising two beds set at right angles, one carrying the work-spindle slide, the other the cutter-spindle slide. This is a simple construction, and one that was probably evolved from the lathe, when the headstock was first utilized for dividing, with the cutter held in the slide rest. In many machines the dividing is done by hand, but the example taken is fully automatic, and is made in several sizes, up to 8ft. diameter capacity. The drawings show a 2ft. machine in detail. Some designs are made for cutting spur gears only, but the present example is capable of cutting, in addition, bevels and worm gears. It is possible to produce worm gears with a straight cut across the face to suit the teeth of the worm, owing to the swivelling movement of the cutter head.

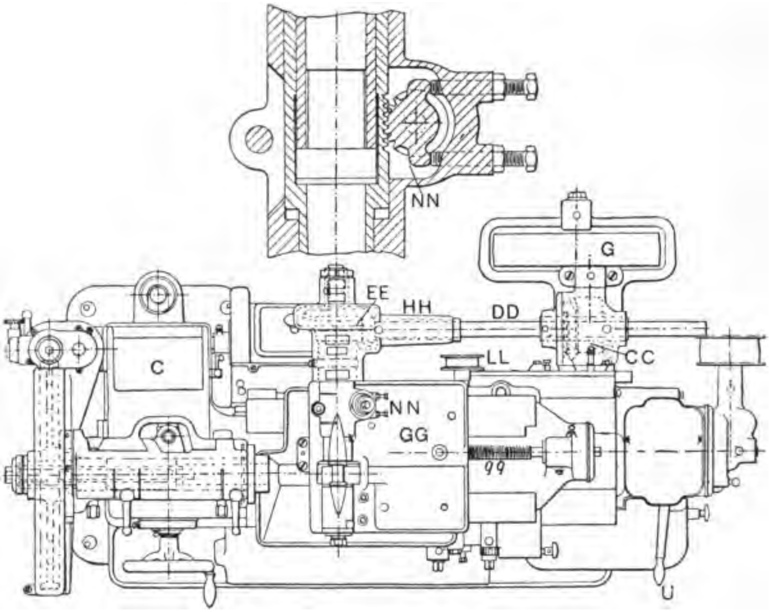


Fig. 151.

This is a type of lessening interest, because the present tendency is almost wholly in favour of more specialized machines, but it should be illustrated, because many of these machines remain in use, and in the general class of shops where only a moderate amount of gear-cutting is done, they fill a useful place.

The Headstock.—Fig. 154 illustrates the construction of the work headstock. It slides on the bed A, fitting by a narrow guide, the gib portion embracing only the right-hand shear, thus securing the same assumed advantages as in lathes fitted with narrow guiding ledges. The gib strip is pulled up by a couple of bolts, tightened from above. There is a clearance at the edge of the other shear, and a plain gib below simply serves to prevent lifting. The head is adjusted along the bed by a screw B, having a micrometer fitting.

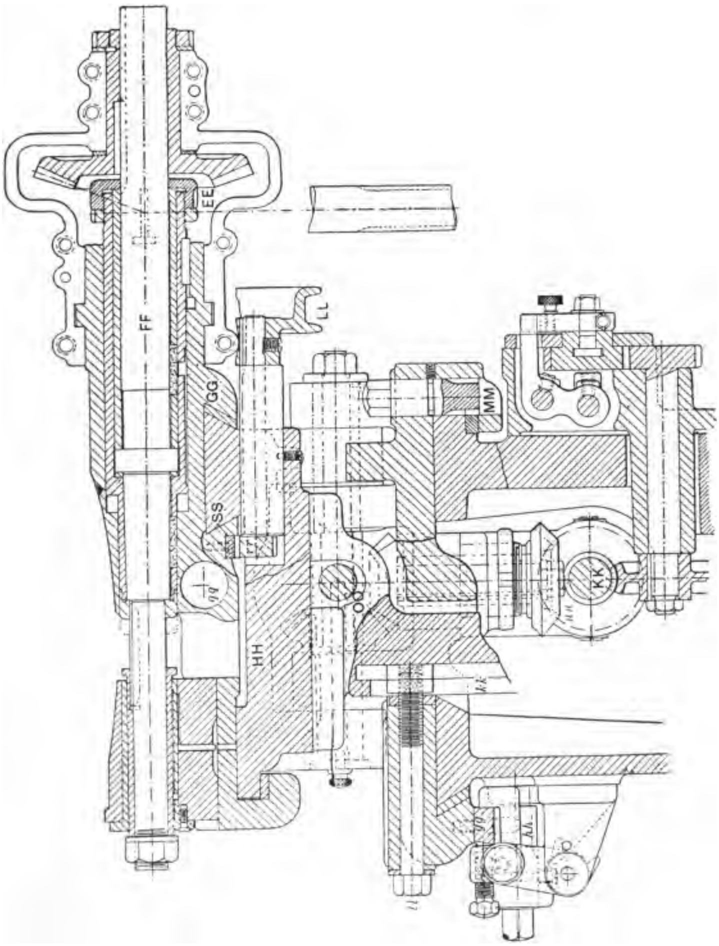


Fig. 152.

C is the work spindle, turning in split bearings in the head; it has a slotted face plate D bolted on at the front, to carry large gears. The arbor E, interchangeable for different bores in blanks, is driven by a short key and held in by a long bolt. The outer end of E is turned to run in a steady bearing, bolted to a slot on the under side of A.

On the end of C is keyed the worm wheel F, 26in. outside diameter with 180 teeth of 7 pitch and 1½in. face. The worm box G is arranged

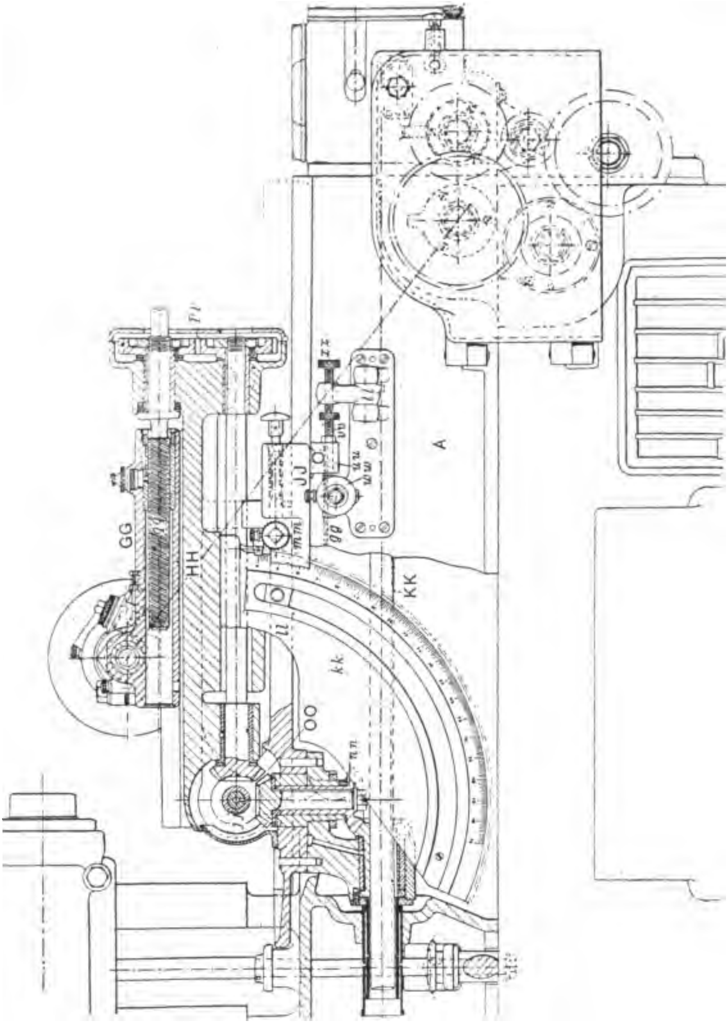


Fig. 153.

to drop slightly, on releasing the set screw below, so that the wheel may be spun freely, for testing the running of blanks previous to cutting.

The Cutter Head.—The details of the cutter head are given in

Figs. 155 and 156. A base plate H slides on the bed, being guided by a tongue in one tee groove. It is clamped by four bolts, and adjustment along the bed is effected by a screw J, turned by the hand wheel *a* (*see* Fig. 157). Micrometer settings are rendered possible by the dial *b*, of large diameter. This is interchangeable on the end of the screw B, so that it may be used there for accurate setting. The dial can be loosened and rotated to bring it to zero after each movement. Upon H turns the head K, the base of which is graduated into degrees, and which may be swivelled to any angle and clamped by the bolts. On the face of K a swivel slide L is fitted, capable of angular setting to 12° on either side of the horizontal for cutting worm gears, this being graduated also on the periphery. Sliding on L is the cutter slide M, moved by a screw N and hand wheel. The cutter spindle O runs in a split adjustable bearing, and is hollow to receive the cutter arbor P, of suitable size for the bore of cutter. This arbor can be adjusted up or down in the spindle by nuts at top and bottom of the spindle, to bring the cutter exactly central with the work, without the use of packing washers. The cutter is held on the arbor by a nut, and just below this is a coned guard collar spreading out over the lower bearing, which may be removed to put on or take off cutters. A split tapered bush in the bearing enables a perfect fit to be made with the tail of the arbor.

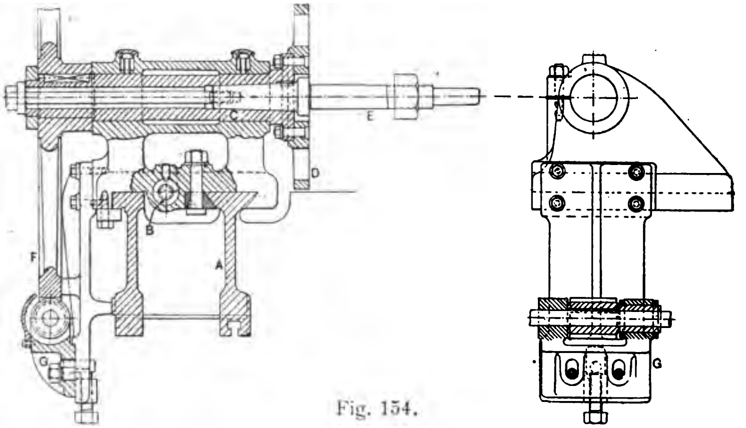


Fig. 154.

Cutter Spindle Drive.—The method of driving the cutter spindle is seen on comparing the three figures—155, 156, 157. A pulley Q is the first element, turning the shaft R, splined to pass through a

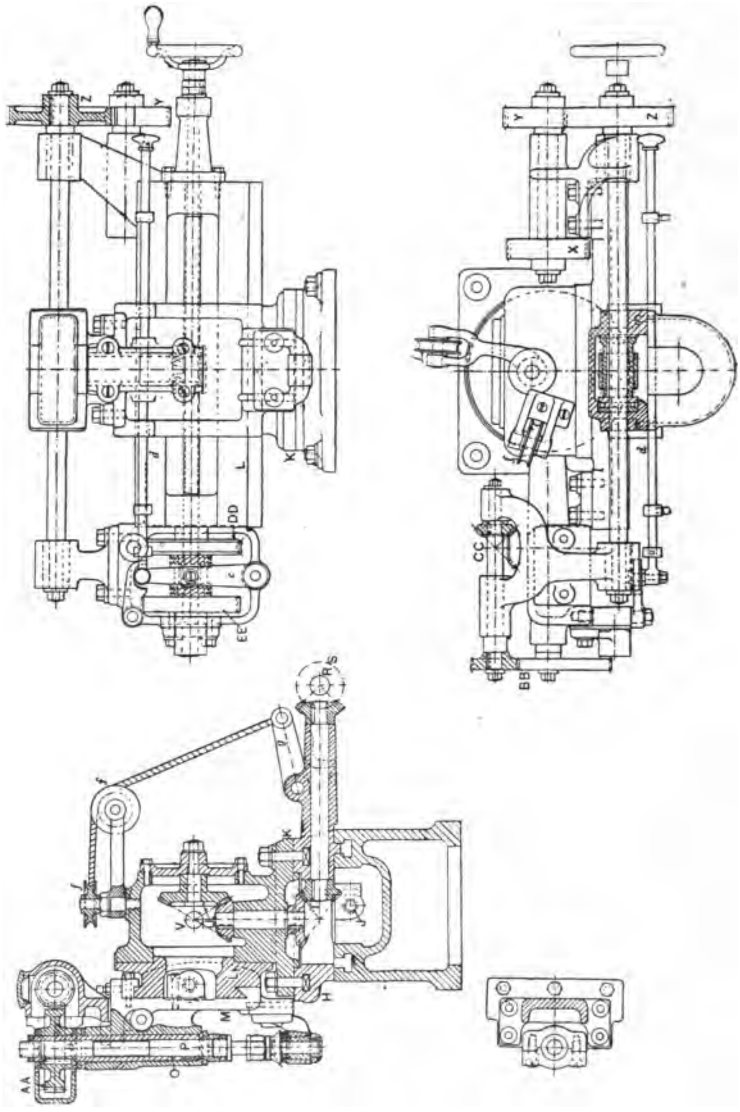


Fig. 155.

mitre gear S, engaging with a companion which transmits the drive through a long bearing in the base H, thence by bevel gears T and U (there being three of the latter) to a horizontal shaft V. Keyed on this is a spur gear W, engaging constantly with another gear X, on a short shaft that carries a gear Y, meshing with one Z. The latter are readily removable, so that other wheels may be substituted, there being six gears in all, giving with the two-speed countershaft twelve rates of speed to the cutter spindle, varying from 20 revs. to 200 revs. per minute. The shaft on which Z is mounted passes through a bearing on the slide M, and drives a steel spiral pinion meshing with a phosphor-bronze wheel AA, giving a ratio of 4 to 1. A smooth drive without chattering under heavy cuts is obtained in this manner. End thrust of the pinion is taken by a ball-thrust washer, seen in plan, Fig. 155.

Feed of Spindle.—The feed of the cutter slide M is derived also from the shaft V, through change gears BB, of which a set is provided to give ten changes varying from ½in. to 5in. per minute traverse; thence by mitres at CC, and worm gear DD. Quick return at a constant speed is effected through the gears EE. Either DD or EE may be made to drive the screw N, in reverse directions, by throwing the claw clutch between them to mesh with one or the other, a lever *c* performing this movement. Automatic reversal is obtained through the medium of the stop rod *d*, which has a pair of adjustable collars upon it, struck by the moving slide M, so throwing the rod along, and moving the lever *c*.

On looking at Fig. 156, a lever *e* will be observed, keyed on the same spindle as *c*, thus partaking of its movement. The object of *e* is to release the dividing mechanism at the appropriate time—namely, when reversal takes place. A chain or cord is attached to the hook of *e*, and passes around a couple of guide pulleys *ff*, down to a lever *g*, on the shaft FF (*see* also Fig. 157). Looking at this drawing, it will be noticed that the shaft R, previously mentioned, is continued into the dividing motion box, and drives a spur gear GG through two intermediates, seen in the plan view to the left. GG runs loose on its shaft, and only drives it when a claw clutch HH is in mesh. From the shaft a train of gears JJ connects to a shaft on which a dividing plate KK is keyed, and mitre gears LL connect through the swing plate MM and suitable change gears to the shaft NN, which passes alongside the machine through the worm box G (Fig. 154). The gearing JJ is so arranged that the first

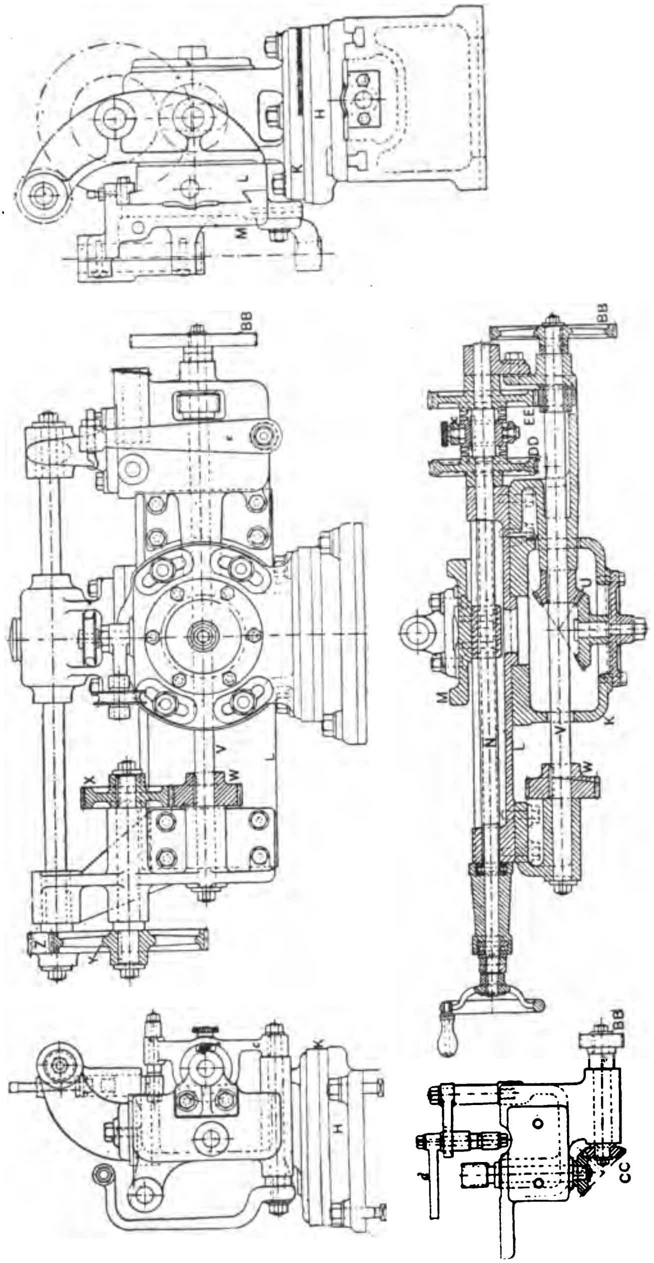


Fig 156.

change-gear shaft, passing through the central axis of the swing plate, makes two revolutions, and the shaft on which GG is mounted three revolutions to one revolution of the dividing plate KK, so that when cutting low numbers of teeth the first change gear on the stud projecting through the swing-plate boss requires to be one-half as large as if it ran at the same speed as the dividing plate, thus easing the strains on the change wheels and shafts.

Turning again to the shaft FF, this is seen to have three levers OO, PP, and QQ, the first keyed on, the others fitted freely. When the shaft is turned, OO lifts PP by means of a pawl *h*, which has a heel *j* that strikes against the end of the screw *k* when PP has been raised to a sufficient height, and thus causes the pawl to release the levers PP and QQ and drop them. PP has a catch that engages in a notch in a ring cast on the side of the plate KK, and the end of QQ fits into a notch on the edge of the plate. On the side of PP is a cam *l* which engages the pin screwed into the clutch HH, and normally holds the latter out of mesh with gear GG.

Operation.—The method of working is as follows:—After the cut has been made in the blank, the cutter slide returns, and when it has reached the end of its travel and reverses again for the next cut, the chain comes into operation, lifting the lever *g*, and consequently raising OO, PP, and QQ. QQ is thus clear of the notch in the plate KK, and PP is clear of the notch in the ring also. The cam *l* is likewise moved away from the pin on HH, and the latter, under the coercion of the coiled spring behind it, drops into gear with GG, and thus starts the dividing motion. The chain becomes slack again after a considerable interval, and the releasing pawl *h* is introduced so that on the chain starting the mechanism, a further pull causes the heel *j* to strike the end of the screw *k*, and thus to drop levers PP and QQ, and these fall on the ring and the edge respectively of KK, and rest there till the notch in the ring comes round and allows lever PP to fall down and bring cam *l* in front of the pin in HH. This is thus drawn out of gear with GG, stopping the motion just as the notch comes round to the catch on QQ, which then falls into the notch and completes the dividing motion in a positive manner. The object of the lever *m* is to raise the levers by hand, when effecting adjustments and trials; it is secured to the boss of PP by screws.

The two discs at *n* are fitted to enable the dividing worm to be turned slightly to adjust the exact position of the blank if necessary,

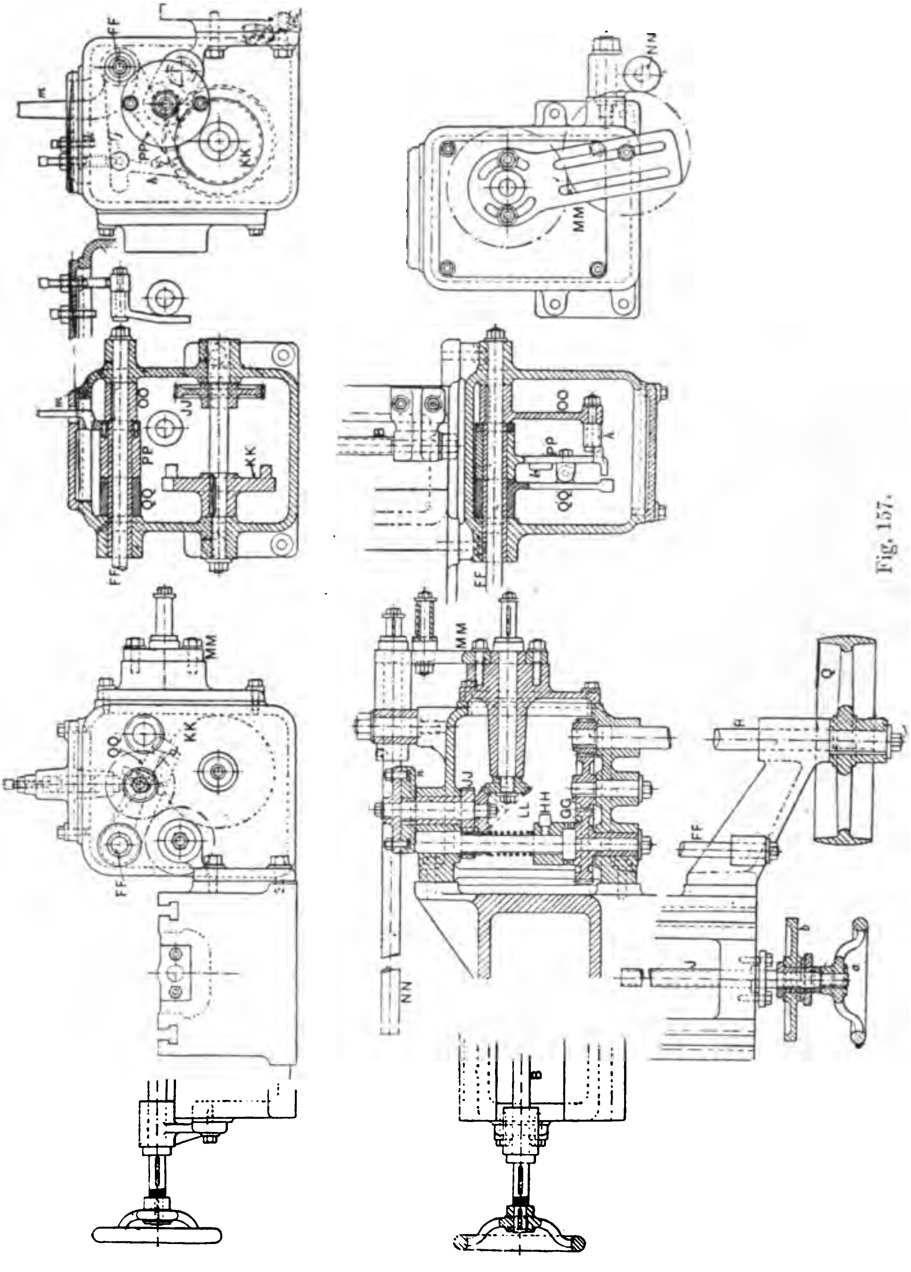


Fig. 157.

and more especially for giving a certain amount of set to the blank when cutting bevel gears where the cutter is made narrower than the width of space at the small end of teeth, and the blank is turned slightly to cut the extra width at the large end. The discs are graduated to show the amount of set-over, so that it can be recorded for duplicate work. With the change gears provided, all numbers of teeth from 6 to 100, and all numbers from 100 to 600, excepting the prime numbers and their multiples, can be cut.

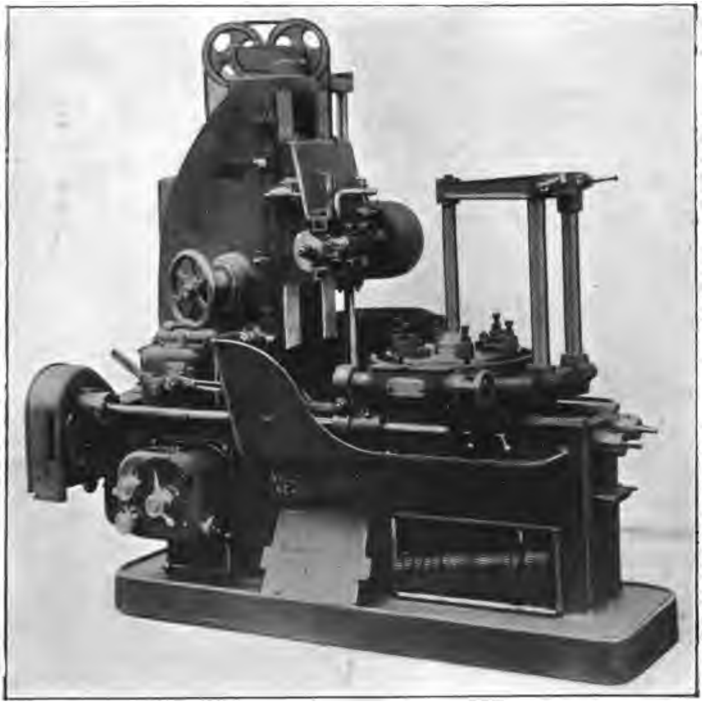


Fig. 158.—A Gould & Eberhardt Universal-gear Cutting Machine.

The Gould & Eberhardt Machine.—Fig. 158 shows one of the latest machines by Gould & Eberhardt, of Newark, N.J., for cutting spur, bevel and face gears, and particularly for roughing-out automobile bevel gears preparatory to being finished on a bevel-gear planer (*see* also p. 326). It has a capacity of cutting 36in. diameter and 12in. face spur gears, and 24in. diameter and 8in. face bevel gears, both up to 3 diametral pitch in cast iron, and 4 diametral pitch in steel.

Cutter Slide.—The cutter slide moves in a vertical plane and the work lies in a horizontal position on a table, arranged on the anvil principle. In the present design, the reciprocating parts are mounted upon the vertical upright portion of the main frame and the feeding arrangement is of the draw-cut principle, there being a separate and direct drive to the feed-screw mechanism. This vertical cutting feature eliminates the disadvantage, when cutting bevel gears, of tilting the cutter slide with all its many moving parts. The latter method is undesirable because it lacks rigidity for heavy cutting, particularly since the advent of high-speed steel. The old method also necessitates the mounting of the reciprocating parts upon an angularly adjustable slide to which motion is transmitted through a circuitous route from the prime mover to the cutter spindle and feed screws.

Work-table.—The work-table is adjustable to any angle and is so mounted that all the strains imparted by the cutter are transmitted to and taken by the main base of the machine. This arrangement of the cutter and work slides eliminates one sliding surface and only requires two sliding surfaces. When the table is in this angular position it is supported by a segment arranged centrally and provided with a worm-wheel sector, whereby the table is adjusted. In addition to these forming a lock, the sector has a strap in the segment that makes a rigid lock for the table when properly adjusted. A crank handle on the front end of the machine enables the operator to adjust the work-table by degrees and minutes, one turn of the crank handle corresponding to one degree, and a sector vernier graduated in degrees shows the amount of the angle given to the table. This table is controlled by an indexing mechanism which is positive, simple in construction, and is practically the same as is usually found in some of the latest spur-gear cutting machines. The last mover of the indexing train is a worm wheel of large diameter, arranged to be close to and rigidly secured with the work-table upon which the gear blank that is to be cut is fastened. Upon this table is also mounted a triangular-shaped work mandrel outer support which sustains the gear blank at its outer end in any angular position.

Feed.—The feed to the cutter slide is controlled by a variable feed-gear box with direct reading, permitting of a rapid changing of the feeds. It provides for sixteen different feeds ranging in geometric progression from 1in. to 15½in., and each of the different feeds is obtained by the operation of two hand wheels. The cutter slide is counterbalanced and is provided with a rapid return. The feeding, indexing and return movements are controlled by fewer hand levers than are usually employed, and these are interlocked.

Cutter Drive.—The cutter-driving parts are separate and direct, as in the firm's spur-gear cutting machine. The cutter is driven by a steel worm and bronze worm wheel with a smooth-running and powerful drive. A solid cutter arbor is made of chrome nickel steel, which can accommodate different-sized bushings for the various holes of cutters. A number of speeds to the cutter, from 23 to 145 R.P.M., are arranged in geometric progression obtainable by means of change gears furnished. There is only one driving pulley for the running of the entire mechanism of the machine, and all gears are entirely enclosed with guards.

The machine weighs 3600lb. and may be furnished with electric motor drive. It is especially adapted for roughing-out nickel steel automobile driving gears and pinions, and has been placed in many of the prominent automobile factories.

Fig. 159 shows the use of the index head to manipulate a small bevel gear on a Brown & Sharpe universal milling machine. This involves sundry settings to produce the nearest approximation to the correct shape of teeth which is possible with a rotary cutter used thus. The settings comprise the partial rotation of the gear by turning the index plate and the adjustment of the table bodily towards the column, and subsequently away from it, reading the amount in thousandths by the micrometer dial.

Double-helical Bevel Gears.—In the Polanowski patents the method adopted for cutting helical bevel gears imparts a parallel motion to the cutter and the rotary motion to the blank, these being so synchronized as to produce the helical curves. The essential elements of the mechanism are shown by Figs. 160 to 162. It is designed on very similar principles to the Stückgold machine (*see* pp. 153 to 156), the principal difference consisting in the arrangements of a pin rack and its gears. This is a modification of a previous design for cutting single-helical bevel gears, and therefore having no reversing motion, which the illustrations here shown have.

Driving.—All the movements are driven from the stepped cones A, Fig. 160, back-geared (though not indicated) driving to the first shaft B, which shaft drives primarily the cutter spindle C, through spiral gears *a* and *b*. The spindle is carried in a head D which is fed along the bed E, at the end of which the driving gear A is carried.

Fig. 161 shows the work head, in this case fitted with two work spindles, for larger and smaller gears, either of which can be brought round by the circular movement of the head on its base F, provided for setting the gear blanks at their correct bevels. The entire movements of the head are actuated from the shaft B, thence through gears to the shaft G and back through change gears H to the screw shaft J. The screw shaft J is used to feed the base F of the head along the bed, and bevel gears transmit the rotations to the cutter spindles through the vertical shaft K. The transmission takes place through two spur gears, one of which is seen at *e*, thence to the spindles through one worm and wheel *f*, and

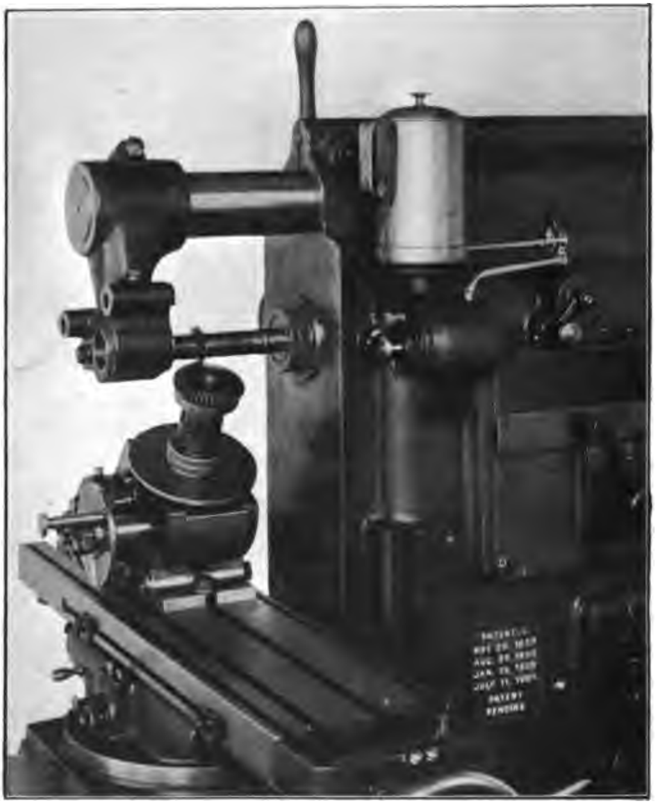


Fig. 159.—Cutting a Bevel Gear on a Brown & Sharpe Universal Milling Machine.

another pair *g*. On the gear *a*, a division plate L with index peg provides for indexing the gears that are being cut.

Cutter Head.—The movements of the cutter head D, including its reversal, are provided for thus: a spur gear H, Fig. 161, one on the end of the shaft B, drives a train of gears, the last of which, *j*, actuates the screw spindle M which travels the head B. Its reversal is effected by three swinging gears (not shown) which transmit the forward or the backward travel as desired. The bed E which carries the cutter head D is moved transversely on a short length of bed N towards or from the work head by means of a screw spindle O.

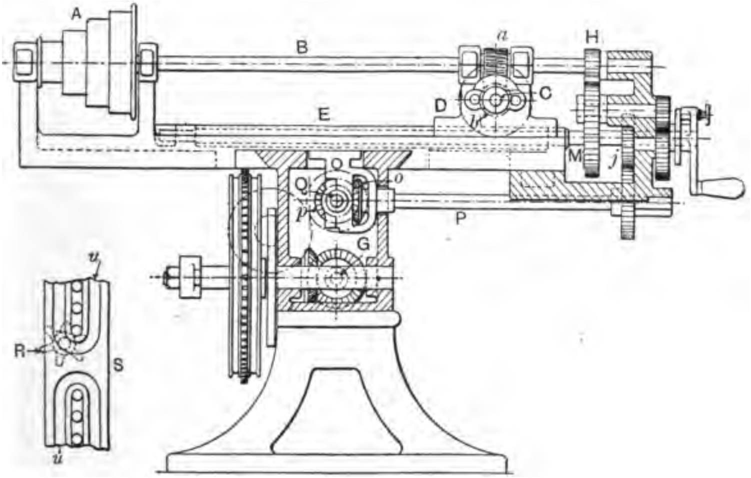


Fig. 160.

Operation.—In operation, the gear blank is swivelled by the movement of the head on the base F until the axis of the cutter stands perpendicularly to the pitch cone of the gear. The cutter revolves, and moves in a straight line. The blank receives an arc movement which is made to correspond with the angle desired, suitably to the pitch, by the change gears H. When a tooth space has been completed, the blank is indexed by the plate L and its peg.

Provision for Double-helical Cutting.—Thus far the machine is suitable for cutting single-helical bevel wheels. For cutting double-helical teeth, the additional mechanism comprises provision for changing the direction of rotation of the blank. The latter is given a forward rotation until the cutter reaches the middle of the tooth, and then a return movement while the second half of the tooth is being cut, the cutter travelling straightforward throughout. The additional arrangements involve the use of a star wheel and pin teeth with endless grooves by which the blank is moved forwards and returned. This is illustrated in Fig. 161.

Commencing with the shaft P, which, through bevel gears, communicates motion to the shaft Q and thence through fixed gears and change gears to the work-head, the bevel gears *o* effect reversal (compare with Fig. 162). The gear *p* is mounted on a sleeve with a second gear *q*, which can be slid together along the shaft Q, by which they are rotated so that either one can mesh with the crown gear *o*. Though therefore the shaft Q always rotates in one direction, the bevel gears provide means for reversing the direction of rotation of the blanks as follows:—

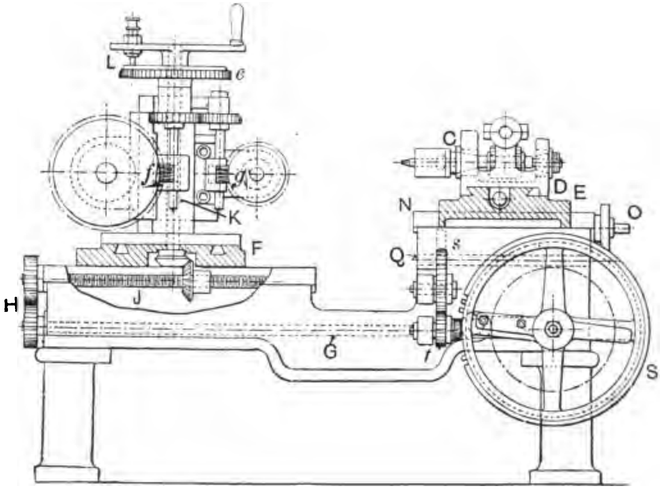


Fig. 161.

The rotation of the shaft Q is transmitted through spur gears *r*, *s*, *t*. On the shaft of *t* a star wheel R is keyed, the teeth of which engage with the teeth of a pin wheel S on the shaft T, to which it is feather-keyed. The object of this is to permit the pin wheel to slide on its shaft, in order that the star wheel rotating in one direction constantly shall impart a reverse motion to the pin wheel. This is accomplished by flanking the zone of pin teeth with annular grooves *u*, *u*, meeting and returning into one another at the ends (*see* the detail to the left of Fig. 160). In the groove *u* the end of the shaft engages which carries the star wheel R. When this end reaches one of the turning points in the groove *u*, the shaft of the star wheel slides the splined pin wheel S along its shaft T, so causing the reversal of its direction of rotation. The wheel therefore makes one revolution in one direction and then one in the opposite direction. The arms *v*, Fig. 162, are a kind of compensating motion to transmit the movement imposed on the rim to the boss without leverage. These alternating rotary motions are transmitted through spur gears *w* and *x* to the bevel gear *y* which actuates the gear *z* on the shaft U, the motion of which is transmitted through the change gears H to the worm gears *f* or *g* which revolve the blanks. The ratios of the gearing which operates the pin wheel are selected so that during one travel of the cutting tool the pin wheel will revolve the blank first in a forward and then in a backward direction, in a similar manner to that mentioned previously.

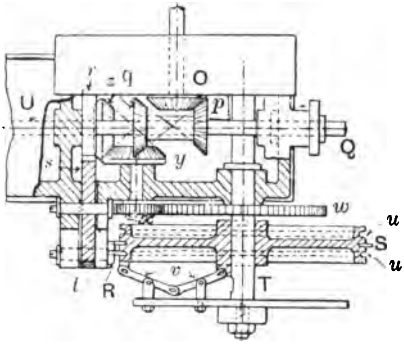


Fig. 162.

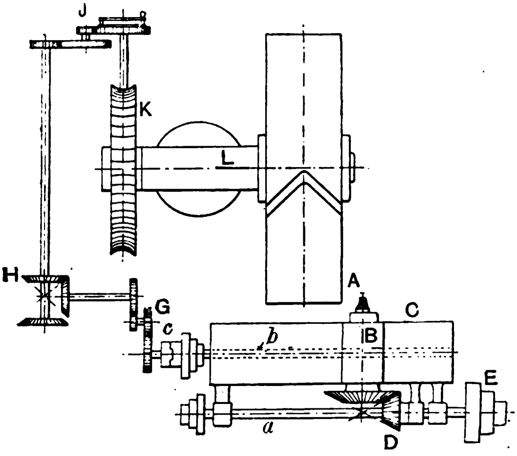


Fig. 163.

End Milling Machine.—A diagrammatic view of the essential elements of a machine using end mills is given in Fig. 163. It provides for the rotation and reversal of direction of rotation of the blank indexing, and for variable feeding of the cutter. The end mill

A is carried in the saddle B, which is traversed along the bed C. It is rotated by the bevel gears D from the shaft *a*, driven by the stepped cones E. The longitudinal feed of the cutter is imparted by the screw *b* driven from the shaft *a* through change-speed gears, cones, or other equivalent methods, thrown in by a sliding clutch *c*. Motion is transmitted from the feed screw *b* to the blank F, through change gears, reversing bevels H, dividing change gears J, to the worm and wheel K, and to the work spindle L.

During the rotation of the mill from one side to the centre, the blank is rotated through the angle of the spiral until the centre is reached. The movement of the saddle then actuates a stop which in turn actuates the reversing mechanism H, so causing the blank to turn in the opposite direction at the angle of spiral, when the cutter completes the tooth space. The cutter saddle is returned quickly by mechanism (not indicated) and the dividing at J is done by hand.

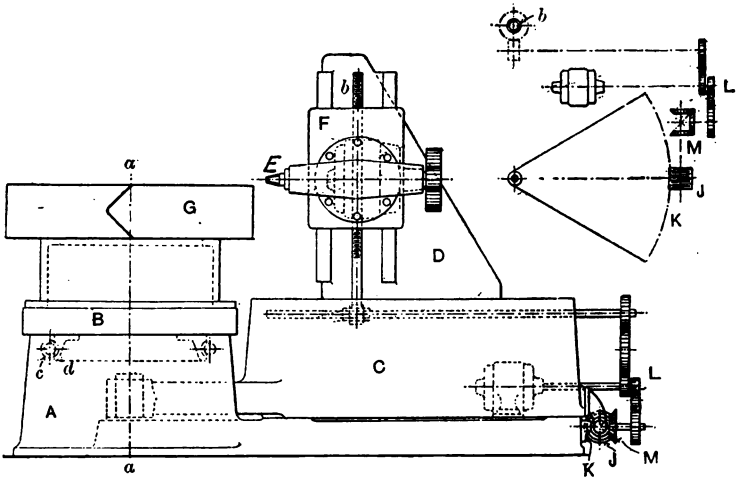


Fig. 164

In another design of end milling machine, the helical movement is not imparted to the blank, but to the cutter, the blank being indexed only. The method is applied to both spur and bevel double-helical gears. Fig. 164 shows in outline diagram the essential mechanism of the helical spur-gear cutting machine.

Design.—The main bed A of the machine carries the work-table B at one end, extended at the other to receive an auxiliary bed C, on which the cutter head D is carried, the whole being movable round the axis *a-a* of the work-table. The cutter E is supported on a saddle F which is travelled up and down the head D by a screw *b*. D also has a longitudinal adjustment on its bed C to suit wheels G of different diameters. The radial motions of the bed and cutter round the centre *a-a* is produced by worm gearing J, K. This circular motion is connected with the vertical motion of the cutter slide F by means of change gears L, so producing the resultant spiral path of the cutter. The direction of motion is changed by reversing mechanism M. *c, d* is the hand-dividing mechanism.

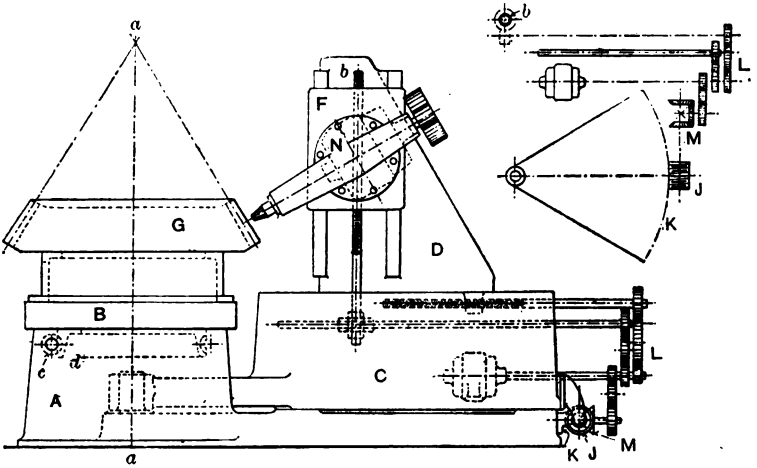


Fig. 165.

Fig. 165 shows the modification necessary for producing bevel helical teeth. The circular plate N provides the means for angling the cutter until its axis stands at right angles with the pitch cone of the blank. In other respects the movements by change gears and the reversal are the same as before, the same reference letters being retained. Fig. 166 shows a pair of bevel gears cut by the Power Plant Co., Ltd.

The Citroën Gears.—These are cut at the works of the Citroën Gear Co., Ltd., Paris, with end mills, including the spur (*see* pp. 163, 164), and bevel helical kinds. The teeth of the Citroën gears are shorter and have a higher pressure angle than the standard, the angle being 45°. Fig. 167 illustrates a gear being cut in the shop.

In cutting helical bevels with end mills, one must modify preconceived ideas gathered in cutting straight bevels with rotary cutters, in which the gears are rolled, or the cutter is set over. The increased width of section in the helical bevels is produced in the deepening of the cut by the enlarged section of the cutter entering deeper into the metal in traversing across from the smaller to the larger diameter. Messrs Citroën have explained the method which they adopt, thus:—

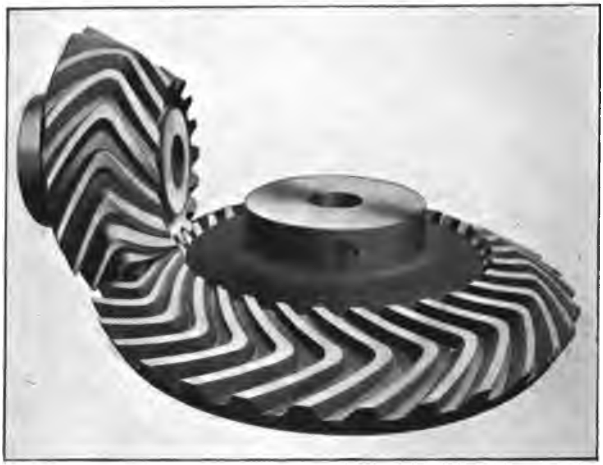


Fig. 166.—Double-helical Bevel Gears Cut by the Power Plant Co., Ltd.

" We have two pitches, the circular or apparent pitch in the plane of the pitch circle, and the real or normal pitch taken in a section perpendicular to the helix. The normal pitch can be obtained from the circular by multiplying the latter by the cosine of the angle (A) of the two planes, which give the two sections of the tooth above referred to.

" In the teeth of our bevel wheels the main point is to have a real (normal) pitch of equal dimensions all along the surface of the teeth. The apparent (circular) pitch will vary, but the real one must not. This is obtained by the peculiarity of the curve of our bevel tooth.

This curve is defined by its projection on a plane perpendicular to the axis of the bevel wheel. This projection is an Archimedean spiral.

"If we consider the angle (B) made at a point of this curve by the radius, and the tangent to the curve in the Archimedean spiral, this angle increases when the distance from the point of the curve to the centre increases. As this angle (B) is proportional to the angle (A) just referred to, the latter increases from the interior pitch circle to the exterior one. Therefore the cosine of this angle decreases, and in the value of the real pitch; this decrease of the cosine compensates the increase of the apparent pitch. This compensation is completed by the tool cutter entering deeper into the metal as it travels from the interior of the pitch circle to the exterior one.

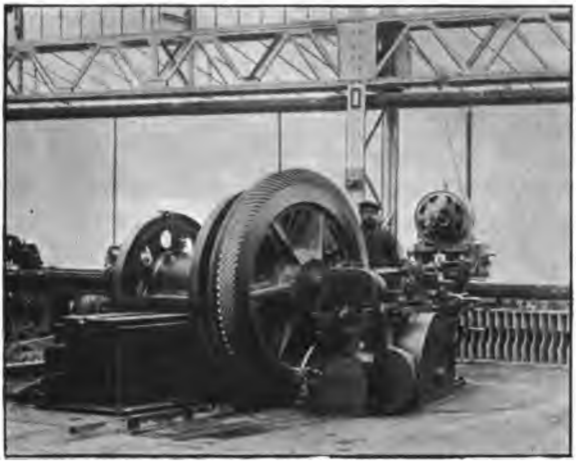


Fig. 167.—Cutting a Citroën Double-helical Bevel Gear.

"Thus our gears do not require a corrective cut by rolling the gears or by setting over the cutter. Our end mill cuts the teeth in the solid in one operation from one side of the face to the other. The apex of the tooth is formed in a second operation."

CHAPTER XII

FORM PLANING

THOUGH the form-planing methods appear to have been menaced by the advance of the generating bevel-gear planers, they nevertheless occupy a very strong position. The theory of a generating machine suggests the ideal, but the actual work done with a form three or four times larger than the large end of the tooth of a bevel leaves but an infinitesimal degree of error. We therefore find that form planers are made in very large numbers for bevel gears, and in larger numbers and in more diverse designs on the Continent and in the United States than they are here. Many shape both sides of a tooth at the same time, or, rather, in alternating double strokes; many are of horizontal type—that is, the arbor moves round in a horizontal plane instead of in a vertical angle.

A Newton Machine.—An illustration is given in Fig. 168 of a very heavy spur gear being cut by the aid of formers. Here there is an undoubted advantage in adopting this method, because the gears are much too massive to be put on any of the ordinary machines of any type. It is really a portable slotting machine attached to a bed which carries a dividing table on which the gear is mounted, and which has in this case a capacity for gears up to 40 feet in diameter. The templets are fitted on each side of the tool slide. When the head receives an outward feed at the top of the stroke it is also simultaneously fed laterally. This last movement is imparted to a star wheel which pulls the head to one side against the templet. A friction device allows the feed to slip to follow the curved form of the templet. This machine was built by the Newton Machine Tool Works, Inc., of Philadelphia, and is shown in use at the Vulcan Ironworks, Wilkes-Barre, Pennsylvania, U.S.A.

Fig. 169 shows a tool which is sometimes used on the Gleason machines for teeth of 2-pitch and smaller, for cleaning out the whole tooth shape without employing separate tools for sides and fillets. A similar expedient has often been adopted with ordinary cast gears on the slotting machine, using a notched dividing plate for pitching the gear around.

The Gleason Bevel-gear Form Planers.—These machines are of the horizontal type—that is, the axis of rotation of the blank is horizontal. This is preferable to the design in which the blank is adjusted for angle against the vertical face of the main framing of the machine. In the Gleason, the whole of the blank-carrying and indexing mechanism, as well as that of the tool-holder, is supported on a bed which is steady under the heaviest cuts. These machines are made in several designs, including those for the cutting of spurs as well as bevels. The example illustrated is a 24in. machine, shown in front and rear views by Figs. 170 and 171, made at the Gleason Works, Rochester, N.Y., U.S.A. Machines of 18in., 36in., 48in., 54in. and 77in. capacity are also built.

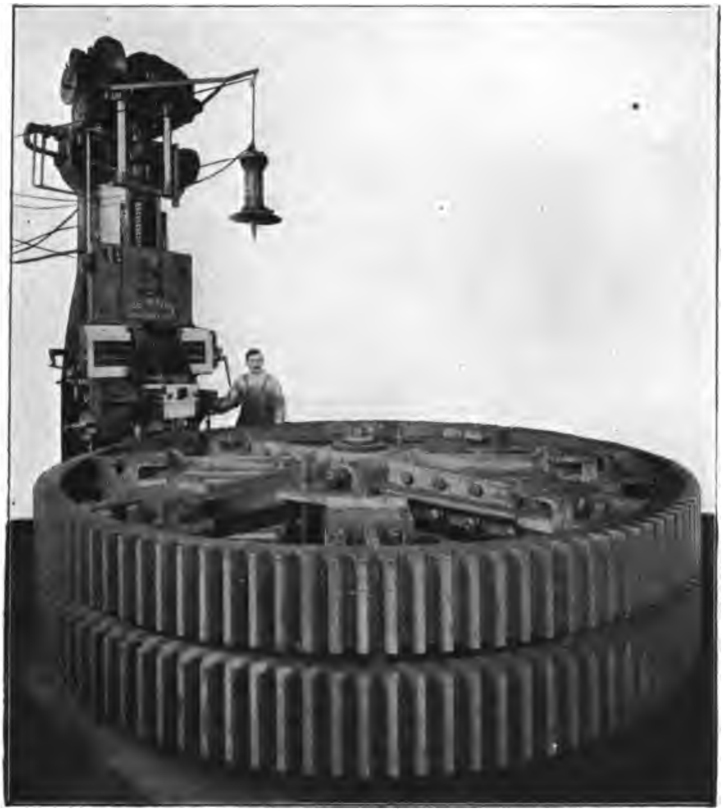


Fig. 168.—Newton Form Planing Machine for Spur Gears.

The gear is, as seen, mounted on a horizontal spindle, and is adjusted for cone distance (*see* p. 70) by moving the head which carries the spindle along the bed until the apex of the gear coincides with the centre of the machine. The arm on which the tool-holder travels is rotated around this centre in a " turret " in a horizontal plane. The arm is also hinged on the rotating head in a vertical plane over the centre of the machine to impart the vertical movement as it is fed over the form, in order to give the correct travel to the tool and impart the correct cone angles to the flanks.

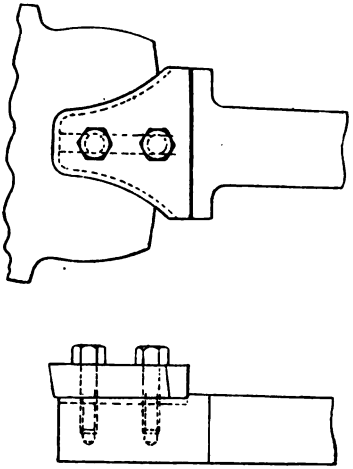


Fig. 169.

The Forms.—Three forms are used. One set is for roughing, one for finishing the upper side of the teeth, and one for the lower side of the teeth. They are carried in a revolving holder so that they can each be brought into position without resetting. They are shaped for involute gears, but cycloidal forms are supplied when desired. This is an obvious advantage of the form planing machines which the generating machines do not possess.

Setting up the Blank.—The action of the machine can be understood if we describe the different parts in connection with the setting up for operation. The angles of the gear must all be obtained first. There are the pitch angle, the angle of the face or point, and the root angle of the teeth (*see* p. 70). The circle of the full depth of the teeth should have been struck round on the large diameter or back angle of the blank before putting it in the machine.

A, Fig. 171, is a centring former or pitch-line plate. This has a notch or tapered slot in the centre of its upper edge, in which the form roller is seen resting, touching both sides of the slot. This plate is attached to the form-holder B, in place of the parallel roughing form required later.

The Turret.—The angular throw of the turret which carries the tool-holder is next set. This throw is equal to the difference between the root angle and the face angle of the gear to be cut, to which 1½in.

must be added for the clearance of the tool. This is done by loosening a taper gib by means of the crank handle C, Fig. 171, and adjustment is made so that a pointer stands at the throw of the turret required. There are graduations at D, Fig. 170, for the purpose of this adjustment.

Next, the bolts E in the form-holder bracket in Fig. 171 must be loosened, also the bolts F in the same figure in the turret. Then with a wrench on the stud G, Fig. 171, turn the turret until the pointer *a*, Fig. 170, stands at the pitch angle of the gear to be cut. Next tighten the bolts in the form-holder bracket H, Fig. 171, and the bolts F in the turret. Loosen the set screws *b* in the connecting-rod strap J, Fig. 170, and pull the tool-post slide K, back out of the way, being first sure that the tool-post L, Fig. 171, is parallel with the top of the arm M in Fig. 170.

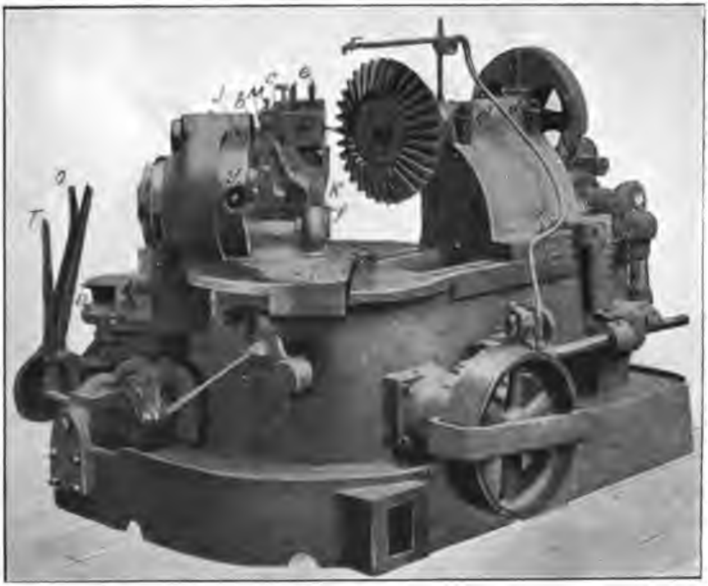


Fig. 170.—Gleason Form Planing Machine for Bevel Gears (Front View).

The Tools.—The proper tools, which are box tools, Fig. 172, have now to be selected. The cutting point is like that of an ordinary planer tool, but the tool body has an adjustable stop for convenience in setting. These stops are placed at a fixed distance behind the cutting point each time the tool is ground. Then, when the tool is put into its holder, the necessary adjustment is provided by the stop without any further trouble to the operator. The slide which carries the holder is driven through triple gears.

In selecting the tools, the thickness of the roughing or stocking tool at A, Fig. 173, must be such that it will clear the tooth space at the minor diameter. The depth at B must clear the depth of the teeth at the major diameter. The finishing tools must be of the proper thickness at the point to pass through the small end of the teeth without touching the opposite side.

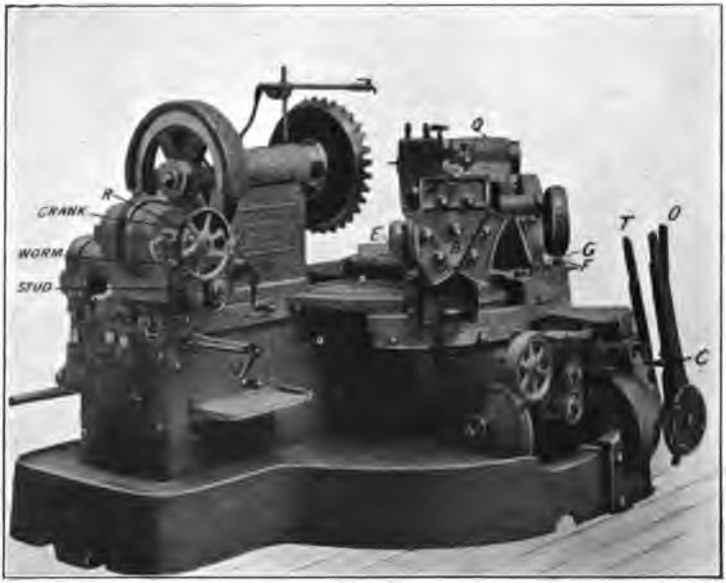


Fig. 171.—Gleason Form Planing Machine for Bevel Gears (Rear View).

The stocking tool is first placed in the clapper block N, Fig. 170, and set by gauge so that the edge of the gauge is even with the centre of the tool, and the end of the tool is even with the end of the gauge. The box tool-holder is so constructed that the tool can be readily adjusted into position by means of a wedge for height, and the stop nut *c*, Fig. 170, for length.

After the roughing round of the blank is done, the stocking tool is removed from the clapper block and replaced with the lower finishing tool. This is set so that the cutting point of the tool is even with the gauge, and the round on the tool, if any, projects beyond the end of the gauge. The upper finishing tool is set at the same time and in the same manner, and left in the machine ready for making the next adjustment.

Gauging.—The gear blank may now be mounted on an arbor suitable for it and fastened securely in the spindle of the machine. Then the head binder bolts *d d d,* Fig. 170, are loosened, and the vernier slide in the cone distance gauge, Fig. 174, is set to read the depth to the pitch, or to the addendum of the tooth to be cut. Then the cone distance gauge is placed on the tool-slide arm M, Fig. 170. The gauge and head are moved by means of the crank handle N, Fig. 171, until the pointer C, Fig. 174, on the vernier slide comes in contact with and is even with the outside edge or crown of the gear, the vernier slide A having been set to read the same as the pitch depth, or the addendum of the gear to be cut. When the indicator B points to zero, the gear blank is in its proper position—that is, the cone centre of the gear coincides with the centre of the machine. The head is now bolted securely to the machine frame, and the cone distance gauge tried again. Then the pitch-line plate A, Fig. 171, is removed and replaced by the straight form, or parallel.

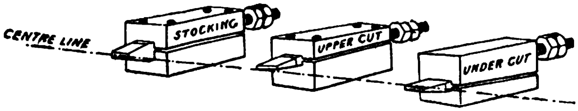


Fig. 172.

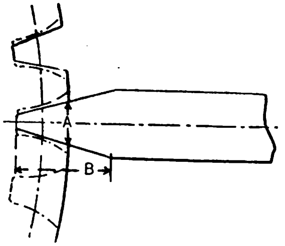


Fig. 173.

Feed.—The turret is next swung in towards the gear blank as far as it will go, by means of the feed cam lever O, Figs. 170 and 171. If the end of the tool when brought against the gear blank is outside of the full-depth circle, the bolts F, Fig. 171, in the turret must be loosened, and the cam turned with the feed cam lever O until the roller is engaged at P in Fig. 170, which swings the turret to its innermost position. Then the bolts F, Fig. 171, are tightened, and the feed motion is set. But if the turret is swung inwards and the tool reaches the full-depth circle before the turret has finished its feed motion, then the turret must be stopped, leaving the point of the tool on the full-depth circle, and the bolts F, Fig. 171, be loosened. Then the feed cam is turned until the roller is engaged at P, Fig. 170, and the bolts F are tightened. Next the upper-cut finishing tool is taken out, and the roughing tool put in its place, and set so that the end of the tool reaches the full-depth circle. The stop nut *c*, Fig. 170, on the box-holder can be adjusted for this purpose. All three tools should now be tried to see that the end of each stands at the full-depth circle, the roughing tool being left in place, and the set screws *e*, Fig. 170, tightened. Now the turret is swung out clear of the blank by means of the feed-cam lever O. The crank-pin bolt *f*, Fig. 170, is loosened, and the stroke of the tool is set by the adjusting screw *g*, which stroke should be about ⅞in. longer than the face of the gear to be cut. This allows ¾in. for clearance at the back of the teeth, and ⅛in. for clearance at the inner end. The crank plate is graduated for these adjustments.

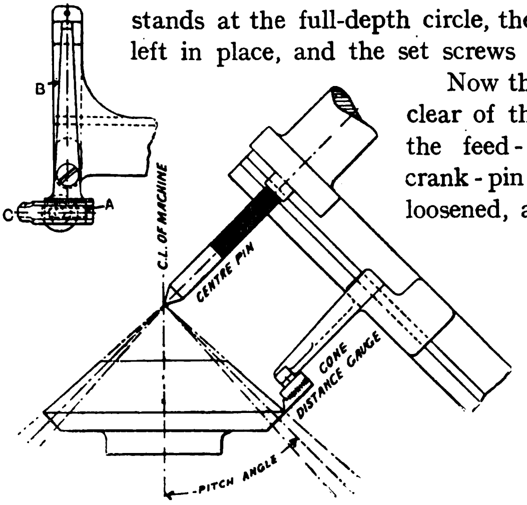


Fig. 174.

The tool-post trips on the return stroke as in an ordinary planer. The stop Q, Fig. 170, must be set so as to bring the clapper block to its proper position before taking each cut. T is a lever for throwing out the power feed.

Tables are provided for the change gears for speeds and feeds, and an indexing table for spacing the number of teeth required. The "worm" and "stud" gears, and the "crank," are indicated in Fig. 171. The clutch R, on the index mechanism, is marked with four equally spaced lines, and the part of the clutch which is keyed to the shaft is provided with a zero mark. These clutches must be set on zero, and now the machine is ready for the roughing-out.

Cuts.—After the roughing-out has been done, the clutch R, Fig. 171, is released. With the crank wrench on the worm shaft *h*, the shaft is turned in the direction shown by the arrow to the left hand, one quarter turn, less one notch. Tightening the clutch R, now, note that the clutch stands even with the zero mark, one notch away from a mark 1. On the machine, on one side of the mark 1, an L is stamped, and on the other side an S. This shows the operator that by setting the clutch on one side the tooth will be large, and on the other side the tooth will be small. The clutch is now set for the semi-finishing cut—that is, one notch large.

The top side form is now swung under the form roller in Fig. 171. The roughing tool is removed and replaced with the top finishing tool, and a cut is taken over the top sides of all the teeth. Next the clutch R is released and set to mark 1, which brings the pitch point P, Fig. 175, up to the centre of the machine. Now a finishing cut is taken over the top sides of all the teeth. Next the clutch R, Fig. 171, is released, and the worm shaft turned in either direction until zero is at one notch from mark 3 on the L side. Then swing the form for the lower side under the form roller, take a semi-finishing cut over all the teeth; release the clutch and set up one notch, so that mark 3 comes directly opposite the zero mark, and take a finishing cut over one tooth. Then measure this tooth with a thickness gauge, or with an adjustable tooth caliper. Solid gauges are supplied for a range of pitches from 16 to 2 diametral pitches (*see* p. 29).

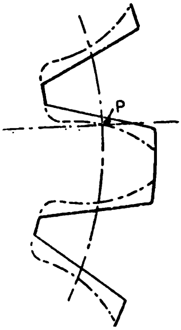


Fig. 175.

Fig. 172 shows the tools in the box, and their respective relations to the centre line. The diagrams in Fig. 176 illustrate the action of tools in cutting roots and flanks, and imparting a large radius to the roots.

A clutch at S, Fig. 171, is used only when a very fine adjustment is wanted. It has 300 teeth. The outside is marked with lines equally spaced, about ⅛in. apart, which help the operator in making an adjustment. The amount by which one notch will set up a tooth depends on the diameter of the gear. The larger the gear the more will the tooth be set up. For example, one notch will move a tooth of a 24in. gear 0.002in., but will move a 12in. gear only one-half that amount, or 0.001in. When the first tooth has been planed to correct thickness the cuts can be taken all round with this setting, finishing the gear.

The photograph, Fig. 176, A, shows a special internal bevel gear which was cut by Messrs E. Arnold Pochin & Bros., of Manchester.

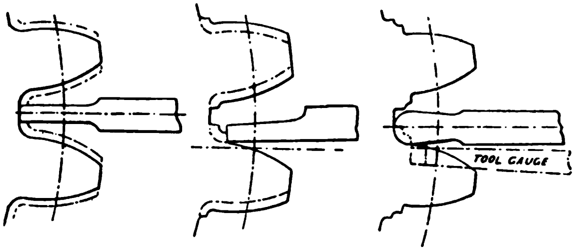


Fig. 176.

As it had not to be dealt with in quantity, an emergency rig-up was arranged on a former-type Gleason bevel-gear shaper. The gear was mounted with the teeth towards the headstock of the machine—the opposite to the usual way of mounting blanks, and a special L-shaped tool was made with the short length sufficiently long to clear the full width of the face of the gear. The clapper-box was tightened securely so that it could not move, and this entailed hand feeding.

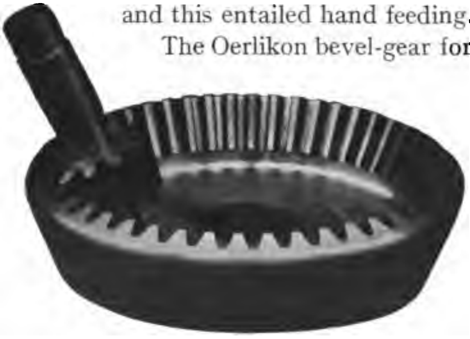


Fig. 176, A.—Internal Bevel Gear Cut by Messrs Pochin.

The Oerlikon bevel-gear form planers have been manufactured for a great many years in several different models. The latest 1913 design is shown by the photograph, Fig. 177. Subsequent figures illustrate the method of operation of the machine and the tools, with the mountings.

The leading features of the Oerlikon planers are these: They are built rigidly, to eliminate vibration and secure a large output. Two tools travel simultaneously, finishing each side of a tooth. The tooth shape is produced by the guidance of one edge of a templet or form representing one flank of a tooth, enlarged. One templet only is required for all gears of the same number of teeth and of the same pitch-line angle, no matter of what pitch and diameter the wheels are. With a set of 35 templets all gears from the smallest up to the largest within the limits of the machine, and a ratio of 1:1 up to 1:8, may be cut. The shaping of all the teeth of a gear is effected automatically, the work of the operator being limited to the setting up and observation of the machines, the removal of the gears and replacing them with blanks. The machine gashes the gears preliminary to finishing them. These machines are made in five sizes, with a capacity up to 6ft. 6in. pitch diameter. They are of the horizontal type. The whole of the mechanism is carried on a box base in the smaller sizes; in the larger ones it is placed on a floor plate, and is driven by the fast pulley (with a loose one adjacent) seen at the left of Fig. 177, making 500 revolutions per minute.

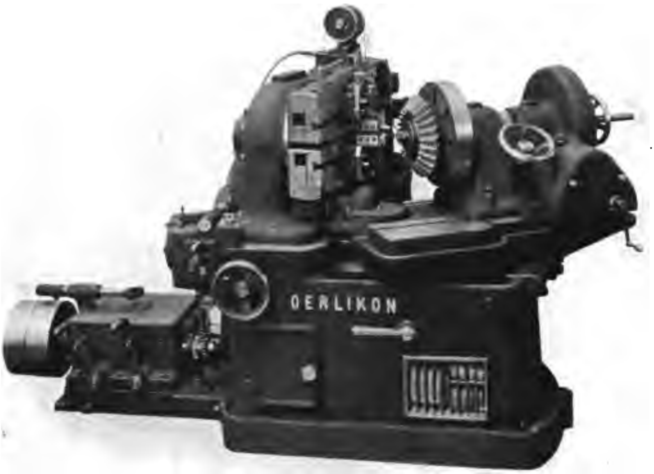


Fig. 177.

The work-head or poppet A, Figs. 178, 179, is carried on, and adjustable along a swivel base B, Fig. 181, swivelling around the cone centre *o* of the machine. The tools are carried in holders C, C′, Figs. 178, 179, in slides D, D′, movable along the guides E, E′ which are adjustable for flank angles on the tool-head F, and are counter-weighted to relieve the load on the form, and to reproduce correctly the form of the tooth flank. The other references on the drawings will be noted in connection with the setting up of the work and the tools.

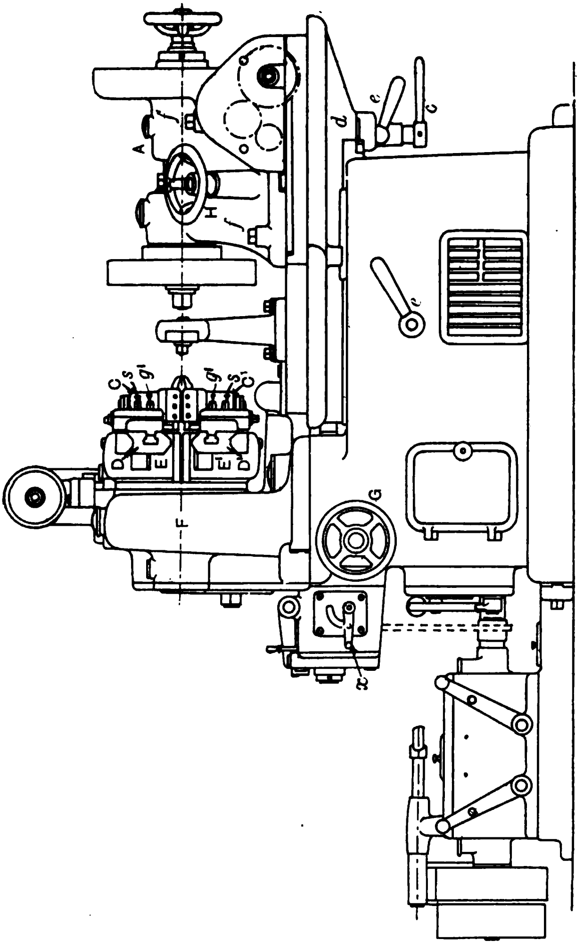


Fig. 178.

The blank has to be turned to the correct dimensions, especially to the outside cone angle *d*, which corresponds with the angle of the addendum of the tooth and the outside diameter, Fig. 181. Also the angle of the pitch circle *b*, and the angle of the root or dedendum *c*, must be known as in all bevel-gear blanks.

Mounting the Blanks.—The blank being mounted on the mandrel in the head A carried on the swivel base B, is duly centred. A grip *a*, Fig. 179, is unscrewed, which sets an eccentric bush free and thus disengages in turn the worm of the dividing wheel by the aid of the lever *b*. The dividing wheel is then turned, the wheel is centred and locked, the worm of the dividing wheel being engaged by the lever *b* screwing up the grip *a*.

The plan or pitch angle of the gear is set by moving round the work-head by the handle *c*, reading the degrees at *d* within about ½′. The head is set by the screws *e e e*, Figs. 178, 179. The machine is set to the cone distance thus:—The tool slide F is set to the angle of the addendum of the gear by the hand wheel G, Figs. 179, 180, reading the degrees at *g*, Fig. 181. The point of the tool is set by means of a gauge, which is fixed on the lower tool-holder. The work-head is advanced by the hand wheel H, Fig. 178, until the blank touches the gauge, Fig. 182, and sets the position of the point as in Fig. 181. When the zero lines *o* fall exactly the one over the other (Fig. 181) the head is secured by bolts *f*, Fig. 178, and the gauge is removed. When setting the machine to the cone distance the tool slides always have to be placed in their horizontal positions.

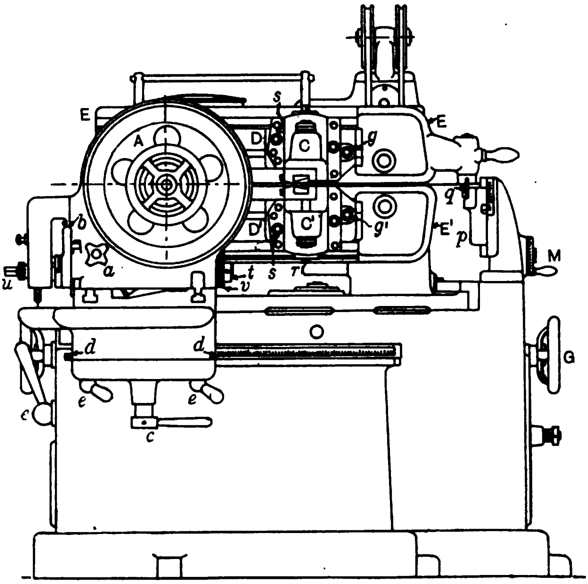


Fig. 179.

The Tool Slides.—The means of regulation of the course of the slides on their guides is seen in Fig. 180. By trial the pin N has to be set so that the travel of the slides is about ⅜in. more than the total length of the teeth to be cut. The slides are brought into the extreme position in front to the point of reversal of the movement. The tool-holders have to be advanced until the tool will clear the blank sufficiently (Figs. 183, 184). Then the tool-holders are fixed by the bolts *g*, *g*,′ Fig. 179. After that the slides are brought into the extreme hinder position to the point of reversal of the movement. The templet slide *p* has then to be lowered by means of the little hand wheel M, so that the pin can pass over it freely.

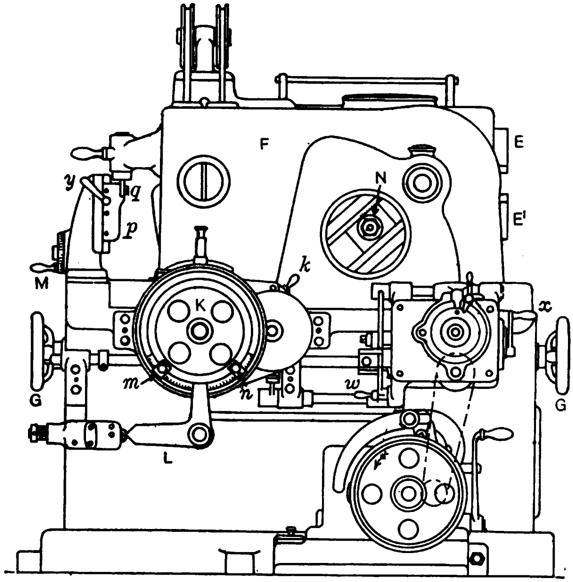


Fig. 180.

Depth of Cut.—To regulate the depth of cut of the teeth, a ratchet *k*, Fig. 180, is lifted. The tool-head F is advanced by the hand wheel G until the scale is set to the angle of the root *c*, Fig. 181. The dog *m* is then moved in the slot of the wheel K, Fig. 180, governing the depth of cut as far as the lever L, and fixed. The tool-head is returned as far as the angle of the addendum of the tooth *g*, Fig. 181, plus the clearance necessary to pass the tools over. For this the prevention of contact depends on the clearance *a*, Fig. 184, which can be regulated by moving the dog *n* round the slot in the wheel to the lever L. The clearance must not be too large. It is necessary to fix the dog *n* securely. The speed of cutting is regulated by sundry levers and gears, the latter being enclosed.

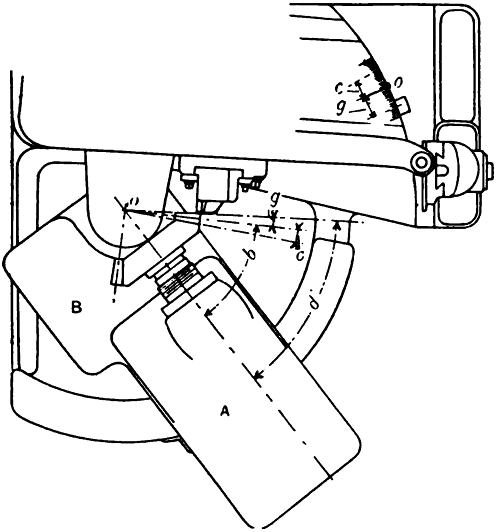


Fig. 181.

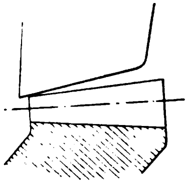


Fig. 183.

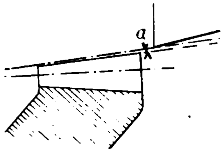


Fig. 184.

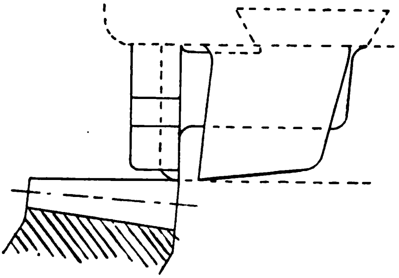


Fig. 182.

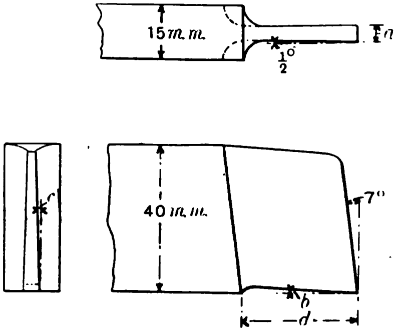


Fig. 185.

Gashing.—Fig. 185 shows the parallel gashing tool for straight cutting, with the cutting angles adopted. Fig. 186 is one used for gashing approximately to the tooth curves, cutting in steps. Its dimensions depend on the depth of the teeth measured on the minor diameter, and leaving enough material for the correct finishing of the teeth. The depth of the teeth is drawn on the interior and exterior diameters, as indicated in Fig. 186. This represents a tracing of a gear of 30 teeth, module 6, with a pitch cone of 45° (mitres). A gashing tool suitable for all gears of the same module is obtained by drawing it out for mitres of the largest possible number of teeth. For module 6, for instance, $\frac{500 \text{ m/m max. dia.}}{\text{module } 6}$ = 80 teeth, drawn out for the interior diameter and taking the length of teeth to be equal to the module multiplied by 15. The height of the serrations or steps is given by the distance *k*, Fig. 186, normal to the radius *r*.

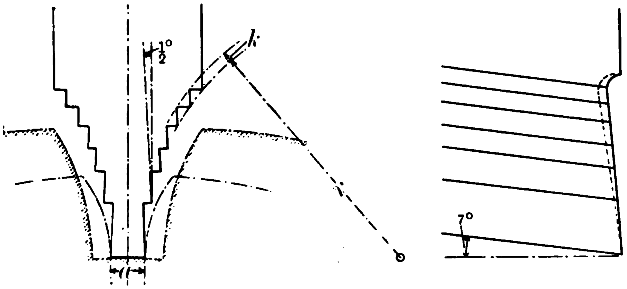


Fig. 186.

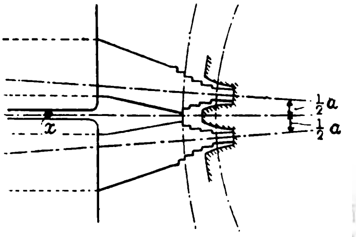


Fig. 187.

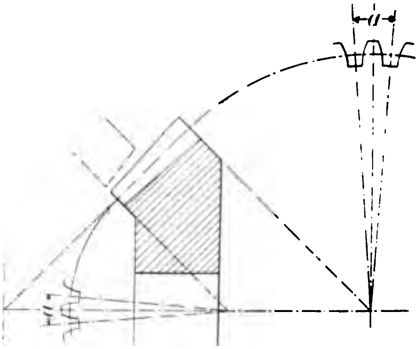


Fig. 188.

When two tools are used for roughing, as in Fig. 187, they are drawn out for the minor diameter, and are set or bent to half the angle made by the centres of adjacent tooth spaces with the cone centre (Fig. 188). To obtain the most suitable shape these are made for a mitre gear. The tools are bent thus in order that the tool-holders shall not foul each other in their movement towards the cone centre. A space of 2mm. is left between the tool-holders at *x*, Fig. 187, in their positions of nearest approach. It is advisable to make sheet-iron templets to shape these tools by. In place of the stepped tools straight ones may be used, as shown in Fig. 185.

The gashing of the teeth can be done by two different methods. In one the straight tools are employed thus (Fig. 189):—The first cut removes a parallel thickness of metal, 1. The next takes off the two angles, 2, with two tools, to about half the depth of the teeth. Another cut goes to the depth of the tooth, 3, leaving about .012in. as a margin for finishing. The fourth, 4, finishes to the exact thickness of tooth. The first three operations may be effected at a high speed and with a large feed, while for the finishing cut a lower speed and less feed are recommended to get a smooth surface on the tooth flank. For the same reason oil is employed as a lubricant. The other method employs the serrated tools, which are to be preferred for cutting tough material and for pitches up to 10 modules. The gashing may also be done with two tools, as in Fig. 187.

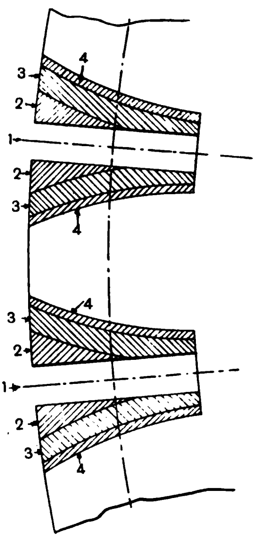


Fig. 189.

To set the point of the plain gashing tool (Fig. 190) the adjustable guides E E′, Fig. 178, are approached one toward the other into their horizontal positions by lowering the slide of the form holder *p* by means of the hand wheel M, Figs. 179, 180. The roughing tool is fixed in the lower tool-holder C′, afterwards drawing back the upper tool-holder C horizontally.

Next the setting gauge is fixed to the guide E′, and the lower sliding tool-holder D′ is clamped by the screw *g*′. Afterward the tool is set as shown in Fig. 190, so that the axis of the tool corresponds exactly with the horizontal plane of the gauge, by regulating the height of the tool by means of the screw *r*.

The stepped gashing tools are set in the same way as the straight tools. When roughing with two tools simultaneously they are set separately, the first tool set being pushed back out of the way while the second is being set.

Finishing.—The tools for machining the flanks are seen in Fig. 191. The amount of bend is about ⅝in. (16 mm.), being bent or hollowed sufficiently to clear the teeth. The cutting angle is 7°. The points are rounded at *a*, Fig. 192, to the gauge. Oil is recommended for lubricant when cutting.

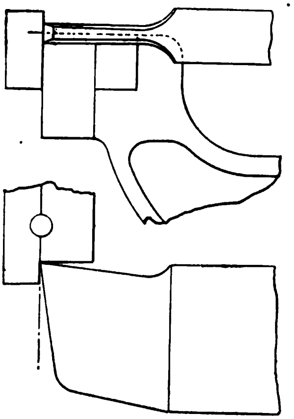


Fig. 190.

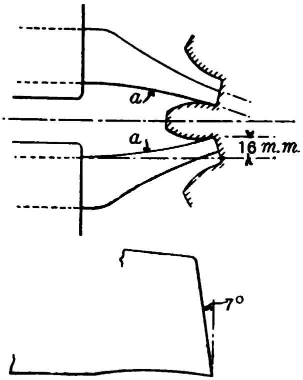


Fig. 191.

The tools which finish the teeth have to be set with great care, thus:—The upper tool is set in the horizontal direction, Fig. 193, as indicated by the dotted lines. Afterwards the gauge for setting the point is advanced until the point of the tool is located over the point of the indicator, and the tool-holder is lowered until the zero line (*o*) of the indicator corresponds exactly with the zero line (*o*) of the gauge for setting the point. The lower tool is set in relation to the upper one so that there is no space left between the two.

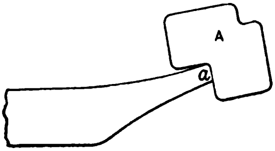


Fig. 192.

When the wheel blank has been roughed out by a single tool the gash is made in the horizontal direction (Fig. 194). To finish the wheel it has therefore to be moved round until the tooth is set exactly between the two finishing tools. This should *not* be done by turning it on the arbor or on the face plate, but by turning the worm of the dividing wheel. Means are provided to release any undue pressure of the dividing apparatus during this operation. Now the machine is started by shifting the lever *x*, Fig. 178, until one division is completed, so as to eliminate any backlash that may be in the gearing. Afterwards one or the other of the operations previously described follows for cutting the teeth.

The thickness of the chip is regulated by the hand wheel M. One graduation indicates a vertical displacement of the form; each arrow on the disc represents .004in. The actual displacement of the tool can be calculated by the formula:—

$$\text{Variation in thickness of the tooth} = \frac{2 \times \text{Displacement of the form} \times \text{Distance } a}{\text{Distance A, of the form}}$$

(*see* Fig. 195).

If the form is set correctly, and for standard involute gears, that is if the distance *a*, Fig. 196, taken on the pitch circle, represents exactly one-half the tooth thickness *b* + the height *c*, the thickness of the tooth will be produced correctly on the machine. The thicknesses are given in a table for different numbers of teeth and different modules. As these are right for spur gears the teeth have to be projected for bevel gears on the back cone distance by the method described in a former chapter (p. 68).

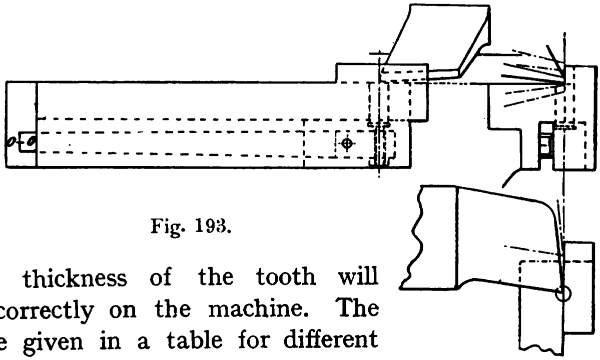


Fig. 193.

The roughing-out with two tools simultaneously is preferable when manufacturing wheels in quantities, as the output of the machine can be much increased. In this case all the wheels are first roughed-out and afterwards finished one after the other, so that the time for setting the machine is reduced.

Thirty-five forms are supplied, selection of which for any gear is made from a table provided. Fig. 197 shows the method of setting up. Thus, to plane a pair of gears of 26 and 57 teeth the relation gives the cone angles of 24° 31′, and 65° 29′. The nearest angles to these in the table provided are a No. 11 form for the pinion, and a No. 11, A form for the gear. The first has a limit of angle of 22° 18′ to 24° 47′, the second of 67° 42′ to 65° 13′. The angle is set by the guide 1, Fig. 197. The setting angle for the pinion, as given by the table, is 19° 25′, and that for the gear is 3° 45′. After setting by the marks on the guide 1 the screw 2 fixes it. Usually the form is not utilized along its whole length but a radius is cut at the root of the teeth. To permit this a guide limit 3 is employed. The tool is thus stopped at $\frac{1}{12}$th of the depth (Fig. 197), by stopping the advance by the guide limit 3. For the vertical setting of the form-holder 4 the set screw 5 is used.

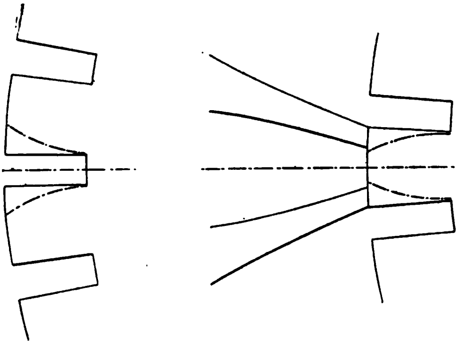


Fig. 194.

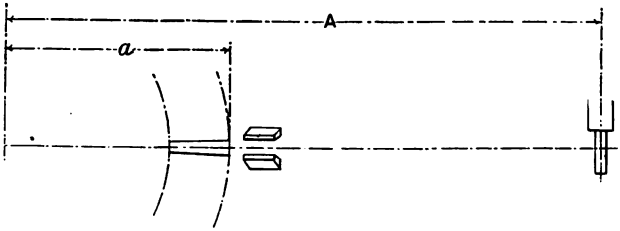


Fig. 195.

Fig. 198 illustrates the setting of the gauge B on the tool-holder A to the half angle of *a* (Fig. 188), where it is fixed by the screws. The upper tool-holder rests in place in front, and Fig. 199 shows how to cut the tooth exactly in the centre between the two tools.

These machines are designed and manufactured by the Schweizerische Werkzeugmaschinenfabrik Oerlikon (Société Suisse de Machines Outils), at Oerlikon, near Zurich (Switzerland).

The Greenwood & Batley Machine.—The form planer of Messrs Greenwood & Batley, Limited, of Leeds, represents a type which has been familiar for many years. Originally indexed by hand, it is now fully automatic. Fig. 200 shows the entire machine, and Fig. 201 the tool box and some details adjacent.

Elements.—The essential elements in the machine illustrated are the headstock, with its work spindle, adjustments and feeds, the dividing mechanism, and the reciprocating tool slide. The axis of the work spindle is always disposed radially towards the centre around which the headstock swings—which centre is the apex of the bevel gear. The planer tool slide located above reciprocates in a strictly linear path. The tooth curves are produced by the rolling of the blank between the reciprocations of the tool in conjunction with the feeding of the blank, which rolling is under the control of the enlarged form.

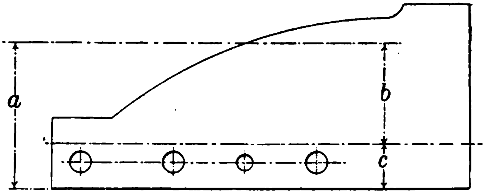


Fig. 196.

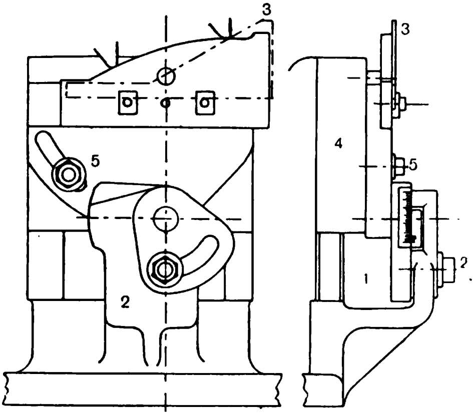
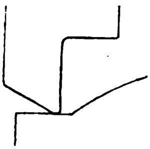


Fig. 197.



Details.—The details of the machine, most of which are apparent from the views given, are these: —The headstock—which carries the automatic dividing mechanism at its lower end, and the work arbor at its upper—is adjustable on the face of two superimposed sector plates which swivel around their pivoted centre—the apex of the bevel gears to be cut. The gear blanks are mounted on the arbor, which is secured with a draw-in rod. The adjustment of the smaller sector plate on the face of the larger one provides for setting the gear blank to the correct cone angle, which adjustment is effected by the hand wheel to the right operating a worm and a quadrant rack. The dividing worm and the change gears are disposed at and about the lower end of the spindle.

Movements of the Blank.—The blank, having been set at the angle corresponding with the tooth points, is fed upwards towards the reciprocating tool by the radial adjustments of the larger or hinder sector plate. This feed is derived from a crank disc at the right-hand end of the machine, having a connecting rod with provision for an adjustable stroke, actuating the pawl and ratchet below, whence through worm gears a pinion engages with the teeth cut on the periphery of the large sector. An automatic trip is combined with this mechanism, which, by means of the adjustment of a dog in the annular groove on the face of the worm wheel, seen in Fig. 200, can be set to automatically disengage the feed as soon as the tooth has been cut to the desired depth. Where the feed mechanism is disengaged by the trip, the sector, counterbalanced for the purpose, returns to its original position ready for the commencement of the next tooth, at which stage the feed is automatically re-engaged.

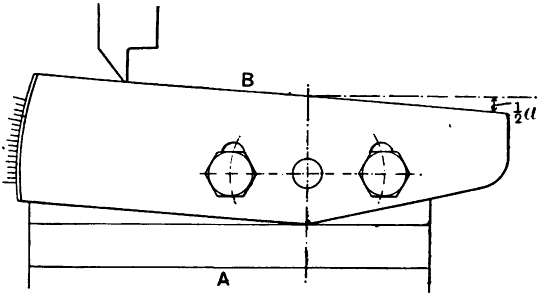


Fig. 198.

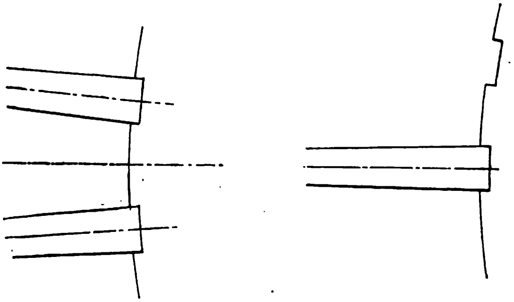


Fig. 199.

While the feed is in operation the blank is also being rolled in the intervals of cutting by the coercion of the form acting around the axis of the spindle and blank. It is seen in both illustrations bolted to a bracket at the upper end of the radial quadrant bar. The edges are pulled into contact with one of the two guide plates, which correspond with the two sides of the teeth, by the tension of the weighted cord passing over the guide pulleys seen, attached to the sliding bracket in which the guide plates are held. The radial bar is adjustable in the socket in which it is carried in order that the height of the form may be regulated to suit the size of the gear blank being cut.

Indexing.—The dividing mechanism is driven by a pulley at the rear of the machine, and therefore not visible, through a friction disc and change gears. It is controlled by a locking cam, which is released on the return movement of the sector plate, a trip lever operating against an adjustable stop which is carried on the radial bar, seen near the bottom of the machine. When a dividing operation is completed the mechanism is automatically locked with the blank in the correct position for cutting the succeeding tooth.

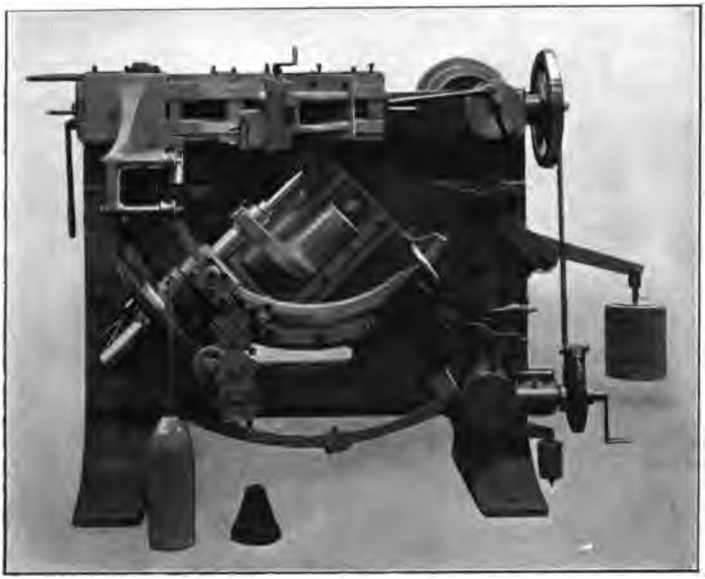


Fig. 200.—The Greenwood & Batley Form Planing Machine.

Tool Box.—The tool box seen in both photographs is reciprocated by a crank disc, having provision for varying the stroke to suit the gear being cut. A driving cone at the rear of the machine provides three changes of speed. The tool box has vertical, horizontal, and angular adjustments, and is hinged to relieve the tool. A fixture is supplied, seen in Fig. 200, by means of which the cutting edge of the tool is set to coincide with the centre of the pin, around which the sector plates swivel. It therefore always planes towards the cone apex. The machine is made in three sizes, the largest gears cut in each being 8in., 18in., and 25in. in diameter respectively.

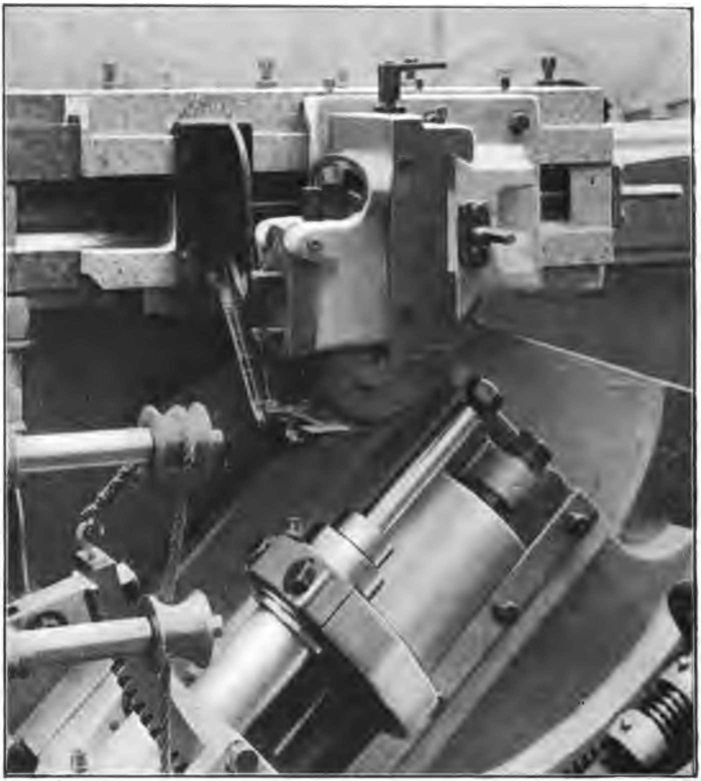


Fig. 201.—Tool Box of Greenwood & Batley Machine.

The Bouhey (French) Machine.—This (Fig. 202) planes single-helical bevel teeth. It is a modification of a type of machine which is very common in France and Belgium, and which is represented here by the machine just described, made by Messrs Greenwood & Batley, Ltd. With minor differences in detail its general construction is as follows:—

It is a form machine, the teeth being planed under the guidance of an enlarged tooth form, the curve of which is about three times larger than the size of the teeth of the gear at the major diameter. The whole of the mechanism is carried by a frame, on top of which the tool and its holder has a reciprocating motion only. On the side of the frame below the tool the wheel blank is carried, receiving its movements from stepped cones. The work spindle, seen standing out at the front with the blank, the indexing worm gear below it, and the dividing apparatus are carried in brackets attached to a large sector which swings in guides on the vertical face of the main frame around the axis which is the apex of the gear, and by which cone angles are adjusted. The blank is set along the work spindle to adjust its cone distance. The indexing mechanism is carried by a frame which is swung under the control of the templet or form. This is fastened to the upper end of an arm on this frame, and bears on a pin or roller fixed in the main frame, the two being held in contact by a counterweight suspended from a cord. Teeth are cut on the outer edge of the sector which swings on the main frame, by a worm, through feed gears below to the right, driven from belt cones. This imparts the angular feed to the blank. At the same time a rocking movement is given to the entire indexing mechanism and worm under the control of the templet or form by which the tooth curves are produced, by the reciprocating cutter. When a tooth has been planed to the full depth the feed is tripped,

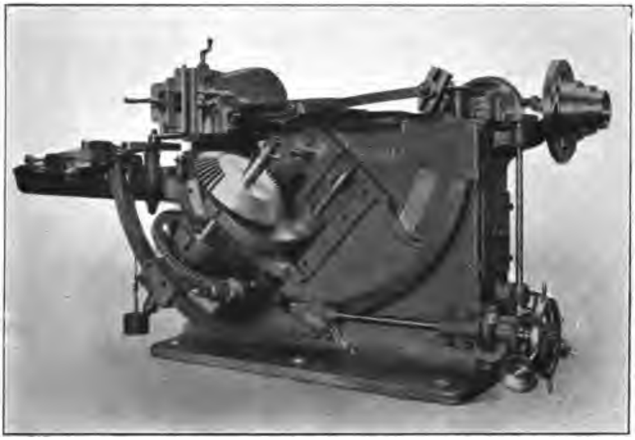


Fig. 202.—The Bouhey Helical Bevel-gear Planer.

and the sector is returned to its original position, the blank is indexed and the cycle recommenced. All the teeth are finished on one side first, and then all their other opposite sides.

The Helical Attachment.—Thus far the Bouhey machine is like many other form planers. But it has in addition mechanism for cutting the helical teeth. A connection is made between the indexing mechanism and the crank shaft which drives the connecting rod to the tool slide. This consists of change gears by which the work is rotated slightly in a definite ratio to the crank during the cutting, with the result that the curved tooth space is cut by the planer tool. The amount of twist can be varied by the change gears. Teeth of the reverse hand to mesh with the first are cut by reversing the connection between the crank and the work.

CHAPTER XIII

MACHINES THAT GENERATE BY PLANING TOOLS

THE SUNDERLAND SPUR-GEAR GENERATING MACHINE.—This is manufactured by Messrs J. Parkinson & Son, of Shipley, Yorkshire, and is designed to include the cutting of spiral as well as spur gears, by which its value is greatly enhanced. The illustration (Fig. 203) shows it performing the first of these two functions, and this will enable an accurate understanding to be gained of the general design and method of its operation, the principle of which was described on p. 94.

Relation of Cutter to Blank.—The relative action of the cutter and the blank is as follows: The blank, being mounted on an arbor in the spindle of the dividing wheel, is controlled by a rotational and a dividing motion. The rack-shaped cutter (*see* p. 94) has a linear motion corresponding with the rotational movement, and at intervals it has a reciprocating motion for cutting. In the first position of the blank and cutter (*see* Fig. 204) the latter, mounted on a vertical slide, is brought into contact with the top of the blank. Then the latter is fed to the proper depth, while the tooth spaces are being shaped by the reciprocating cutter, the slide carrying which moves upwards while the cutting is being done to the proper depth, which operation terminates with a movement of the blank equal to the pitch. At this stage the blank is withdrawn from the cutter, and its rotation arrested, while the cutter is returned to its starting-point, which takes place through a distance equal to the pitch. The blank is then returned to the cutter, and its rotary motion recommences with the upward movement of the cutter slide, when the previous process is repeated. This goes on automatically until the blank is completely cut. This in bald outline states the relative operation of the cutter and blank. The methods by which these are accomplished, and the general design of the machine, may be studied in the drawings and diagrams along with the photographic illustration.

Method of Driving.—All the motions except the return of the cutter slide are obtained from the four-stepped cone A, Figs. 205 and 206. It drives first the cutter slide B across the face of the blank, through the medium of change gears, and a worm gear drive to the crank disc, from which a connecting rod actuates the slide B. The details immediately following are not shown in these drawings, but they will be readily understood. A rack behind the reciprocating slide engages with a pinion which is driven by a rack and pinion at the other end of the pinion shaft. This second rack derives its motion from the crank disc, being cut on one side of the connecting rod. It is so arranged that the distance from the disc centre to the point of connection with the rack can be varied, thereby altering the length of stroke to suit the width of the blank which has to be cut. The revolving disc is encircled by a bearing carrying adjustable bronze shoes, and is driven by a Whitworth quick-return motion, the connecting pin being in the worm wheel, which has a quick lead. The driving worm shaft is connected to the cone shaft by two pairs of gears, either pair of which can be used. These double the changes given by the cone, and as the latter has four steps, eight speeds are obtained for cutting, in geometrical progression.

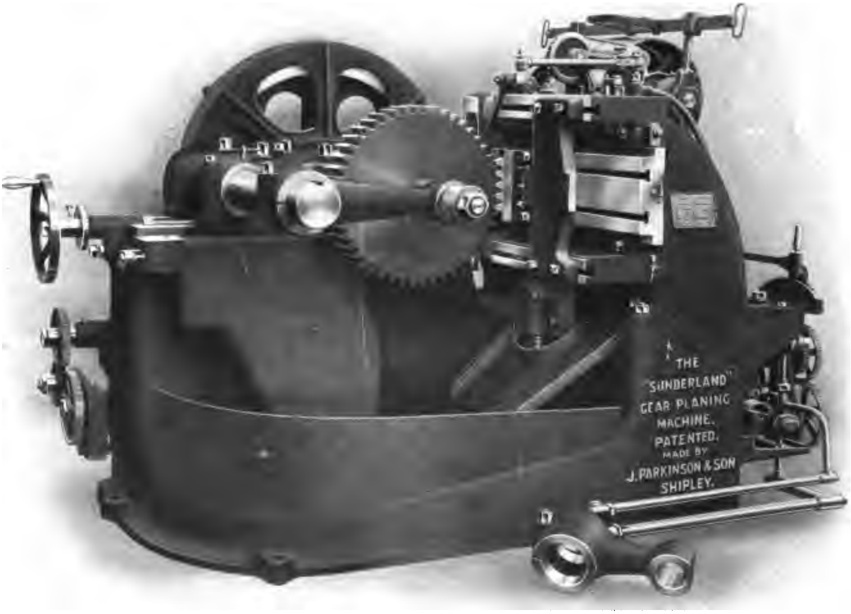


Fig. 203.—The Sunderland Spur-gear Generating Machine.

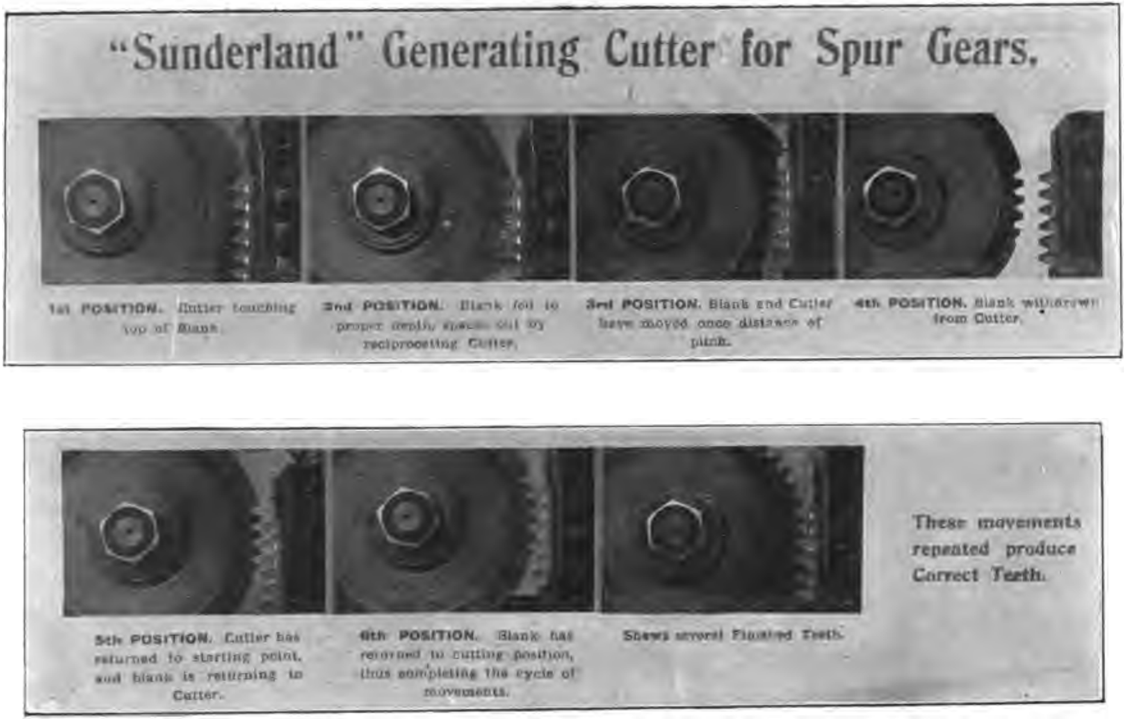

"Sunderland" Generating Cutter for Spur Gears.
1st POSITION. Cutter touching top of Blank.
2nd POSITION. Blank fed to proper depth, space cut by reciprocating Cutter.
3rd POSITION. Blank and Cutter have moved once distance of pitch.
4th POSITION. Blank withdrawn from Cutter.
5th POSITION. Cutter has returned to starting point, and blank is returning to Cutter.
6th POSITION. Blank has returned to cutting position, thus completing the cycle of movements.
Shews several Finished Teeth.
These movements repeated produce Correct Teeth.

Fig. 204.—Illustrating the Sequence of Movements of Cutter and Blank.

Mandrel Drive.—The blank is mounted on an arbor which is driven by the worm wheel C and worm D. The worm can be disconnected to permit the spindle to be revolved rapidly in order to test the truth of an arbor or a wheel blank. The hand adjustment of the slide is provided for by the hand wheel E. An outer support is provided for the blank arbor, and the rim is stayed by an adjustable support on the cutter slide. In addition to these supports the slide carrying the work spindle can be securely locked in position after adjusting for depth of cut, and the slide which gives the in-and-out motion to the blank after each tooth has been cut is held by a dead stop when in the cutting position.

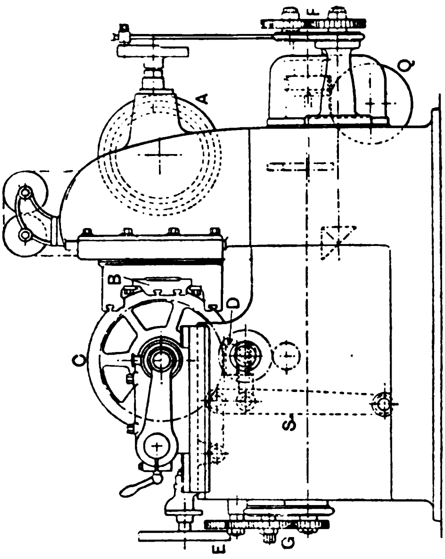


Fig. 206.

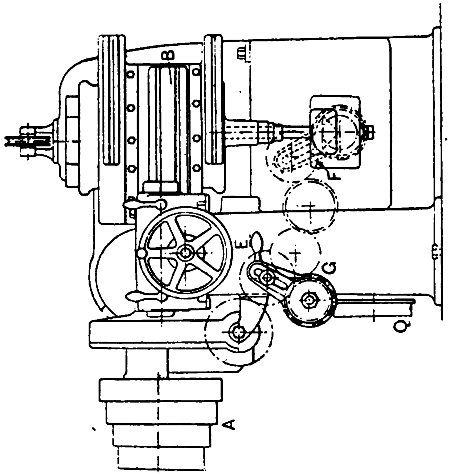


Fig. 205.

When the machine is arranged for cutting spiral gears, the cutter-carrying slide can be swivelled to any angle up to 60°, the action of the machine being exactly as for spur gears, except that the change gears must be selected to suit the "real pitch," not the "normal pitch."

Movements of Cutter and Blank.—The foregoing are comparatively simple movements. More intricate are those by which the relative motions of the cutter and blank are produced precisely as though they were in rolling contact. Other movements are those by which the blank is withdrawn at the completion of a pitch and tooth, and returned; and another, the movement by which the cutter slide is returned to its starting position. These can only be understood by a study of the developed diagram (Fig. 207), some portions of which can be compared with the drawings (Figs. 205, 206), and with the photograph (Fig. 203).

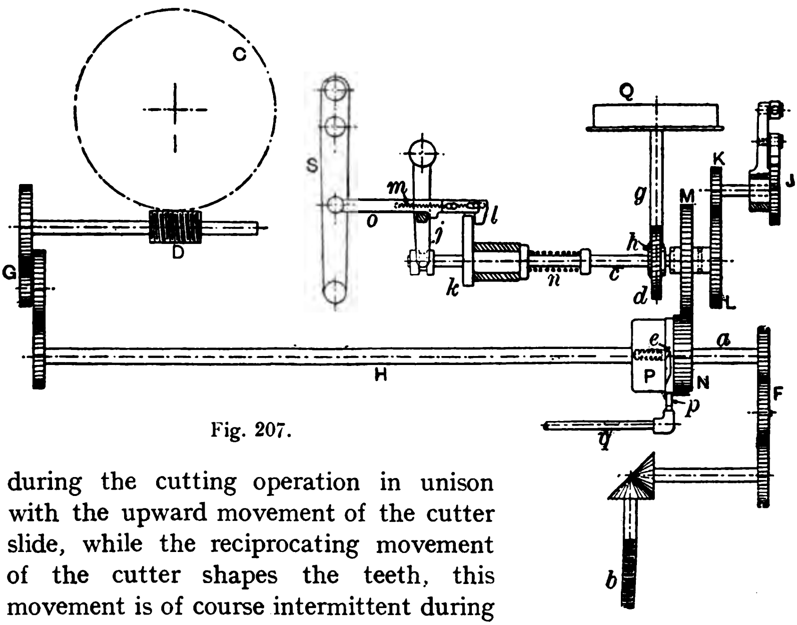


Fig. 207.

Though the general statement is made that the blank rotates during the cutting operation in unison with the upward movement of the cutter slide, while the reciprocating movement of the cutter shapes the teeth, this movement is of course intermittent during the cutting. The feed imparted to the blank and cutter occurs only during the return stroke of the latter, being inoperative during the actual cutting. This, as well as the rate of feed, is provided for by a ratchet and pawl in the train, by which the movements of the blank and cutter are geared together. The ratchet feed rod is seen at the right hand of Fig. 206. Thus, in the diagram (Fig. 207) the change gears F for the cutter motion are seen to the right, and the change gears G for rotating the blank are at the left. They are connected by the shafts H and *a*; but the ratchet wheel J, with its pawl, controls their movements and renders them intermittent. The shaft which carries the ratchet J carries the gear K at the other end, which drives gear L. Gear M, which can be clutched to L, drives N on a short shaft *a*, and the latter carries one wheel of the change-gear train F through which the screw *b* is driven, by which the vertical movement of the cutter slide is imparted, the amount of which movement corresponds with the pitch of the gear being cut. These motions are connected with the motion of rotation of the blank, which is driven by the worm wheel C (compare with Figs. 205 and 206), through the change gears G, which rotate the blank through an arc corresponding with the number of teeth to be cut.

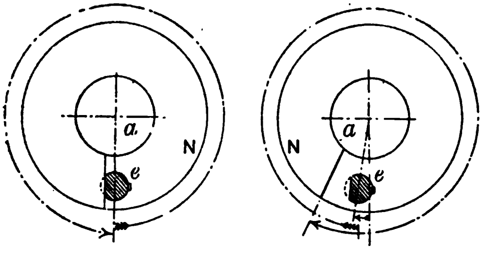
The feed from the ratchet wheel J is operative only when gear M is clutched with L, as shown. It is slid along with its shaft *c* for clutching either with L or with a worm gear *d*, the latter producing the return of the cutter slide, to be noticed directly. The gear M with its shaft *c* may also occupy a middle position without engaging with either L or *d*.

The gear N driven by M is provided with a notch in one side to receive the end of a spring plunger *e*, which is carried in a boss P fixed on the end of the shaft H, which has its axis in line with the axis of the short shaft *a*. While the feed is in, the shaft *a* thus drives shaft H by the plunger *e* as indicated in the enlarged face view (Fig. 208, A).

Return of Cutter Slide.—The belt pulley Q is driven directly from the countershaft at a constant speed, and is the means of returning the cutter slide. Its shaft *g* carries a worm *h* which drives the worm wheel *d*, with which gear M can be clutched while still remaining in mesh with the wider gear N, but being out of action as regards the feed ratchet wheel J. The worm wheel *d* rotates in the opposite direction from L for the purpose of returning the cutter slide. When this motion is in, then the direction of rotation of gear N being reversed, the plunger *e* is forced out of engagement with its notch in N, with the result that shaft *a* no longer drives the shaft H, and the motions of the dividing gear and the blank are therefore arrested. The plunger is forced out because one side of the notch is chamfered as indicated in Fig. 207. This reversal of direction of drive is indicated in the enlarged face view (Fig. 208, B).

The clutching of M to *d* for the return of the cutter slide is accomplished thus:—The movements taking place in the mechanism which lies between the work-slide lever S, and the sliding-clutch gear M. A lever bar *o* is attached to the work-slide lever, having an offset piece which engages with a projection on the lever *j*, which last lever is thus swung to and fro by the movements of the work-slide lever S. The withdrawal of the blank from the cutter occurs when the work-slide lever is pulled over to the left. It then pulls over the lever *o*, which pulls *j*, the forked end of which draws the shaft *c* along and so clutches the gear M with the worm wheel *d*, and thus renders operative the motion from the pulley Q for returning the cutter slide.

The release of the return stroke and the throwing in of the feed again are produced thus:—The shaft *c* carries a cam *k* which rotates with *c* but does not partake of its endlong movement. A sliding hook *l* on the lever *o* lies against the side of the cam *k* when the work slide is withdrawn and the motion of the cutter slide is being effected, the hook *l* being held in contact with the side of the cam by the tension spring *m* until the lever *j* has clutched the gear M to *d*, and the return of the cutter slide has commenced. The hook *l* is clear of the cam *k* during the cutting. As the cam rotates, a gap in its edge comes opposite the hook *l*, when the spring *m* pulls *l* over the edge of the cam, the further rotation of which lifts *l* and *o*, so releasing the lever *j*, when the shaft *c* is pushed along by means of the compression spring *n*, thus clutching the gear M to L for driving the feed. These movements are effected by one revolution of the shaft *c* in each direction.



A Fig. 208. B

The Blank and Tool Slide.—We will now notice the mechanism by means of which the motion of the ratchet wheel J is transmitted to the blank and tool slide. The gear N, which is smaller than M, carries the projecting pin *p*, which at a definite stage of the feed movement engages a spring catch, carried in a lever on the shaft *q*, producing an intermittent oscillatory movement of the latter. This occurs only during the feed and not during the reverse motion. The shaft *q* carries a lever at its other end (*see* Fig. 209), which, during the feed, holds up the bar R. But when the shaft *q* is oscillated, the bar R drops and releases a pawl *s*, which is then pulled by a spring into engagement with the ratchet wheel *t*—being carried by a shaft that rotates constantly. The pawl is pivoted on a disc T, and has a gear *u* keyed to it; *u* is in mesh with *w* of twice the size of *u*. The result is that when the pawl *s* engages with the ratchet *t*, the disc T and gear *u* make one revolution, and *w* makes half a revolution. In the revolution of *t*, the projection *x* on the disc T lifts the bar R to its original height, where it is sustained by the lever *r*. When the revolution is completed the tail of the pawl *s* strikes an inclined face on the bar R, and is thus released from engagement with its ratchet. The half revolution of the gear *w* is transmitted by an eccentric on its shaft through links and lever to the work slide U, on which the blank is mounted. The half ratio just now noticed gives one withdrawal from, and one return of the slide and blank to, the cutter during one cycle.

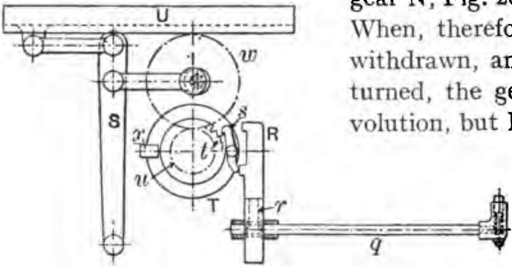


Fig. 209.

Backlash.—Backlash in the cutter slide is taken up thus:—The gear N, Fig. 207, is smaller than M. When, therefore, the work slide is withdrawn, and the cutter slide returned, the gear M makes one revolution, but N more than one; yet, as already explained, without transmitting motion to the shaft H. The notch in the face of gear N, in which the plunger *e* enters, is chamfered on one side, as already stated, in order to force the plunger out when the cutter slide is being returned, and allowing the shaft H to remain inoperative. The cam *k* then releases the shaft *c* as already explained, its spring putting gear M into engagement with gear L and commencing the upward motion of the cutter slide to take up the backlash. This movement continues until the projection *p* on N produces the dropping of the bar R, with the resulting return of the work slide to the position for cutting. Then the plunger *e* enters the notch in the gear N, and the feed of the blank and of the cutter slide again take place.

The Fellows Gear Shaper.—This is made by the Fellows Gear Shaper Company, Springfield, Vermont. It is shown by the four views, Figs. 210 to 213. Fig. 210 is a front elevation, Fig. 211 a plan view, Fig. 212 a rear elevation, Fig. 213 a portion of one end enlarged to show certain details of the mechanism. The principle of its operation is described on p. 92. The general design of the machine is as follows:—

Method of Driving.—It is driven from a countershaft to the stepped cone A on the shaft B. The movements of the cutter are derived from this through a pinion C of 30 teeth on this shaft, which is brought into engagement with the internal gear D for quick motion, or with the external gear E through an intermediate gear C^1 for slow motion (compare with Figs. 211 and 213; the relations of these gears are outlined in the latter). The internal gear D is thrown into and out of engagement with C by an eccentric fitting of its shaft. These movements are produced by the yoke F, which is provided with a handle, and has its eyes fitting over the shaft G, which is rotated by the gears D and E.

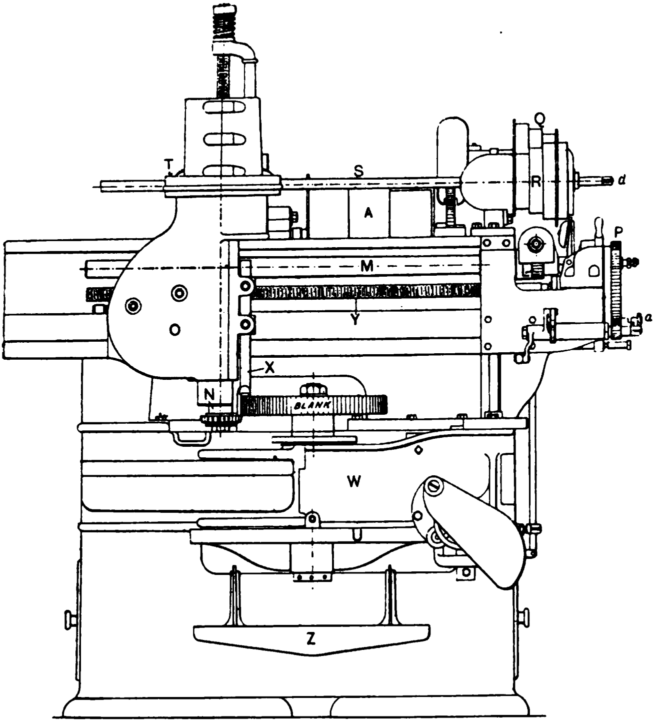


Fig. 210.

A slotted crank H at the opposite end of the shaft G transmits motion to a rack bar J, the length of stroke of which is adjustable in the slotted crank by the screw K turned by its squared end, and clamped by the block seen encircling K. The length of stroke is set by the graduations on the face of the crank seen in Fig. 213, which has divisions of the inch on one edge and millimetres on the other. In this way the length of stroke of the cutter is varied to suit gears of different depths of face. Allowance is made by the graduations for the clearance of the cutter above and below the blank, which is much less than that required for circular rotating cutters.

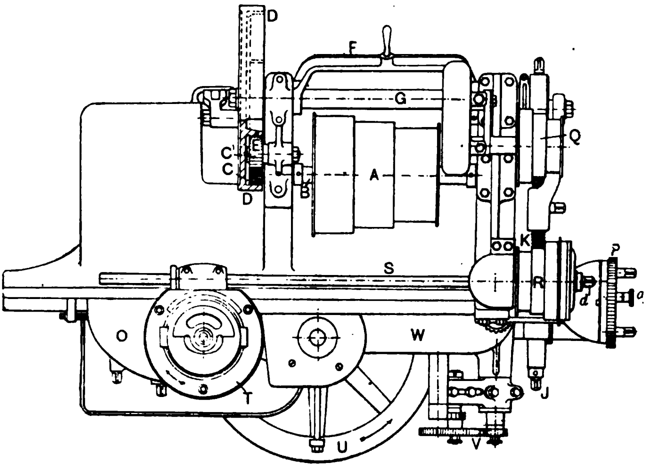


Fig. 211.

The movement is transmitted from the rack bar through a pinion L engaging with the rack teeth on a sleeve encircling the bar J, Fig. 213. The pinion is on the end of a rock shaft M, at the opposite end of which another pinion actuates a rack (not seen) behind the cutter slide N which reciprocates in the cutter guide O. The rate of reciprocation is varied by placing the belt on the different steps of the cone A, and by the engagement of the first pinion C with the ring of teeth D directly, or with E through the intermediate C^1. A downward movement of the yoke F puts in the slow motion with gear E; an upward movement throws in the quick motion with the gear D. Square-headed binding screws over the bearings hold the yoke fast in either position.

The depth of cut, that is, for the depth of teeth, is set by graduations on the periphery of a pilot wheel P, the settings being given in proportions of the pitch or module. On withdrawing a locking pin *a* the pilot wheel can be moved round until an indicator shows the graduations required on the rim.

Feeds.—There are two depth feeds provided—a coarse one for the small cutters and a finer one for the large cutters. The adjustment is effected by a lever behind the pilot wheel P. The cutter is fed towards the blank by moving the head O bodily by the screw Y until the proper depth is reached. Then the cutter and gear blank begin to revolve, the cutter maintaining its reciprocating motion. The feeds are tripped by the revolution of the work arbor. A spider which operates the feed trip is on the hub of the lower index wheel U, to which it is clamped. When it comes in contact with a stop it trips the feed rod, and the mechanism which it controls.

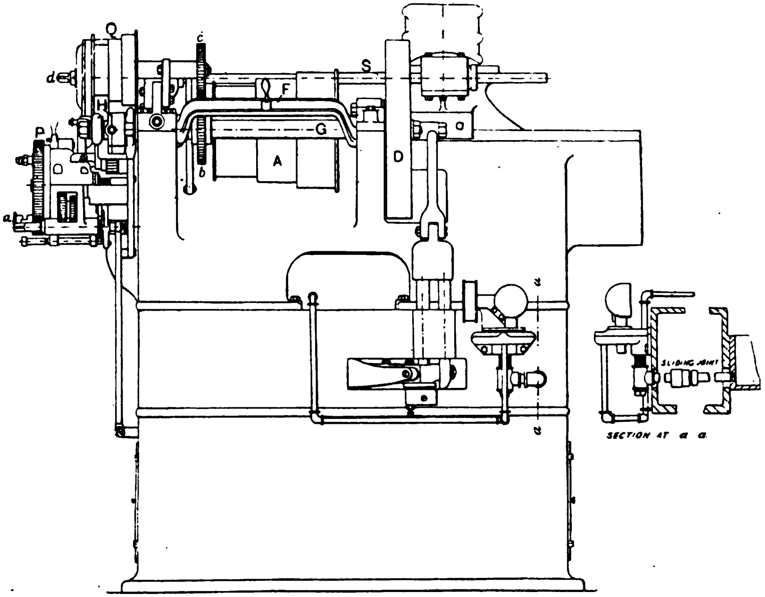


Fig. 212.

The rotations of the cutter have to synchronize with the rotation of the blanks, and these are derived from the crank shaft G through gears *b* and *c* driving to the cones Q, and thence to the reverse cones R, the latter being on one end of a worm shaft S. At the opposite end of the shaft S is a worm engaging with an upper index wheel enclosed at T, which is fastened to the top end of the cutter spindle N, so imparting an arc movement to the cutter in addition to the vertical strokes just noted. Hand rotations can be effected by means of a crank put on the squared end *d* of the shaft S.

Indexing.—Motion to the index wheel for the blank—the lower or work index wheel U on the bottom end of the work arbor—is communicated also from the shaft S, but through bevel gears driving through the change gears V. A constant ratio is maintained between one of the gears used, termed the " pitch gear," and the cutter, by making the number of teeth in the pitch gear twice that in the cutter. This is placed on the quadrant stud.

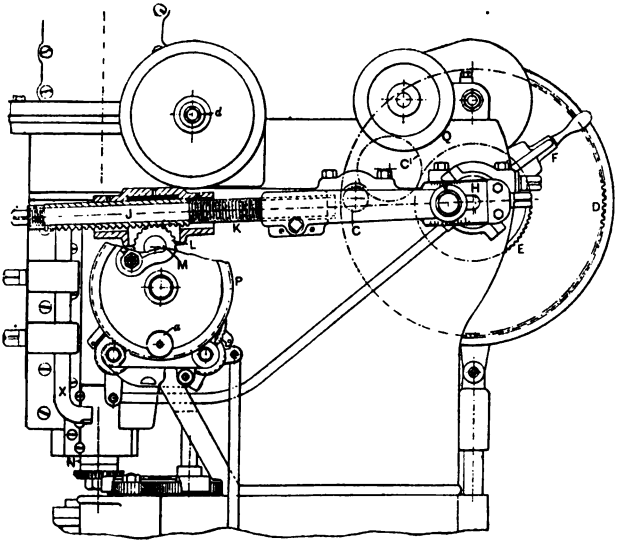


Fig. 213.

The Fellows Company lay stress upon the fact that the indexing mechanism, shown diagrammatically in Fig. 214, has a separate drive, and that the shocks and jerks which may exist to a greater or lesser extent in the working of the cutter are not transmitted to the index wheel and the gears. There is no halt in the running of the index gears, but they continue to revolve quietly at slow speed, connecting the work and the cutter so that they revolve in unison as though they were a pair of gears in mesh.

Work Spindle.—The apron W in which the work spindle is carried is hinged, and has to be swung outwards to permit of putting the blank on its arbor. It is also reciprocated slightly between the cutting and return strokes to relieve the cutter on the latter stroke. There are two plungers at the rear of the machine, by which the apron is locked, and which when withdrawn leave it free. A work support X is set on the top edge of the rim of the blank in opposition to the cutter, which is drawn upwards. Z is a tray which receives oil and chips. The pump arrangements will be observed in Fig. 212.

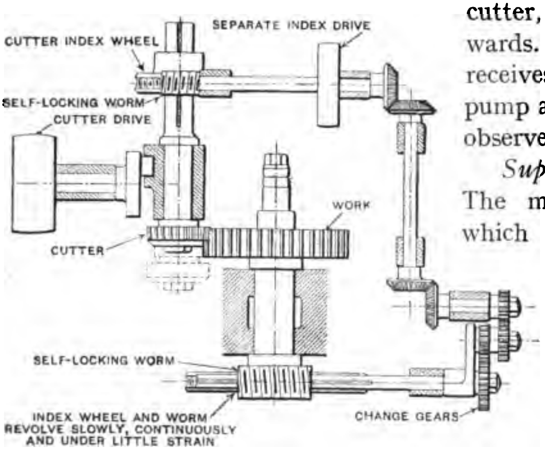


Fig. 214.

Support to the Blank.—The method of support which is afforded to the blank is termed the face-plate system. The arbor has an inverted taper, and being placed in the spindle from the wider side, it is then drawn into its seat an integral part of the spindle. When a face plate is screwed to this arbor, and the work clamped to it as in Fig. 215, the support afforded is very good. The cut taken is when possible a draw cut—that is, in an upward direction, and against the support X (Figs. 210, 215, 216). But the action of the machine can be reversed for cutting internal gears as in Fig. 217. Cutters are supplied in two diameters—3in. pitch diameter for cutters of seven pitch and finer, and 4in. diameter for all coarser ones. These machines are made in several sizes, and some now are motor driven. Designs are modified considerably, with dimensions. The illustrations given are those of a 36in. machine, of the firm's No. 2 model.

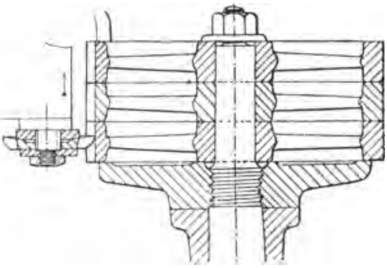


Fig. 215.

The Bilgram Machine.—The Bilgram spur-gear planing machine (Fig. 218), which has been brought out by J. E. Reinecker, of

Chemnitz-Gablenz (for whom Messrs Pfeil & Co., of Clerkenwell, E.C., are the English agents), generates the teeth on the same principle as the older bevel-gear planer (*see* p. 232). Modified by the essential differences between spurs and bevels, they are designed similarly. A roll cylinder is substituted for a roll cone, and the horizontal for the angular position of the work arbor. The tool ram is reciprocated similarly in both, and the tool is of wedge-shaped section, having its flanks inclined at 75° to the horizontal. It cuts on sides and bottom. There are, however, three such tools used, differently ground, as will be described presently. The cost of such cutters is small, while gears with different pressure angles, and with corrected heights above and below pitch line for small pinions, are readily cut by virtue of the mechanism embodied in the machine without the substitution of other cutters.

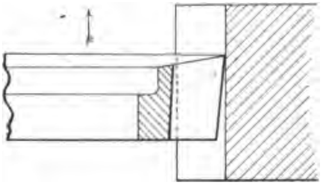


Fig. 216.

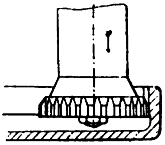


Fig. 217.

Mode of Action.—The outline drawings of the machine (Figs. 219 and 220) will suffice to render its general design and mode of action intelligible. All the motions, which are fully automatic, are operated from the belt pulley A. This is driven from the pulley on the seven-speed countershaft (Fig. 221). A drives the ram B carrying the tool *a* at its opposite extremity. The length of stroke is adjusted for gears of different widths of face across a slotted disc within the pulley, the ram being slotted as seen in Fig. 219 for the adjustment of the bolt which clamps the connecting rod. The movement of the ram is constant along a straight line parallel with the base of the machine. The tool *a* with its holder is lifted from the work on its return stroke, when the dividing is also effected. A cam recess C within the pulley A receives a roller which actuates the cam shaft D which is connected to the tool box and lifts it.

The Blank.—The mechanism for dealing with the blank is carried on the knee E, which is adjusted vertically by the screw *b*, and locked in position by means of the screw *c*. The screw *b* has a graduated dial by which the correct height is set before the cutting commences.

Supported on the knee is the cross slide F, provided with both hand and automatic traverse, and on which the arbor for the gear blank is carried, *d* being the axis of the arbor. The necessary movements which are imparted to the blank are of a dual character, comprising the cross feed, and the movement of the roll cylinder G, which is derived from the cross feed and connected by the change gears H, the relationship of which is determined by formulæ supplied.

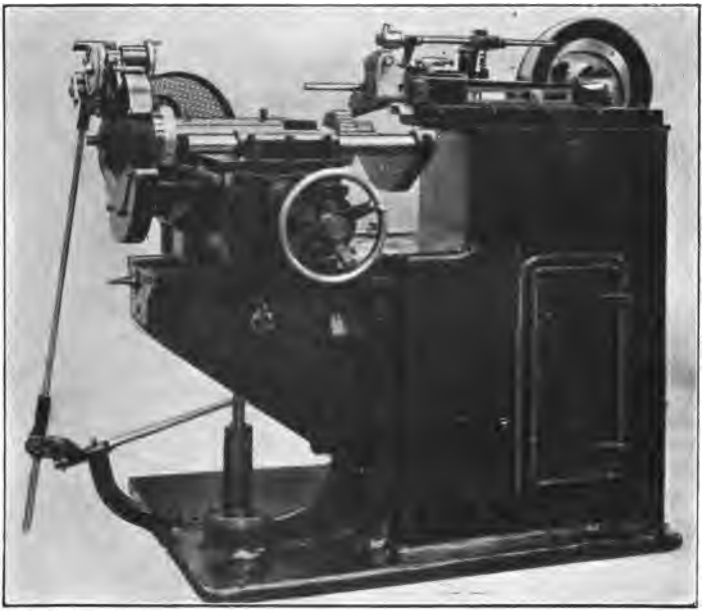


Fig. 218.—The Bilgram Spur-gear Generating Planer.

The roll cylinder G is the essential generating element in the machine, corresponding, as already stated, with a roll cone of the bevel-gear generator. Two steel bands united to it (not shown) coerce its rolling motion to right or left according as the cross slide F is moved in one direction or the other. In this way the roll is imparted to the gear blank on the axis *d*, producing the correctly generated form of tooth. Hand adjustments are made by the screw *e* and a nut. The feed of the cross slide and the consequent movement of the roll cylinder are imparted by an eccentric movement from a slotted disc at J through gears K, and engaged by tightening the binding screw *f*, turning it to right or left according to the direction required, a point which will be reverted to again. K indicates ratchet gears which work the cross feed of the slide.

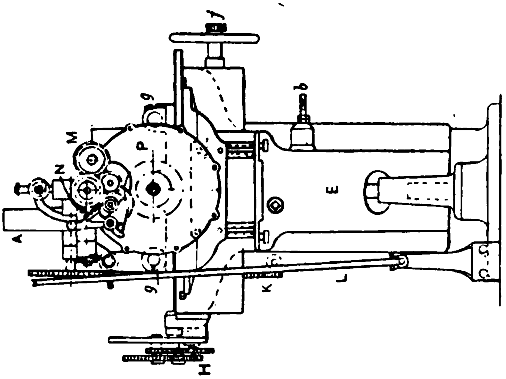


Fig. 220.

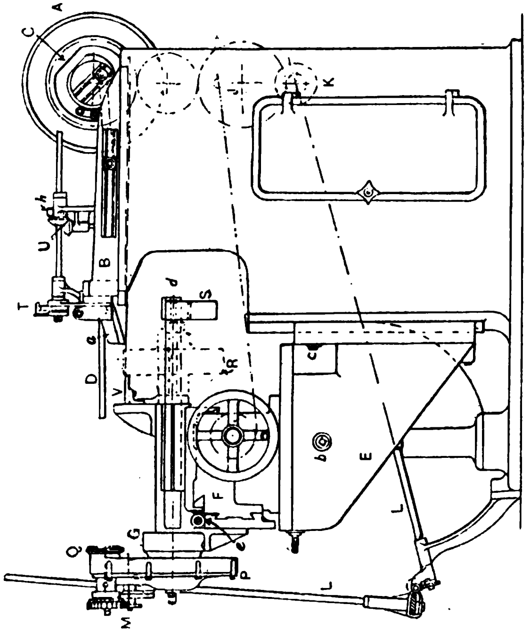


Fig. 219.

Indexing. — The dividing movements to the blank are transmitted through the universal jointed shaft L to the other end of the machine, where the dividing gears M and anchor wheel N with its pawl are carried. The function of the latter is that of holding the blank in position during the forward stroke of the ram, which corresponds with the cutting. The dividing or pitching change gears seen at M are set up to give the correct number of teeth. P is a gear box, the function of which is to convert the constant rotation of the shaft L into an intermittent one for operating the anchor wheel and the dividing gears, at an interval which is regulated by the speed of the driving pulley A and the stroke of the ram B.

Pitching Tooth Spaces.—To set accurately the distance between the teeth, a division plate or disc, which is fixed, having 99 holes, and a second plate, movable, having 100 holes, forming together a vernier adjustment, are seen at Q. The second plate operates upon the work arbor and blank. Correct setting is located by a pin put through the two discs. This setting, which is varied to give stronger teeth to small pinions, is an important feature which will be referred to again.

Rounding.—There is a rounding device at T for imparting a slight convexity to the tops of teeth (adjustable for various teeth by means of a graduated screw) by preventing the tool box from falling to its normal position when cutting above the pitch line, in increasing proportion as it approaches the top of the tooth. This is determined by a positive connection with the cross slide by means of bevel gears U. On the boss of the last bevel gear there is a series of holes taking a pin which presses an electric push *h* at any desired point in the traverse of the cross slide F, so that a bell rings and warns the operator of the completion of the necessary travel of the slide.

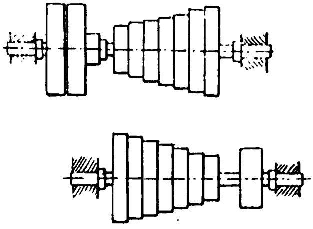


Fig. 221.

Operation. — The action of the machine may now be summarized. The blank, outlined at R, being set on its arbor, the outer end of the latter is carried in the support S, which is itself held by two shafts clamped by the split bearings *g g*. The dividing change gears at M, and the change gears at H connecting the cross slide and roll cylinder, and the vernier adjustment at Q, having been correctly placed and set, the ram carrying the tool is reciprocated by the rotation of the pulley A, and the tool *a* makes a cut while the anchor wheel with its pawl holds the gear in position. The thrust of the tool is resisted by a steady bracket V when large gears are being cut. On the return stroke of the ram, with the lifting of the tool *a*, effected by the cam within C, the anchor wheel releases the gear blank, and the division to the next tooth takes place.

The method of feed by the cross slide F and roll cylinder G is like that of the Bilgram bevel-gear generator in this respect—that there is no vertical feed. Also, it is of no consequence whether the cross slide travels from right to left or from left to right. The explanation of this fact is as follows:—

There is a line engraved on the cross slide, and a corresponding line on its bed. When these two lines are opposite, the centre of the work arbor is in the same vertical plane as the centre of the tool, and this is the lowest point to which the tool can reach when cutting the blank, and in this position, therefore, the depth of the tool is set. A movement of the slide in either direction rolls the gear blank gradually into a position in which the tool can pass right over the blank. It will be seen, therefore, that if the ram is now started, and the feed engaged, operating in that direction which will gradually cause the lines on the slide and its bed to meet, the tool will go deeper with every stroke until the lines coincide, when the tool will have reached the lowest part of the tooth depth. The feed being continued, the tool will gradually come to the top of the blank on the other side until it is clear of the blank. The cross slide will then have travelled to the same distance from its central position on the farther side as it occupied on the near side when the cut commenced. Although, when the slide was in its central position, the tool was at the lowest depth of the tooth, the tooth was not completed until the cross slide had completed its traverse.

The Vernier.—The width of the space to be cut is set by means of the vernier Q. A table is provided for this, because the width of the space is varied according to the numbers of teeth required. Considerable stress is laid on this feature, since it makes the teeth of the small pinions stronger than they would otherwise be. It is obviously undesirable to have a pinion which has to make, say, five or six revolutions to one of its mating wheel, of the same thickness of tooth. By the provision made at the vernier the thickness of the wheel teeth is reduced, and that of the pinion teeth increased, the amount taken from one being added to the other. A table supplied with the machine gives vernier settings for corrected thicknesses for numbers of teeth ranging from 11 to 100.

In setting the position of the blank for cutting, the method is to put the vernier over by half the distance given in the table supplied with the machine, when roughing-out. The vernier is then put back to zero for the right-hand cut, and is set over the full distance for the left-hand cut.

Tools.—With regard to the tools and their method of setting, there are, as stated, three tools used—a roughing-out tool ground at right angles with its length, a finishing tool ground at an acute angle to its right side, and another finishing tool ground at an acute angle to its left side, the three being shown in Fig. 222. The roughing tool cuts on two sides and the bottom, and is used for roughing-out only, but the finishing tools cut only on one side.

The gripping of the first tool for the roughing central cut is done by mounting it in the manner shown, with the cutting end against the block A (Fig. 223). The central position is assured by the blocks C and D (Fig. 224). The setting for cutting the flanks is done by other blocks located on one side only, which ensures the positions relatively to the zero line and the flanks just now noticed. The blank to be cut is set on its arbor by means of an indicator by which it is centred accurately.

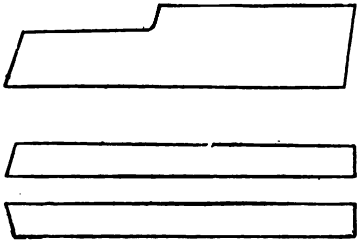


Fig. 222.

The formulæ for different pressure angles take account of the following:—15°, 17° 30′, 20°, 22° 30′, and 25°. Formulæ are also given for setting the machine for cutting gears with corrections where small pinions engage with large wheels, and interferences would occur without such corrections. These are not effected by modifications in the curves of the flanks as when disc rotary cutters are used. But the gears are sized by formulæ which without altering the pitch lines increase the length of faces of the pinions, and lessen the length of their flanks, and correspondingly lessen the faces of the wheels and lengthen their flanks. The important point is that whatever pressure angle, or whatever the relative sizes of gears, all teeth are correctly generated without any subsequent corrections. Some of these features are described more fully in connection with the bevel-gear generator, *see* p. 242.

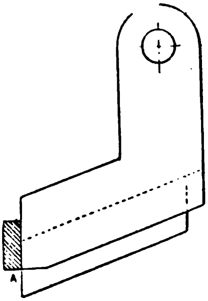


Fig. 223.

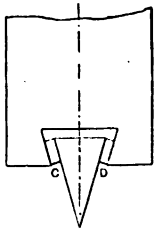


Fig. 224.

CHAPTER XIV

MACHINES THAT GENERATE BEVELS BY PLANING

The Principle Stated.—As being helpful to the clear understanding of the subject the relation of a master gear to the gear being cut is postulated. That is, every bevel gear being cut is related to the crown bevel through a master gear meshing with the crown gear and carrying on its axis the gear to be cut (Fig. 225). The relations are those of pitch cone angle and number of teeth, but not of pitch. If the crown gear is rotated to roll the master gear, a planer tool corresponding with the flank of the crown gear tooth will produce an involute-shaped tooth on the gear being cut, operating under the coercion of the master gear, and partaking of its movement. This principle is adopted in several machines though not in this particular form. It is sufficient if the rolling action of the pitch surface is produced without tooth engagement. In the Bilgram bevel-gear generator, for example, the roll cones represent the pitch surfaces of master gears, and the flat surface they roll on is the pitch surface of the crown gear. And instead of a fixed angle such as would be maintained in a rigid frame the vertical angle is adjusted in quadrant plates. Again, in order to produce the curve of the teeth, the master gear or its equivalent and the blank must be rolled laterally around the axis of the crown gear or alternatively the latter must be rolled, the master gear and the gear being cut rotating on their axis only. In the Bilgram machine the blank swings around the vertical axis of the imaginary crown gear.

The Bilgram Machine.—Among the bevel-gear generating machines the Bilgram is properly accorded the first place, because it was the first in point of time, and is still more largely used than any other. For many years it was made semi-automatic only (Fig. 226), the indexing being done by hand, but it is now made of fully automatic design.

Principle of Action.—The principle of the machine is that one rack tooth generates all gears of one pitch by a reciprocating action, cutting in the intervals of the movements of the blank which oscillates in the plane of the cone angles, and rolls on the pitch surface (*see*

Fig. 82, p. 96). These motions are produced by the mechanism of the machine, which embodies the generating action, as follows (Figs. 227 and 228, which show the semi-automatic machine in longitudinal elevation, with part sections and in plan respectively). This represents the machine as made by J. E. Reinecker, A. G., of Chemnitz.

Starting with the tool A, which represents a rack tooth, this does not occupy the entire width of the tooth (*see* Fig. 229). But its sides, which do the cutting, are inclined at the angle of 15° (or other angle). It is the form of tooth which is identical in a straight rack, and in the teeth of a crown bevel gear the pitch plane of which is flat. The tooth faces lie at right angles with the angle of pressure of the gear. The tool reciprocates only, it has no lateral feed, and therefore the oscillations and the rolling of the blank provide the feeds. Vertical and lateral adjustments of the tool are provided, but, once set, it reciprocates only, lifting on the return stroke. Sundry adjustments of the tool are essential. That for length of stroke is provided by a slotted disc and connecting rod B (Figs. 227 and 228). It is adjusted bodily in the vertical and lateral directions in its holder C, which is attached to an apron D. The latter is hinged on a pin E, tapered for take-up for wear, and carried at the end of an arm, or ram F, to which the connecting rod is fitted. The tool lifts on the return stroke. The tool is adjusted by the screws A′ and B′.

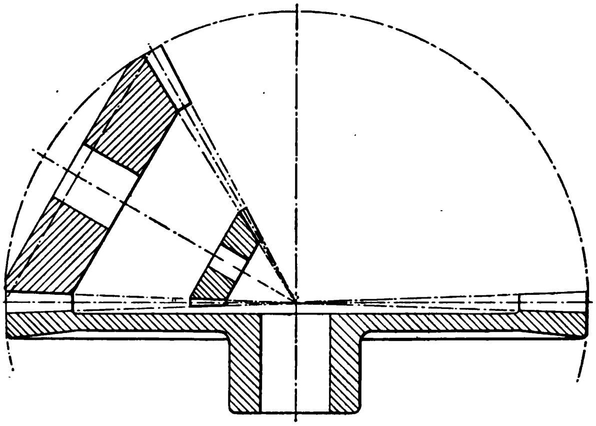


Fig. 225.

The Blank and its Movement.—The mechanism for carrying the blank, and feeding and indexing, comprises the following:—An arbor G carried in a holder having provision for angular adjustments in a vertical plane between two uprights H, H, being clamped therein by the nuts seen. A vernier provides for exact setting. The oscillations of this element provide for the angles of the teeth in the plane of the cone section.

The uprights are carried by a horizontal plate J, which moves about a vertical axis—that which intersects the apex of the cone of the gear. By this means a lateral movement is imparted to the blank corresponding with the flank angles of the teeth. The generation of the involute curves is produced by the coercion of the roll cone K on the upper end of the arbor mechanism, which produces a rolling of the gear corresponding with its diameter. As the amount of roll varies in every different size of gear, a set of twenty-two roll cones is supplied, ranging from 11½° to 75° of angle, which provide exact or approximate angles for a complete range of gears. The cones supplied are marked with the following angles: 11½°, 12° 30′, 14°, 15°, 16°, 17½°, 20°, 22½°, 25°, 27½°, 30°, 32½°, 35°, 37½°, 40°, 42½°, 45°, 50°, 55°, 60°, 65°, 70° and 75°. These provide average angles covering the whole range of gears that may be cut on the machine. Any

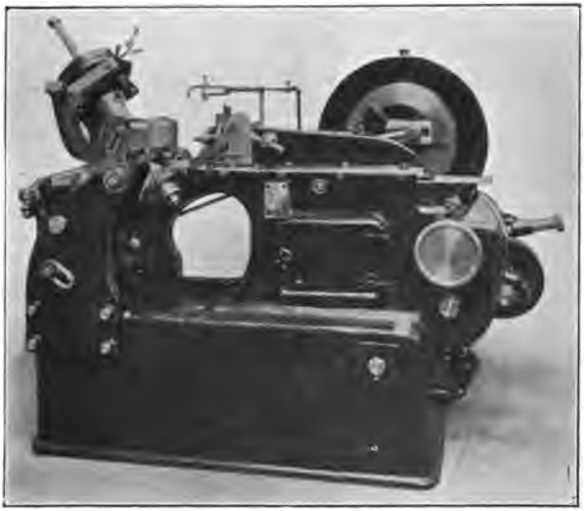


Fig. 226 —The Bilgram Bevel-gear Generating Planer.

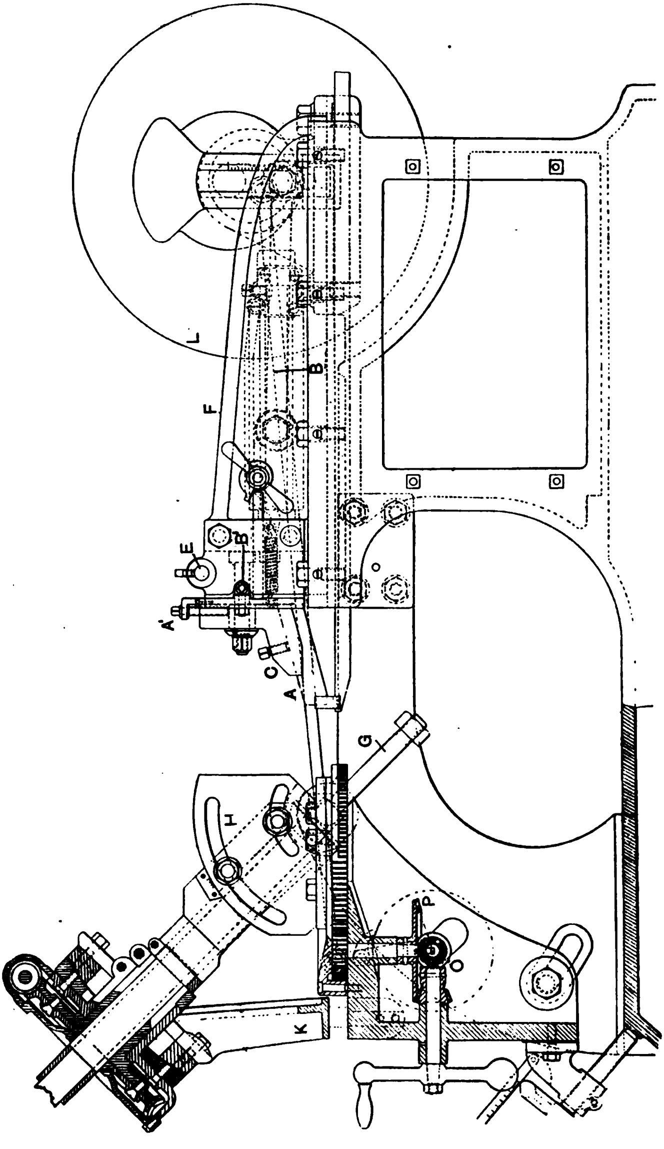


Fig. 227.

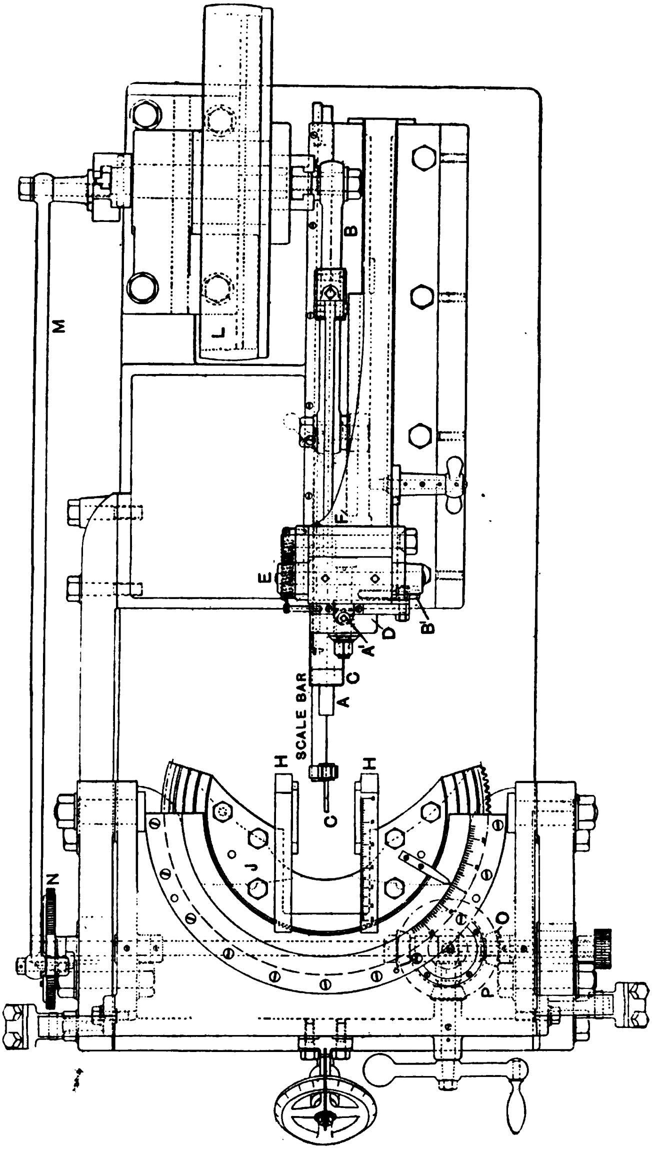


Fig. 228.

intermediate bevel is obtained by taking the nearest roll cone thereto. Each forms a portion of a conic frustum, corresponding with the cone angle of a wheel, and they are compelled to roll on the bands because coerced by the movement of the head to which they are bolted. One of these is shown at K in Fig. 230, resting on a "cone block," with the flexible steel bands *d*, *d* anchored at each end, which coerce the movements of the cone.

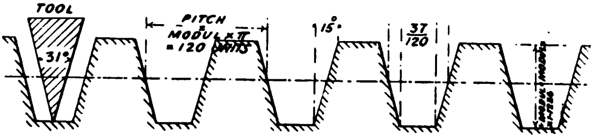


Fig. 229.

The belt pulley L, which reciprocates the cutter, also actuates the feed through a connecting rod M, adjustable in a slotted crank disc. This actuates a ratchet wheel N on the end of a cross shaft, and thence bevel gears O, O, P transmit the arc movements to the plate J, which carries the inclined arbor. These movements are produced to both right and left, according to which sides of the teeth are being planed. At the same time the roll cone is coercing the arbor in its lateral roll. The indexing is located at the upper end of the arbor spindle (Fig. 227); either change gears are used, or an index plate and peg. The worm wheel is carried on a sleeve which encircles the arbor spindle. As the roll cones pull laterally, plates are provided with curved faces seen in Fig. 227, otherwise the mechanism could not accommodate itself to the rolling action produced by the cones.

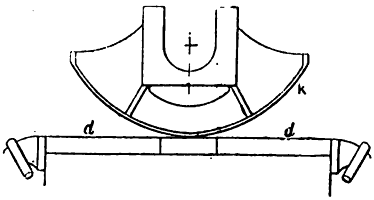


Fig. 230.

Setting Up.—In the fixing up of work on the machine calculations are based upon the standard proportions shown in Fig. 229. The basis is the "modul," or equivalent of the Brown & Sharpe "diameter" pitch "*s*" (*see* p. 36), and which is the distance from pitch line to point—the addendum measured on the major diameter of a bevel gear. The pitch equals the modul × π, so that:—

$$\text{Modul} = \frac{\text{Pitch}}{3.1416}$$

and

Pitch = Modul × 3.1416.

The distance from pitch line to root=Modul × 1.1236, to give root clearance.

The basis rack teeth have an angle of 15° with the perpendicular, or 75° with the pitch plane.

The Blank.—Beginning with the turned blank, the formulæ used are given below for shafts at right angles. Compare with the drawing (Fig. 231).

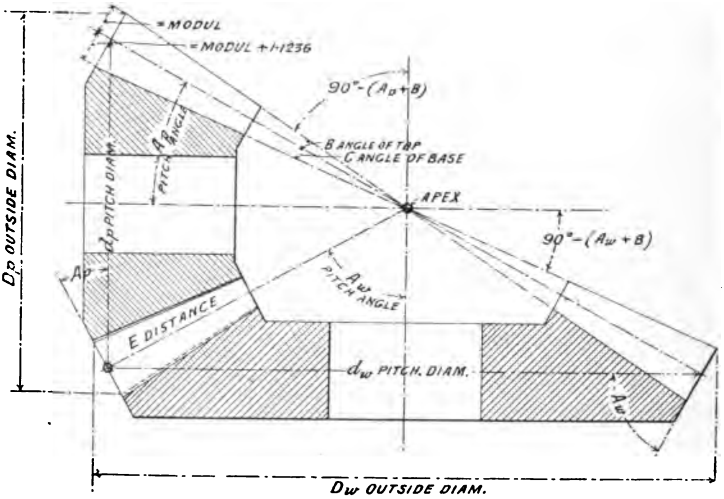


Fig. 231.

Let

m be the Modul.
T_w „ Number of teeth of wheel.
T_p „ „ pinion.
d_w „ Pitch diameter of wheel.
d_p „ „ pinion.
A_w „ Pitch angle of wheel.
A_p „ „ pinion.
E „ Distance from apex to pitch circle.
B „ Angle of top of wheel and pinion tooth.
C „ „ base „ „
h „ Height of teeth (outside).
D_w „ Outside diameter of wheel.
D_p „ „ pinion.

$$\text{Modul}=\frac{\text{Pitch}}{3.1416}; \quad \text{Pitch}=\text{Modul} \times 3.1416$$

$$d_w = T_w \times m;$$
$$d_p = T_p \times m;$$
$$tg\,.\,A_w = \frac{T_w}{T_p};\ \text{Pitch angle } A_p = 90^\circ - A_w;$$
$$E = \frac{d_w}{2 \times \sin A_w};$$
$$tg\,.\,B = \frac{m}{E};\ tg\,.\,C = \frac{1.1236 \times m}{E};$$
$$h = 2.1236 \times m;$$
$$D_w = d_w + (2 \times m \times \cos A_w);\ D_p = d_p + (2 \times m \times \cos A_p).$$

Having only the outside diameter of gear, the modul is obtained by the formula:—

$$m = \frac{D}{T + (2 \times \cos A)}$$

The formulæ being applied as below to a pair of wheels of 40 and 24 teeth respectively, with a modul of .25in., and axis at 90°, the order is as in Fig. 232. The wheel is cut before the pinion.

Example:—To cut a pair of gears with 40 and 24 teeth respectively; $m = .25$in.; centre lines at 90°.

$$d_w = 40 \times .25 = 10\text{in.};$$
$$d_p = 24 \times .25 = 6\text{in.};$$
$$tg.\,A_w = \frac{40}{42};\ \angle A_w = 59^\circ\ 2';$$
$$\angle A_p = 90^\circ - 59^\circ\ 2' = 30^\circ\ 58'$$
$$E = \frac{10}{2 \times \sin A_w} = 5.831\text{in.};$$
$$tg\,.\,B = \frac{.25}{5.831};\ \angle B = 2^\circ\ 27';$$
$$tg\,.\,C = \frac{1.1236 \times .25}{5.831};\ \angle C = 2^\circ\ 45'$$
$$h = 2.1236 \times .25 = .531\text{in}.$$
$$D_w = 10 + (2 \times .25 \times \cos 59^\circ\ 2') = 10.257\text{in}.$$
$$D_p = 6 + (2 \times .25 \times \cos 30^\circ\ 58') = 6.428\text{in}.$$

End angles for turning the

Wheel = 90° - (59° 2′ + 2° 27′) = 28° 31′ and 59° 2′.
Pinion = 90° - (30° 58′ + 2° 27′) = 56° 35′ and 30° 58′.

Workshop Order.

Modul	.25	
Number of teeth . . .	40	24
Pitch angle	59° 2′	30° 58′.
Angle of base	2° 45′.	

Distance		5.831in.
Height of tooth		.531in.
Outside diameter	10.257in.	6.428in.
Change (holes)	38	36

(Referring to the index plates which are supplied with the machine.)

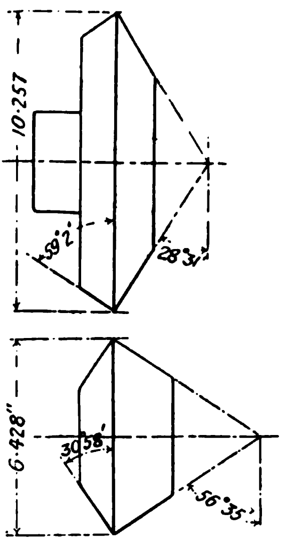


Fig. 232.

Setting Up.—First the index plate is put on. Then the wheel blank is placed on the arbor, which is set to the pitch angle A_w (Fig. 233). The gear blanks are held endwise by means of washers, and by the nuts *a* and *b* (Fig. 234). Final adjustment of the gears is made by inserting an adjusting collar *c* (Fig. 234) between the blank and the spindle head. The roll cone which comes nearest to the pitch angle rests on the cone block (Fig. 230). The pitch angle A_w in Fig. 233 is now reduced by 1°—that is, the arbor is set to the angle A_w—1°, so lowering the roll cone, which will impart a trifle more of rounding to the tooth flank than is theoretically correct, but which is conducive to smooth working. Next the flexible steel bands are clamped on. Then the angle of the base box is set to the angle C_w of the base or root of the tooth, less 1°. This is done with the graduated hand wheel seen at the rear in Figs. 227 and 228, moving a diagonal screw in its boss, the point of which screw rests on a bevelled end of the bed of the machine.

Adjustment of Blank.—The wheel blank has now to be adjusted to the height of the tooth. Bottom blocks of a definite thickness, hardened and ground, are used for this purpose. They range from .275in. to .375in., in steps of .001in. These are laid in the gauge (Fig. 235) at the end of the scale bar in the axis of the machine. The measuring caliper (Fig. 235), which is set to the exact height of the tooth, is then set on top of the bottom block, and the wheel is moved into that position in which the jaw of the caliper will rest on the upper circumference of the edge of the wheel, as shown.

The distance piece is next put into the gauge of the scale bar and moved up to the wheel (Fig. 236). When it touches, notice must be taken of the distance indicated on the scale at the back of the bar (*see* Fig. 228). This scale bar is a valuable adjunct to the machine. It is adjustable by means of a vernier to a thousandth of an inch—three places of decimals; and is used for setting the exact diameter of the rim of the wheel in the manner shown in Fig. 236, by which means the rims of wheel and pinions are insured being flush when put together for gearing. Note is next taken whether the wheel runs truly on the arbor, testing it by means of the indicator in Fig. 237, which magnifies any error largely at its free end.

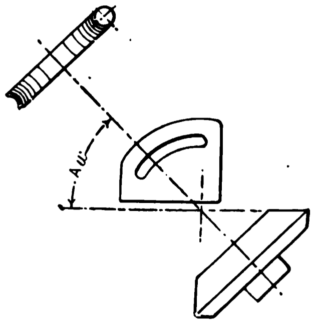


Fig. 233.

Cutting.—Everything is now fixed up ready for cutting. The first tool stocks out the metal with a central cut, followed by finishing the flanks to right and left. For taking the roughing cut, the tool is adjusted vertically by means of the bottom block, which has to pass easily between the tool point and the gauge (Fig. 238, A). The central position is obtained by passing a side block on each side between the tool and the gauge, as in Fig. 238, B. The tool is afterwards fastened for taking the right-hand cut. It is adjusted vertically to the bottom block, and sideways to the side block, as in Fig. 238, C. The exact adjustment of the side blocks is very important, since a little difference would make the tool cut the tooth flanks thinner or thicker than the specified standard. The index plate is turned round eighteen holes to bring the tool into the position I in Fig. 239, and the cutting proceeds. For cutting the left-hand tool flank the tool is fastened for the left hand in the same way as for the right, and the dial is turned back in the opposite direction thirty-seven holes, which brings the tool into the position II in Figs. 239 and 240. The three tools used are shown in Fig. 241, one for roughing-out parallel, and one

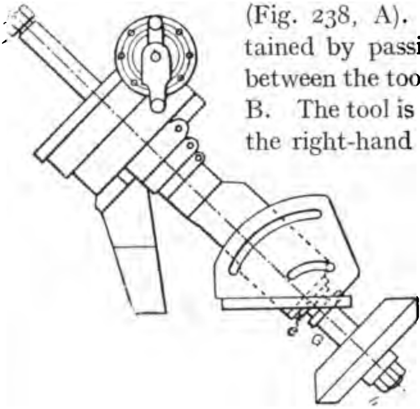


Fig. 234.

right and one left for finishing the flanks. The roughing tool is ground straight across the end, the finishing tools have side front rake. Templet blocks are sent with the machine to ensure uniformity in grinding the ends of the tools, which are the only parts ground. These blocks are recessed to receive the planing tool, the cutting ends alone projecting, to be passed under the emery wheel.

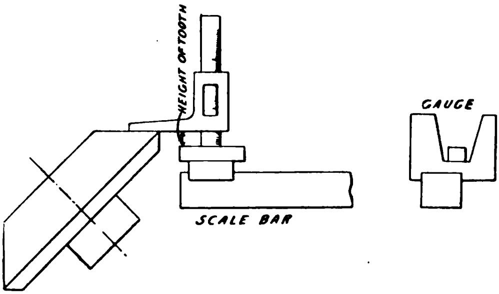


Fig. 235.

Corrected Teeth.—In wheels of less than 30 teeth undercutting occurs (*see* p. 17), and the worm threads must be rounded, or else the wheels be made of larger diameter by increasing the length of the addenda. The latter is adopted in the case of small bevel pinions. In such pinions the undercutting is a source of weakness. This is avoided, and the labour of ascertaining the exact relations of pitch line and addenda required to avoid undercutting is saved by the use of tables supplied with this machine. In gears that have a less number of teeth than those given in the table below, the gears are sized differently than by the formulæ just now given:—

Ratio	1 : 4	1 : 3	1 : 2	2 : 3	1 : 5
Pinion	28	27	24	20	10 teeth.

The height of the wheel tooth above the pitch circle is reduced, and the depth below the circle increased correspondingly, so keeping the total height unchanged. Thus the angle of top B in the previous ormulæ, and of the base C, are reduced respectively by a certain

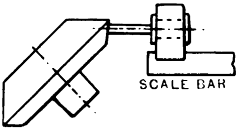


Fig. 236.

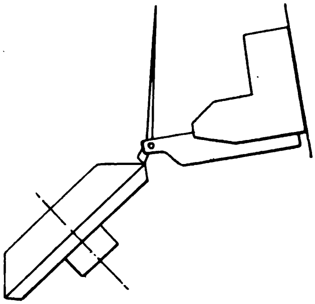


Fig. 237.

correcting angle *a*. In the pinion the opposite condition exists. Thus:—

Corrected angle of top of wheel=ordinary angle of top - correcting angle *a*.

Corrected angle of base of wheel=ordinary angle of base + correcting angle *a*.

Corrected angle of top of pinion=ordinary angle of top + correcting angle *a*.

Corrected angle of base of pinion=ordinary angle of base - correcting angle *a*.

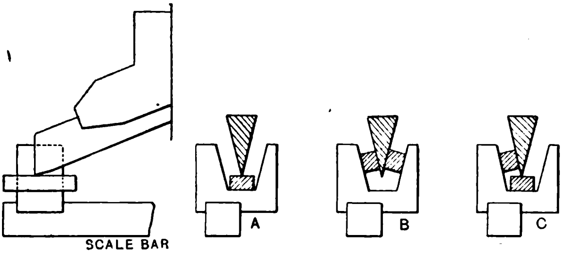


Fig. 238.

Fig. 242, A, shows the ordinary positions of the pitch circles, and B corrected ones. Fig. 243 gives sections through teeth illustrating the ordinary angle in full lines, and the corrected angle in dotted lines varying by the distance *a*. Diagrams are supplied by Messrs Reinecker for these corrected angles, in the form shown in Fig. 244, the curves representing the ordinary angle of the top B in the regular formulæ, and the straight lines the

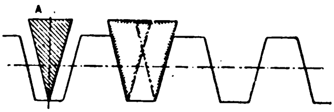


Fig. 240.

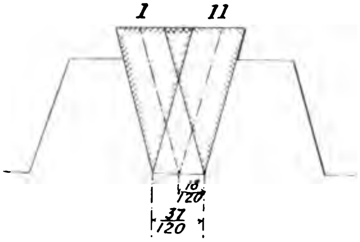


Fig. 239.

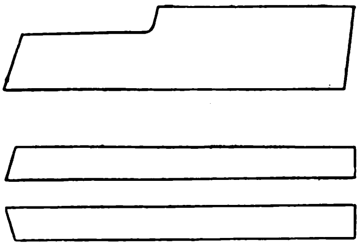


Fig. 241.

reduced angle of wheel, which will not require that the pinions shall be undercut. The difference between the two angles is the theoretical correcting angle. Practically two-thirds of this angle are found sufficient. Fig. 244 is only a diagram, which represents on a very small scale the large diagram of curves and lines and divided edges, all the details of which are omitted from this figure.

Example.—As an example of this correction, take a pair of wheels of 37 and 14 teeth, of ⅝th pitch, with axes at right angles. Referring to the rules previously given:—

$$\text{Modul} = \frac{.625\text{in.}}{3.1614} = .1985\text{in.}$$

$d_w = 7.361$in. $\qquad d_p = 2.785$in.

$< A_w = 69^\circ\ 16'$; $\qquad < A_p = 20^\circ\ 44'$.

$E = 3.935$in.; $\qquad h = .4215$in.

$< B = 2^\circ\ 54'$; $\qquad < C = 3^\circ\ 15'$.

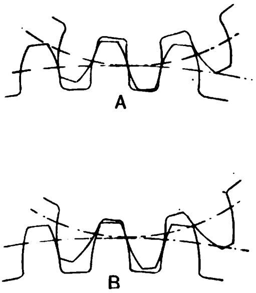


Fig. 242.

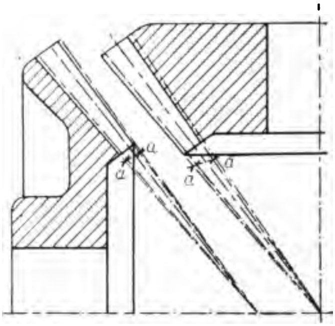


Fig. 243.

To find the correcting angle in the original of the diagram (Fig. 244), follow a certain line through, say, 37 from left to right, and through line 14 downwards. The point where these lines meet lies in the circle which represents the ordinary top angle B of 2° 54′ and is also between the straight lines representing angles of 1° 30′ and 1° 35′. The exact angle would therefore be 1° 33′. Two-thirds of this angle give the correcting angle $a = 0^\circ\ 54'$. Hence:—

Wheel— Angle of top $B_w = 2^\circ\ 54' - 0^\circ\ 54' = 2^\circ\ 0'$.

Angle of base $C_w = 3^\circ\ 15' + 0^\circ\ 54' = 4^\circ\ 9'$.

Pinion— Angle of top $B_p = 2^\circ\ 54' + 0^\circ\ 54' = 3^\circ\ 48'$.
Angle of base $C_p = 3^\circ\ 15' - 0^\circ\ 54' = 2^\circ\ 21'$.

The outside diameters of wheel and pinion are obtained by the following formulæ:—

Wheel— $D_w = d_w + (2 \times E \times tg.\ B_w \times \cos A_w)$.
Pinion— $D_p = d_p + (2 \times E \times tg.\ B_p \times \cos A_p)$.

And the order goes into the shop as below, in conjunction with Fig. 245.

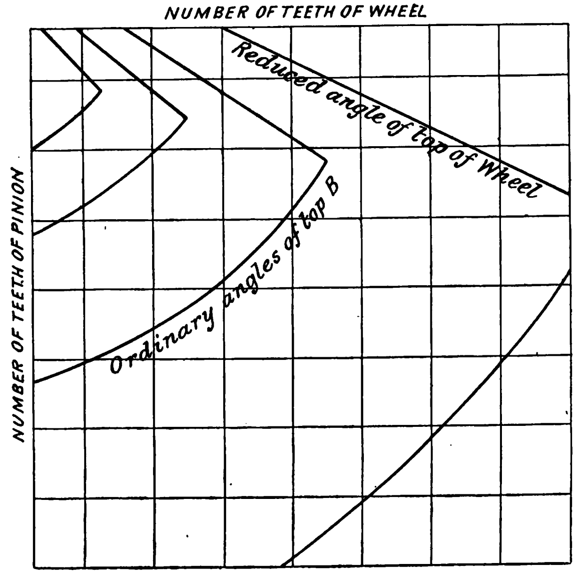


Fig. 244.

Modul	.1985in.	
Number of teeth . .	37	14
Pitch angle . . .	69° 16′	20° 44′
Angle of base . .	4° 9′	2° 21′
Distance . . .	3·935in.	
Height of tooth . .	.4215in.	
Outside diameter . .	7·458in.	3·274in.
Change (holes) . .	39	35

(Referring as before to index plates.)

Sequence of Operations.—When the machine is set up, the duties of the attendant are as follows:—When one side of a single tooth has been cut, and the pointer in the plan view (Fig. 228) indicates on the scale that the proper depth is reached, the operating pinion is disengaged. The worm is then turned round to give the pitch distance for the next tooth. The pinion is then put into gear again by the knob for the automatic cutting of the next tooth flank. When all the flanks on one side have been cut, the other pinion is put into gear, and the same series of operations gone through for the other sides of the teeth. The Bilgram machine therefore, with the exercise of ordinary care and attention, gives results which are as near perfection as it is possible to get them; and for several years a good business has been done in cutting the best gears for the trade on these machines. The machines are supplied with complete accessories in every detail, which enables the user of the same to operate them without having to make any new tools, etc. Most of the machines, as now made, are fully automatic.

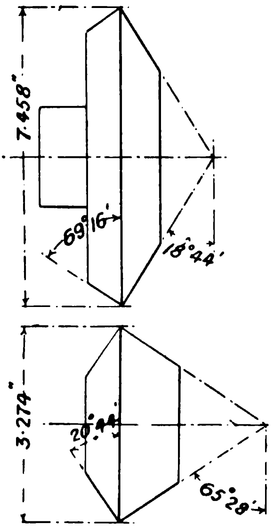


Fig. 245.

The Robey-Smith Machine.—This generates teeth, the curves of which are arcs of circles. The indexing takes place between each stroke of the tools, of which two are cutting simultaneously. The movements of these are linked with those of the blank, so that as the latter is fed inwards for depth, the tool slides open to impart the increase in the thickness of the teeth from points to roots.

General Design.—The machine (Fig. 246, *see* also Figs. 248, 249, p. 253) is driven by a stepped cone, whence a cam actuates the work-table. On this the bearings for the work spindle and the dividing wheel are carried. The table is mounted on a horizontal bed which forms the base of the machine, and has an angular motion on it in a horizontal plane by a circular rack, with a knock-out motion. It is set on the bed so that the reciprocating tools begin to operate at the points of the teeth, and is fed inwards towards the tools at each indexing, until the full depth is reached, when the automatic knock-out motion operates. With each movement of feed the widening of the spaces between the tools is produced by the link motions controlled by the inward movement of the work-table thus:—

To the carriage or work-table a circular bar is attached which has teeth cut at the end nearest the slides, which teeth engage with those of a sector connected by bevel gearing with a slotted arm. This arm is linked to a parallel motion that operates the tool slides, being connected to a lug that forms an extension of a diagonal link, by the movement of which link derived from the connecting rod the tool slides are thrown farther apart as the cutting proceeds. The connection of the links to the slides is through the links seen, being always parallel. Connected to these by pins are two quadrants. To these the tool slides, hinged at the centre or apex, are connected, and driven through the cross head and crank.

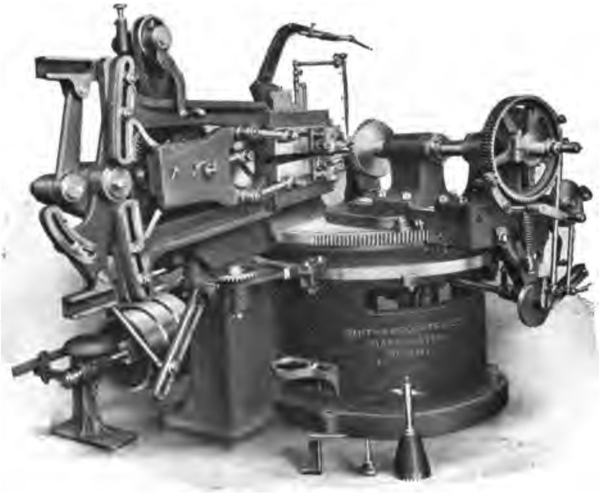


Fig. 246.—The Robey-Smith Bevel Gear Generating Planer.

Instructions for Operating.—A suitable form for the sheet of instructions to be supplied to the workman is here given on p. 248.

(*Smith & Coventry, Limited.*)

No. GRESLEY IRON WORKS,

Salford..................191

	PINION	WHEEL
No. of Teeth....................		
Pitch Angle.....................		
Angle of Base..................		
Distance........................		
Angle of Arms..................		
Outside Diameter...............		
Face		
Turning Angles { Face		
Back...........		

No. of Gears...

Drawing ....

Previous to turning any pair of bevel wheel blanks, the turner should be supplied with a paper containing the following particulars of the wheels:—

(1) Outside Diameter.
(2) Face Angle.
(3) Back Angle.

Two methods of making the calculations necessary to obtain these quantities are given, one worked out arithmetically and the other trigonometrically.

DIAGRAM OF A PAIR OF BEVEL GEARS (FIG. 247),
WITH NOTATION USED IN FORMULÆ.

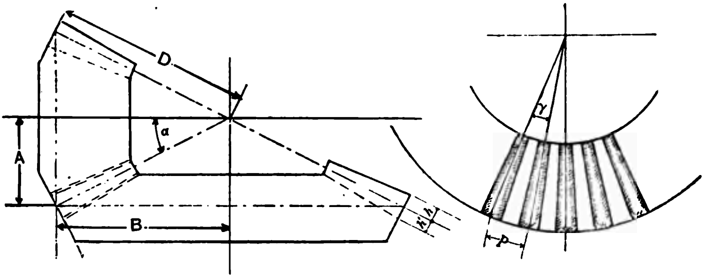


Fig. 247.

Letter n=No. of teeth in pinion.
N=No. of teeth in wheel.
$\alpha=\frac{1}{2}$ angle of pitch cone of pinion.
D=Apex distance.
P=Diametral pitch.
p=Circular pitch.
γ=Angle of circular pitch.
A=Radius of pinion pitch cone.
B=Radius of wheel pitch cone.
h=Height of tooth above pitch line.
h=Depth of tooth below pitch line.

ARITHMETICAL FORMULÆ.

Example: Ratio, 2 to 1. 3 pitch $\left\{\begin{array}{l}\text{Pinion, 15 teeth.}\\ \text{Wheel, 30 teeth.}\end{array}\right.$

(1) *Outside Diameter.*

$$\text{For pinion}=\frac{n+2\left(\frac{N}{\sqrt{N^2+n^2}}\right)}{P} \qquad \text{For wheel}=\frac{N+2\left(\frac{n}{\sqrt{N^2+n^2}}\right)}{P}$$

$$=\frac{15+2\left(\frac{30}{\sqrt{900+225}}\right)}{3} \qquad =\frac{30+2\left(\frac{15}{\sqrt{900+225}}\right)}{3}$$

$$=\frac{15+2\left(\frac{30}{33{\cdot}541}\right)}{3} \qquad =\frac{30+2\left(\frac{15}{33{\cdot}541}\right)}{3}$$

$$=\underline{\underline{5{\cdot}596''}} \qquad =\underline{\underline{10.298''}}$$

(2) *Face Angle.*—To obtain this dimension, there must first

be calculated the pitch cone angle, the apex distance, the angle of circular pitch, and the angle of top of the tooth.

Pitch Cone Angle (5).

For pinion. Tan $\alpha=\frac{n}{N}$ For wheel. ½ pitch cone angle of wheel

$$=\frac{15}{30}=\tfrac{1}{2}=.5 \qquad\qquad =90^\circ-\alpha$$

$$\alpha=\underline{\underline{26^{34}}} \qquad\qquad =90^\circ-26^{34}$$

$$=\underline{\underline{63^{26}}}$$

Apex distance (4).

$$\begin{aligned} D=\sqrt{A^2+B^2}&=\sqrt{2.5^2+5^2}\\ &=\sqrt{6.25+25}\\ &=\sqrt{31.25}\\ &=\underline{\underline{5.590''}} \end{aligned}$$

Angle of circular pitch (γ).

$$\gamma=\frac{180}{P.D}, \text{ or } \frac{57.295p}{D}$$

$$=\frac{180}{3\times 5.59}=\frac{180}{16.77}=10.733=\underline{\underline{10^{44}}}$$

Angle of top of tooth $=\gamma\times\frac{h}{p}$

$$=\left(10^{44}\times\frac{.333}{1.047}\right)=\left(10^{44}\times .318\right)$$

$$=\underline{\underline{3^{24}}}$$

From these data, the face angles of the wheel and pinion are calculated by the following formula:—

(2) *Face Angle of Pinion* $=\alpha+\left(\gamma\times\frac{h}{p}\right)$ (2) *Face Angle of Wheel* $=90-\alpha+\left(\gamma\times\frac{h}{p}\right)$

$$=26^{34}+3^{24} \qquad\qquad =90-26^{34}+3^{24}$$

$$=\underline{\underline{29^{58}}} \qquad\qquad =\underline{\underline{66^{50}}}$$

(3) *Back Angle.*

(3) For pinion $=90^\circ-\alpha$ For wheel $=\alpha$

$$=90^\circ-26^{34}=\underline{\underline{63^{26}}} \qquad\qquad =\underline{\underline{26^{34}}}$$

TRIGONOMETRICAL FORMULÆ.

Example: Ratio, 2 to 1 = 3 pitch $\begin{cases}\text{Pinion, 15 teeth.}\\ \text{Wheel, 30 teeth.}\end{cases}$

(1) *Outside Diameter.*—Previous to making this calculation, the pitch cone angle must be obtained.

Pitch Cone Angle (5).

$$\text{Tan}\,\alpha=\frac{n}{N}=\frac{15}{30}=\tfrac{1}{2} \qquad \therefore \alpha=\underline{\underline{26^{34}}}$$

$$\therefore 90^\circ-\alpha=\underline{63^{26}}=\tfrac{1}{2} \text{ pitch cone angle of wheel.}$$

By using this value in the following formula, the outside diameter is found.

(1) *Outside Diameter.*

$$\text{For pinion }=\frac{n+2\cos\alpha}{P} \qquad \text{For wheel }=\frac{N+2\sin\alpha}{P}$$

$$=\frac{15+2(.89441)}{3} \qquad =\frac{30+2(.44724)}{3}$$

$$=\underline{\underline{5.596''}} \qquad =\underline{\underline{10.298''}}$$

(2) *Face Angle* and (3) *Back Angle.*

Calculate first the apex distance.

Apex distance.

$$D=\frac{N}{2P\cos\alpha}=\frac{30}{6\cos\alpha}=\frac{5}{\cos(26^\circ 34')}=\frac{5}{.89441}=\underline{\underline{5.590''}}$$

If h=height of tooth above pitch line.

β=angle of tooth above pitch line, then $\frac{h}{D}=\tan\beta$

(2) *Face turning angles*=$(\alpha+\beta)$ for pinion. $(90^\circ-\alpha)+\beta$ for wheel.

(3) *Back turning angles*=$(90^\circ-\alpha)$ for pinion. α for wheel.

Note.—Care should be taken in the turning of the wheel blanks, in order that the above quantities (1), (2), (3) shall be exactly to the dimensions given to the wheel cutter. If the outside diameter or the wheel angles are at fault, it is impossible to cut a satisfactory wheel.

For the Wheel Cutter.

The particulars required are:—

(4) Apex Distance (D).
(2) Face Angle.
(5) Pitch Angle (a).
(6) Base Angle.
(7) Number of Teeth.
(8) Angle of Former Arm.

Two methods are given for working out these particulars, as in the case of those for the turner.

ARITHMETICAL FORMULÆ.

Example: 3 pitch {Pinion, 15 teeth.
Wheel, 30 teeth.

The quantities (4) Distance, (2) Face angle, and (5) Pitch angle were required in the calculations for the turning angles, and the formulæ necessary to obtain them were given.

(6) *Base Angle.*

Before obtaining this angle, the pitch cone angle, the angle of circular pitch and the angle of the bottom of tooth must first be known; of these, the pitch cone angle and the angle of circular pitch have already been calculated.

$$\textit{Angle of bottom of tooth} = \gamma \times \frac{h'}{p}$$

$$= \left(10^{44} \times \frac{.3857}{1.0472}\right) = \left(10^{44} \times .368\right)$$

$$\underline{\underline{=3^{56}}}$$

$$\text{(6) } \textit{Base Angle of Pinion} = \alpha - \left(\gamma \times \frac{h'}{p}\right)$$

$$= 26^{34} - 3^{56} = \underline{\underline{22^{38}}}$$

$$\text{(6) } \textit{Base Angle of Wheel} = 90 - \alpha - \left(\gamma \times \frac{h'}{p}\right)$$

$$= 90 - 26^{34} - 3^{56}$$

$$= 59^{30}$$

(8) *Angle of Former Arm.*

The angle of circular pitch (γ) is used also in this formula.

$$\text{(8) } \textit{Angle of Former Arm} = \frac{\gamma}{4}$$

$$= \frac{10^{44}}{4}$$

$$\underline{\underline{=2^{41}}}$$

TRIGONOMETRICAL FORMULÆ.

The quantities (4) Apex distance, (2) Face angle, and (5) Pitch cone angle were required in the calculations for the turning angles and the formulæ necessary to obtain them were given.

(6) *Base Angle.*

If h' = depth of tooth below pitch line.

β' = angle of tooth below pitch line, or root angle.

$\frac{h'}{D} = \tan \beta'$.

Angle of base of teeth, or base angle.

= pitch angle − root angle of tooth.

$= \alpha - \beta'$ for pinion. $(90^\circ - \alpha) - \beta'$ for wheel.

(8) *Angle of Former Arm.*

$$= \frac{2700}{PD} = \frac{2700}{3 \times 5.590} = 161' = \underline{\underline{2^\circ 41}}.$$

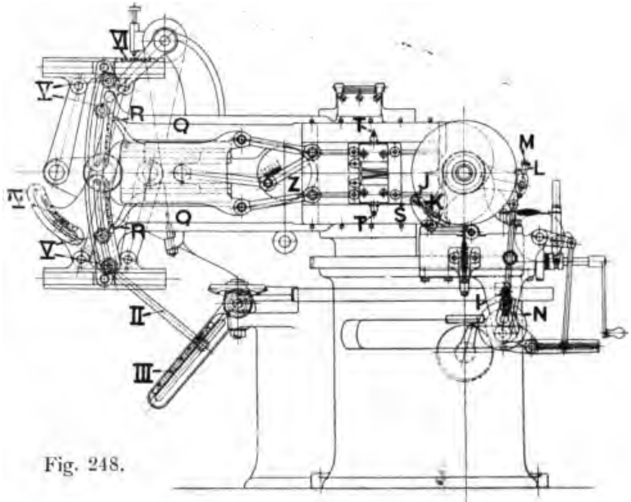


Fig. 248.

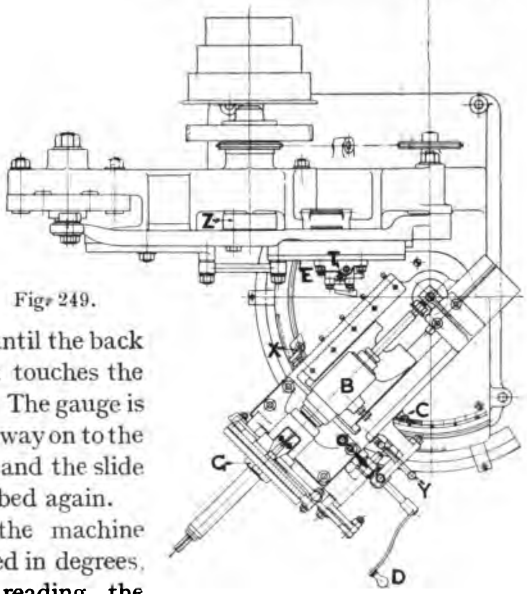


Fig. 249.

Setting the Wheel Blank.—Two gauges are necessary for setting the wheel blank in its proper position relative to the centre of the machine (Figs. 248, 249). The gauge (Fig. 250) is used for this setting, which corresponds with the apex of the wheel. The rod in the gauge is adjusted till the vernier reads the distance (quantity 4) on the scale, and is then locked in position. The wheel blank is set back from the centre by withdrawing the slide B (Fig. 249) until the back face of the wheel just touches the pointer of the gauge. The gauge is then swung out of the way on to the back of the machine, and the slide B must not be disturbed again.

On the top of the machine table is a scale marked in degrees, which is used for reading the pitch, face and base angles of the wheel (quantities 5, 2 and 6). The angle of the blank is read on this scale by vernier C, which is divided to read to five minutes as well as the degrees.

Move now the table slide by the handle D until the vernier C reads the face angle of the blank (2). If the wheel is truly turned, and the blank accurately set to distance gauge, the face of the wheel should be perfectly parallel with the tool slides E. To test this, the gauge (Fig. 251) is supplied, which should just fill the distance between the tool slide and the face of the wheel at every point.

Setting of Dividing Apparatus.—The wheel is divided, with regard to the number of teeth to be cut, by means of a pawl and dividing plate. Twenty-one dividing plates are supplied, and a reference to the following Table will show which plate is to be used for a given number of teeth. The plate is placed in position on the spindle sleeve and screwed up securely by the split nut G. This nut locks together the dividing plate, the split sleeve and the mandrel. If the nut is not securely locked, the blank will move while being cut.

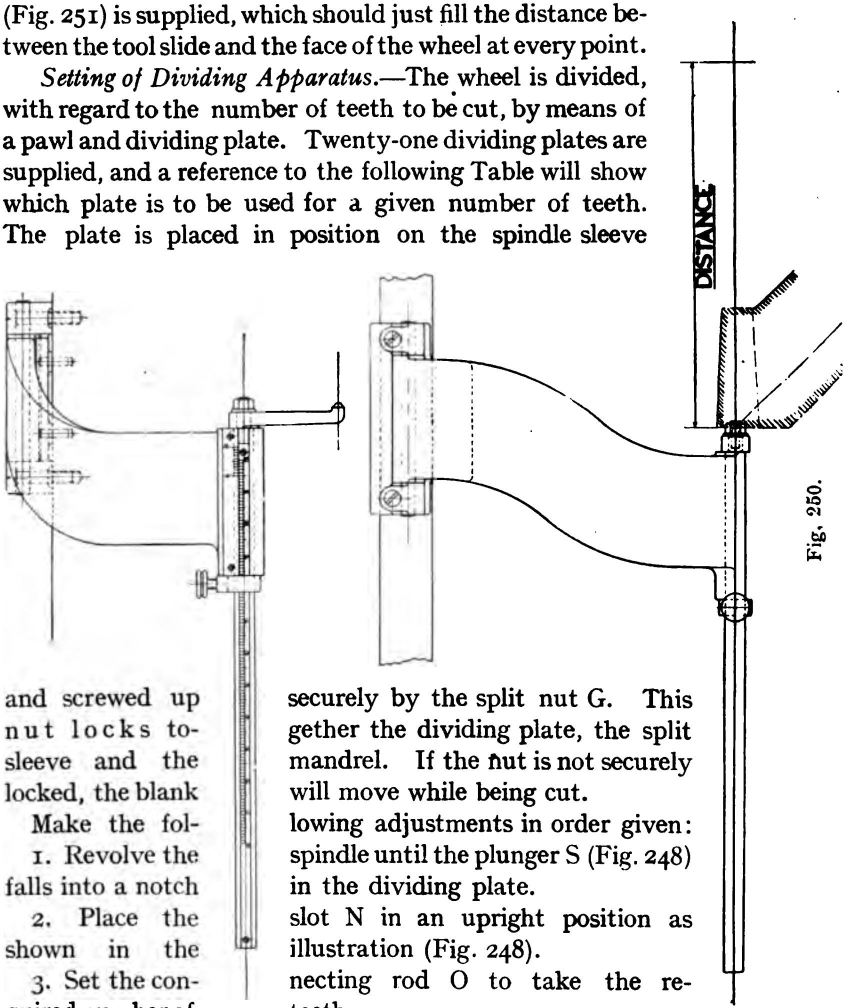


Fig. 250.

Make the following adjustments in order given:

1. Revolve the spindle until the plunger S (Fig. 248) falls into a notch in the dividing plate.

2. Place the slot N in an upright position as shown in the illustration (Fig. 248).

3. Set the connecting rod O to take the required number of teeth.

4. Bring the bevelled end of the slipper plate J right up to the roller K, but not touching it.

5. Adjust the pawl L by means of the screw M until it almost touches the face of the tooth in the dividing plate, which concludes the setting.

DIVISIONS OBTAINABLE BY DIVIDING PLATES SUPPLIED.

No. of Teeth to be Cut.	Use Plate with Divs.	No. of Divs. to be Taken.	No. of Teeth to be Cut.	Use Plate with Divs.	No. of Divs. to be Taken.	No. of Teeth to be Cut.	Use Plate with Divs.	No. of Divs. to be Taken.
10	60	6	32	96	3	60	60	1
11	66	6	33	66	2	62	62	1
12	60	5	34	102	3	66	66	1
13	78	6	35	70	2	70	70	1
14	70	5	36	108	3	74	74	1
15	60	4	37	74	2	76	76	1
16	96	6	38	76	2	78	78	1
17	102	6	39	78	2	82	82	1
18	90	5	40	120	3	84	84	1
19	76	4	41	82	2	86	86	1
20	60	3	42	84	2	88	88	1
21	84	4	43	86	2	90	90	1
22	66	3	44	88	2	92	92	1
23	92	4	45	90	2	94	94	1
24	96	4	46	92	2	96	96	1
25	100	4	47	94	2	98	98	1
26	78	3	48	96	2	100	100	1
27	108	4	49	98	2	102	102	1
28	84	3	50	100	2	108	108	1
29	116	4	51	102	2	116	116	1
30	60	2	54	108	2	120	120	1
31	62	2	58	116	2			

Tools to be Used.

From $\frac{1}{4}''$ to $\frac{7}{16}''$ pitch, use $\frac{1}{32}''$ radius
,, $\frac{1}{2}''$,, $\frac{3}{4}''$,, $\frac{1}{16}''$,,

From $\frac{13}{16}''$ to $1\frac{1}{4}''$ pitch, use $\frac{3}{32}''$ radius
„ $1\frac{5}{16}''$ „ $1\frac{3}{4}''$ „ $\frac{1}{8}''$ „
„ $1\frac{3}{4}''$ upwards „ $\frac{5}{32}''$ „

Setting of Tools.—Place the tools in the holders with the radius edges facing each other, and nip them up lightly by means of the taper bolts at the back of the holder.

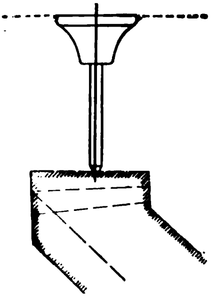


Fig. 251.

The gauge (Fig. 252) is then held against the surface of the tool slide (Fig. 249) and the tools are adjusted forward in the holders by the small set screw until they just touch the surface stamped with the size of radius of the tool being set, then tighten up.

The gauge (Fig. 253) is then placed upon the table of the machine, and the upper tool-holder adjusted vertically by the screw T (Fig. 248) until the cutting edge of the tool just touches the ground surface U of the gauge, then tighten in this position.

Treat the lower tool-holder similarly, setting to the surface P. For convenience in the last setting, the tool-holder may be moved in turn along the slide E, so as to be out of the way of the other tool-holder.

In opening out the former arms for the thickness of tooth, the following deduction from the angle given on the instruction sheet should be made.

For tools with $\frac{1}{32}''$ radius deduct 3 min.
„ „ $\frac{1}{16}''$ „ „ 4 „
„ „ $\frac{3}{32}''$ „ „ 5 „
„ „ $\frac{1}{8}''$ „ „ 6 „

Use of Tools.—The radius edge of the tool is always the cutting edge. All bevel pinions having less than 20 teeth, and of a ratio of 2 to 1 or greater, must be cut with the tool which has its radius on the hollow side.

All mitres and bevel wheels must be cut with the tool having its radius on the straight side.

Setting the Machine.—It is usual to perform two distinct operations in the cutting of any wheel—(*a*) Roughing-out of the space and (*b*) the forming or finishing of the sides of the tooth. The machine has a different setting for each operation.

Roughing.—When roughing-out the spaces, the former arms Q, Q, Fig. 248, are set so that the pointers R, R read 1⅓ times the angle to which the arms are set when finishing the teeth. For example, if the angle of the former arms was given on the paper (quantity 8) as 1° 30′, the roughing operation would be made with the pointers R, R reading 2°. The knock-off motion is then set by winding the saddle round until the tool touches the face of the blank; it will then be found that the vernier C does not read the same angle as the instruction sheet; whatever the difference is, make the same allowance on the base angle. C reads the base angle (quantity 6). When in this position the stop bracket X is moved to touch the end of the knock-off rod, and bolted securely. The stroke of the tool should be 1¼in. longer than the face of the wheel teeth, and may be made still longer in the case of a large, heavy wheel. The stroke is regulated by the position of the connecting-rod bolt in the slot of the disc Z. The disc is marked with a scale by which the stroke can be immediately set to any length desired. The overrun of the tools after cutting through the blank should not be more than ¼in. This completes the setting for the roughing operations; care being taken to see that screw I is unlocked.

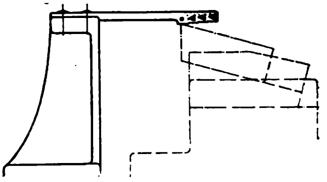


Fig. 252.

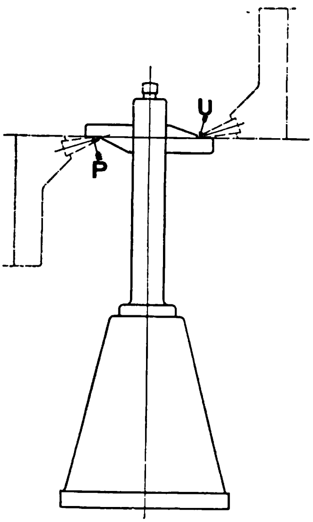


Fig. 253.

Finishing.—When setting for the forming or finishing of the teeth, wind the saddle round until the vernier C reads the pitch angle (quantity 5). Then set the connecting rod II (Fig. 248) to the pitch angles on scales III and IV. Scale VI is moved by means of the lever III until the pointer reads the pitch angle. At this stage lock the bolt I on the saddle. After this is done release the bolts V,V,V,V, and adjust the former arms Q, Q until the pointers R, R read the correct angle of the former arms (quantity 8). To adjust the former arms, use the lever attached to the pulley over which the band for balancing the arms passes, and finally, lock up the bolts V,V,V,V.

To test the accuracy of the settings, wind the saddle outwards and then bring it back to the pitch angle, when the pointers should give the same scale readings as at first. The settings for the stroke and knock-off motion remain undisturbed from the roughing operation. Before starting to cut, the saddle should be wound back to allow the vernier C to read the face angle plus half a degree. If a little more or less curve is required to the form of the teeth, it can be got by changing the position of the connecting rod II on lever III.

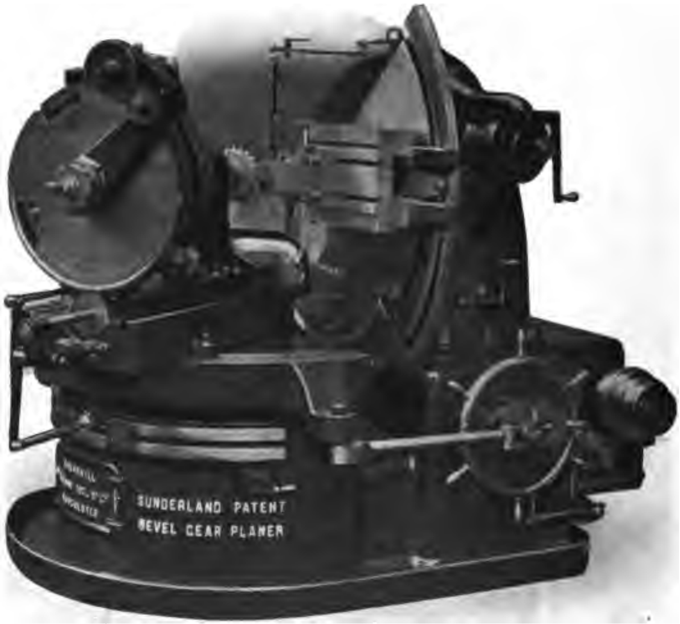


Fig. 254.—Sunderland Bevel-gear Generating Planer (Front View).

The Sunderland Bevel-gear Generator Planer.—This (Figs. 254, 255) is manufactured by the Churchill Machine Tool Co., Ltd., of Manchester. It possesses the advantage that gears can be gashed at one setting before the tooth forms are generated, thus avoiding the necessity for setting them on another machine for gashing. Provision is included for imparting a slight additional convexity to the tooth points in order to avoid risk of interference. The capacity is for gears up to 20in. diameter and up to 2½ diametral pitch in cast iron, and 3 diametral pitch in mild steel, and the range is from mitres, up to bevels having a 6 to 1 ratio. Three speeds of cutting are provided for by change gears, giving respectively 100, 66, and 34 cutting strokes per minute. The photographs, taken at front and back of the machine respectively, will, with the drawings, enable the construction and method of operation to be understood.

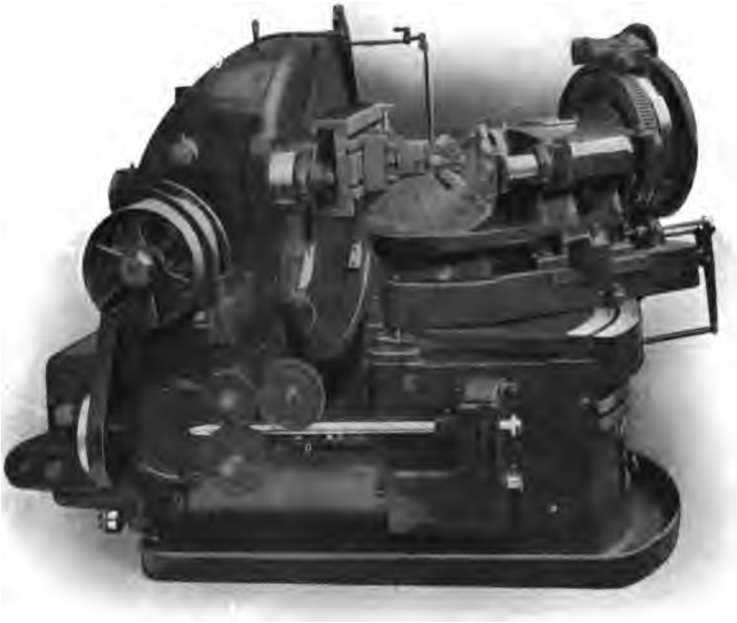


Fig. 255.—Sunderland Bevel-gear Generating Planer (Rear View).

The general scheme of the machine is outlined in Fig. 256. A base A carries at the left hand the work-head B on its swivel slide C, with the dividing gears. The cutter slide D is carried on a swivel E on the face of a fixed head F bolted down on the base at the right hand. The movements of the work headstock provide for the depth adjustments of the teeth, and those of the cutter head for the generation of the tooth curves. The tools are of wedge shape.

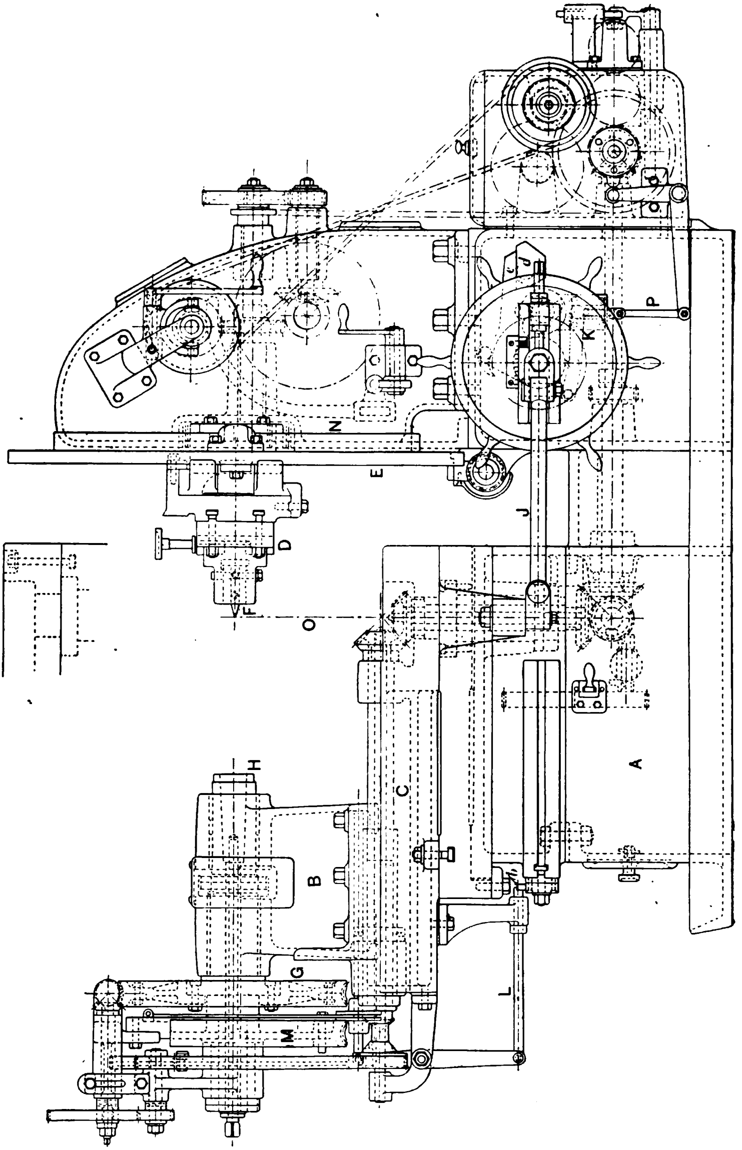


Fig. 256.

By means of slots and stops in the dividing wheel G, gears can be cut having pressure angles of 15°, 18°, 20° and 22½°.

Setting Up.—The setting up and method of operation of the machine is as follows:—The gear blank is carried on an arbor H in the work-head B, which head is carried as stated on the swivelling slide C, the centre of swivel of which coincides with the cone centre O of the machine—the apex of the gear. By the angular movement imparted to this slide the depth of cut of the teeth is adjusted. The feeding-in movement is transmitted by a connecting rod J and crank at the side of the machine and actuated by trains of gears from the single pulley drive. The crank pin for the rod J is adjustable by a vernier.

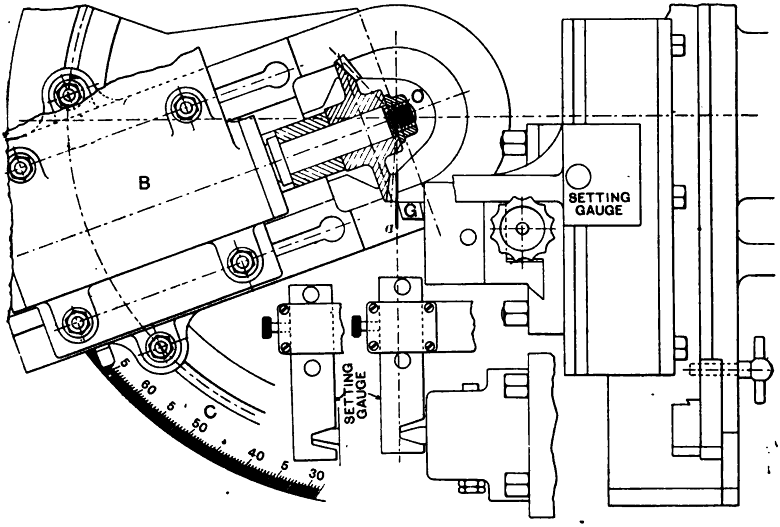


Fig. 257.

Cutting.—The relative positions of the blank and tool at the commencement of cutting are shown in the plan view, Fig. 257. Here the blank has been brought up to the tool by moving the work-head along its slide, and set by interposing a clearance gauge *a* between the blank and the point of the tool. The gashing is done first, followed by the generation. The tool, carried and reciprocated on the slide D, Fig. 256, carried on the vertical swivel slide E, is reciprocated by a connecting rod and crank disc. Graduations are provided on the connecting rod for exact adjustment of the length of stroke. The cut is a draw cut, and takes place from the larger end of the tooth to the smaller. It is necessary in consequence of the large angular movement of the slide when generating to see that the graduations on the crank disc which govern the stroke, and those on the connecting rod which govern the same, should correspond when set. Also the stroke of the tool should exceed by ¼in. or ½in. the length of face of the gear blank. A little more clearance is left on the outside than on the inside, so that the tool can be given full opportunity to return to its cutting position before the commencement of each cut.

Gashing.—In order to perform the preliminary gashing, the tool-slide is locked in a central position with a stop pin seen at the left of Fig. 257. The work-slide which carries the gear is then fed towards the tool automatically by means of a ratchet motion, controlling the plate K and rod J which operates on the return stroke of the tool. When the full depth of the tooth space has been cut, the work-slide is automatically withdrawn from the tool. The operation of withdrawal also puts into action the dividing motion, which takes place simultaneously with the other movements of the machine. By the time the dividing is completed the gear is again being fed forward to the tool, and these actions are repeated until all the tooth spaces have been stocked out.

Indexing.—The dividing motion is put into action automatically by the adjustment of the plunger *b* in the slot in front of the machine, about midway in the stroke of the swivelling slide. This plunger, by pressing out the rod L, puts the dividing motion into action through plates above adjusted so as to withdraw a pin from the stops. The feed is regulated on the disc K through a plate *d* and a feed lever *e*, by which means from one to five ratchet teeth can be passed as required. When all the spaces have been stocked out, the machine should be stopped when the gear blank is at its outer position clear of the tool. The pawl will then be raised out of engagement with the ratchet wheel, after which the finishing cuts are taken. To start the finishing cuts the stop pin which locks the tool-slide is withdrawn, leaving it free to swivel. A plunger above P is also withdrawn. A plunger which governs the fast motion is then operated through the bell-crank lever and friction cam which is operated through the ordinary reversing mechanism for the generating motion. The tool F is adjusted for taking the finishing cuts. This is done by means of a tool-setting gauge (Fig. 257). One side of all the teeth are first finished, and afterwards all the other sides. It is immaterial which is done first, but the direction of rotation of the dividing wheel must be set for the side to be cut, either top or bottom. A reversing handle is provided for this purpose. It is pulled " out " when the top side is to be finished, and pushed " in " for finishing the bottom side. Holes for the different positions of the stops are provided. There are two of these stops, one for the top, the other for the bottom side of the tooth, and the choice of four slots for each, for locating the corresponding side of the tooth in the correct position for generating teeth with pressure angles of 15°, 18°, 20° and 22½°. The generating motion is governed by mechanism at the rear of the machine, arranged with adjustable stops for the amount of " roll " required, which depends on the ratio of the gears being cut. The generating motion is provided by the quadrant segment M. It can be stopped instantly by a lever. It is always stopped on the downward stroke of the tool-slide, and when the swivel-slide which carries the gear is in its outermost position. The interference cam N is changed according to instructions sent with a cam chart with the machine, being governed by the centre angles of the gears being cut. Its object as stated is to slightly round the points of the teeth to ensure quiet running, which it effects by accelerating or retarding the action of the tool.

The Gleason Machine.—The Gleason automatic generating bevel-gear planer (Fig. 258) operates with a pair of cutting tools which shape opposite sides of the same tooth. The cutting edges of these tools represent the adjacent teeth of a crown bevel gear of the same pitch as the tooth which is being planed. The tools and the blank roll together in the relation of a crown gear in engagement with a pinion. They travel along in planes converging to the centre of the cone of the gear being cut, which is also the centre of the machine. They both operate simultaneously, but while one is making the forward stroke the other is on the return stroke. This doubles the speed of cutting, balances the thrust of the cut, lessens vibration, and avoids risk of interference of the tools, which might occur beyond the small ends of the teeth in gears of long faces if the tools were working in the same direction at the same time.

Setting of Blank.—The blank is mounted on an arbor which is a combination of two spindles, one within the other (Fig. 259). Both spindles rotate together except during the act of indexing. As the crown gear is represented by the cutter slide and its reciprocating tools and slides, these are adjustable to give varying included angles, and they are mounted on a " turret " adjustable for cone angles. The varying angles of the pitch cones of different gears are provided for by a series of twenty-eight bevel-gear segments in the 18in. machine, and these being fastened to an oscillating element termed a " yoke," control the movements of the blank and of the cutter slides. These elements, with the other essential parts of the machine, may be studied in the subsequent outline drawings, aided by the photograph (Fig. 258).

In the plan view (Fig. 260), *o*, the centre of the machine, is the cone centre; A is the blank, and *b* one of the cutting tools. The tool slides *c*, *c* are carried on guides *d*, *d* on a turret B (compare with Fig. 261), which swivels about the cone centre *o* in order to bring the tools into the correct cone angular, or sectional relations to the gear blank A. The angular relations in the other direction—those, namely, for the tooth flanks—are obtained by the movements of the slides around the centre *o′* in Figs. 261 and 262, which is plumb over *o* in Fig. 260. The yoke C is the element through which the oscillations of the blank and tools are produced, its centre of oscillation being the axis *e* of the work arbor. The work mandrel D (Fig. 260) is engaged by a feather at the right-hand end of the yoke, which

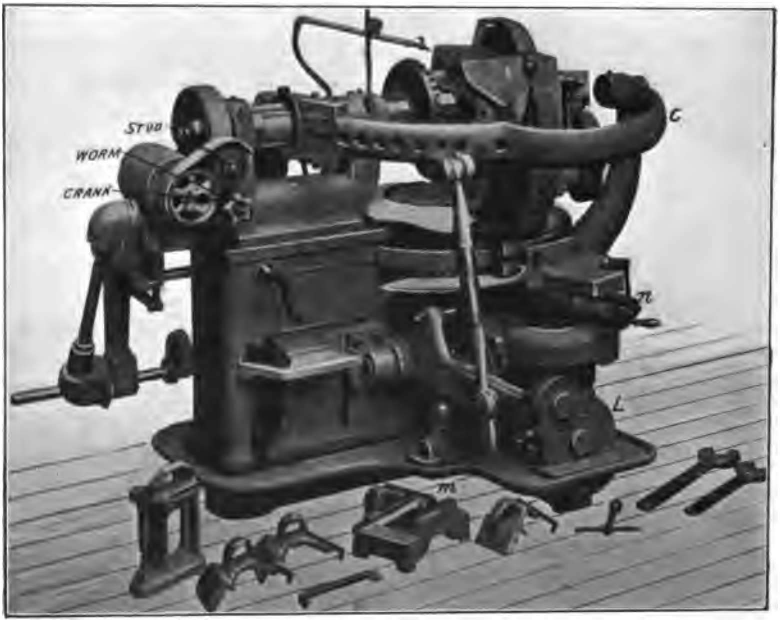


Fig. 258.—The Gleason Automatic Generating Bevel-gear Planer.

is consequently compelled to oscillate with the yoke. This movement is produced by one of the twenty-one bevel-gear segments supplied, one of which is seen at *f* (compare with the photograph, Fig. 258), which vary in respect of the angle, *g*, *o*, *e* (Fig. 260). The one which has the same angle as the gear to be cut is selected and bolted to the inside of the yoke C, so that its pitch line is a continuation of that of the gear to be cut. A series of bolt holes is provided in the yoke (*see* Fig. 258) to accommodate the range of segments. The swinging movement of the yoke and segment is communicated to a crown-wheel segment *h*, which is bolted to the end of the swinging frame that carries the tool-slides. The cutter-slide

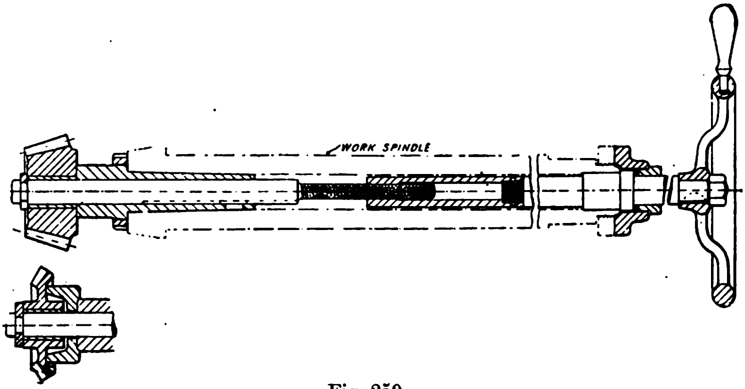


Fig. 259.

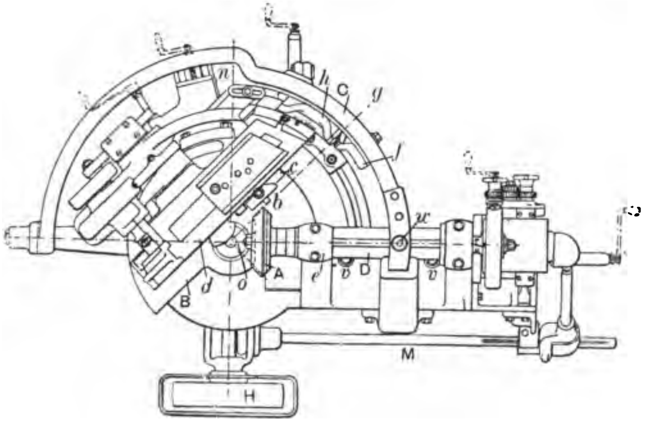


Fig. 260.

and the gear blank are moved at the same velocity ratio. The yoke C which operates the movements is itself oscillated through the connecting rod E (Fig. 261), driven from the slotted radius arm F, which is driven by a cam G behind it. The slot in the arm F permits of variations being made in the length of stroke, and the cam varies the speed, giving a slow motion near the pitch line, where the cut is heavy, and a relatively rapid motion at the ends of the tooth, where the cut is lighter.

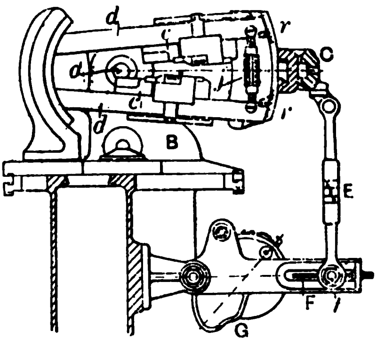


Fig. 261.

The Tools.—The tools are reciprocated along guide bars *d*, *d*, which are attached to blocks that can be swivelled and adjusted about the centre *o′* (Fig. 261), to suit any angle of tooth, which adjustments are made by the turnbuckle *j*. On the axis *o′* an oscillating shaft carries a spur pinion. This pinion engages with a rack on each tool-slide on its top and bottom respectively, and so reciprocates the slides in alternate directions. These are not indicated in the drawings. As the axis of the pinion lies in the axis *o′*, around

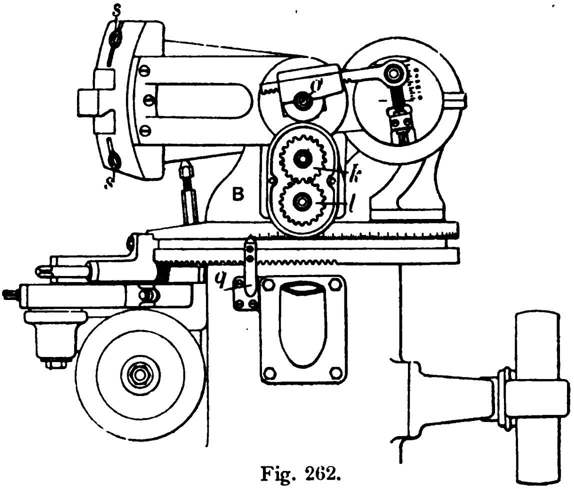


Fig. 262.

which the guide bars *d, d* are adjusted for angle, the racks remain always in engagement at all angles. The pinion is oscillated by an adjustable crank and rack shown in face view in Fig. 262, and by a pinion on the outer end of the tool-slide driving the pinion shaft. The length of stroke is determined by the adjustment of the crank, which is indexed. The tools are driven by a shaft coming up through the centre of the turret, through bevel gears, covered in by guards on the left of Fig. 260, and spur gears *k* and *l* in Fig. 262, these last being change gears for regulating the number of strokes of the tools per minute. The pulley H drives these gears, as well as the work arbor, through the shaft M, and gears and change gears. These are the leading features in the design of the machine. Details can now be better studied by going through the method of its setting up and operation.

Operation.—All the angles of the gear to be cut having been obtained from tables and formulæ, the machine is first started and run until the arms and yoke are on the descending roll. The jack *m* (Fig. 258) is then placed in position for holding the lever arm F (compare with Fig. 261), and when this rests on the jack the machine is stopped. The difference between the root angle and the face angle of the gear to be cut will be noted from an angle chart supplied. One degree is added to this for clearance, and the sum is the amount of travel of the turret desired, which will now be set by means of graduations on the slide *n* (Figs. 258 and 260). Each mark on *n* is equivalent to one degree of travel of the turret. The next stage is the adjustment of the crown gear *h*, and the bevel segment *f*, which has to be done with suitable precautions to ensure the correct cone angle, and the central setting of the yoke. When these have been made, the segment *f* is finally secured to the yoke C, and a thrust brace *p* (Fig. 258) is brought against the crown gear, the pressure being cushioned by leaf springs. The base of the turret is graduated as seen in Fig. 262, and a zero point at *q* enables the angular setting of the turret to be made. The turret is adjusted to the root cone angle of the gear.

Tooth Angle.—The adjustment of setting the arms to the proper tooth angle for the flanks is done by means of the graduations *r, r* (Fig. 261). The binder bolts *s, s* (Fig. 262), and two others, not seen, having been loosened, the upper arm is set by means of the turnbuckle *j* (Fig. 261), and its bolts are then tightened, and the lower arm is set similarly. The arms are set to the exact angle given on a chart supplied, for the finishing cut only. When taking the stocking cut, a few minutes of arc are added to the angle to afford the allowance for the finishing cut. The amount to be added varies according to the cone distance pitch and quantity of material left to be removed. The amount therefore in any case is left to the judgment of the operator.

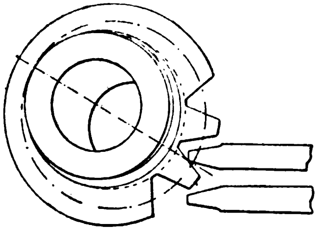


Fig. 263.

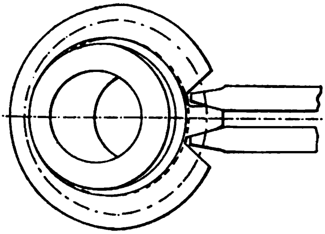


Fig. 264.

The Roll.—The next thing to be done is to set the " roll," or generating motion, which is governed by the position of the crank pin *t* (Fig. 261) in the slot F. The amount of roll is governed by the pressure angle of the teeth, and a pinion requires more roll than its mating wheel. The tools must roll clear of the cut, or cease cutting at each end of the roll. Fig. 263 represents the generating tools beginning their work on a roughed-out blank, the tools being below the centre of the machine. Fig. 264 represents them at their central position, and Fig. 265 above the centre of the machine at their highest position. The blanks, it will be noticed, have been previously gashed out with stocking tools shaped in truncated vee forms.

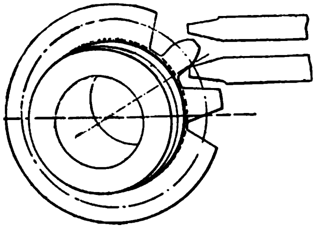


Fig. 265.

The machine is set for the proper amount of roll by adjustments of lines brought opposite to each other, which is done by means of the right and left hand screw E (Fig. 261), and must be done while the arm F is resting on the jack *m*.

Stocking Tools.—The stocking tools are put in their clapper blocks, and fastened in place by tightening binder bolts. Next the tool gauge *u* (Fig. 266) is tried. If the point of the tool either extends beyond or does not reach the gauge, it is adjusted by means of screws. If the cutting slide is too high or too low, the necessary adjustment is made by other screws. When the end of the tool and the cutting slide pass the gauge, touching at both the end and side, the tool is in the correct position. The index gears are selected from a table, and set according to the references, "stud," "worm," and "crank," in Fig. 258.

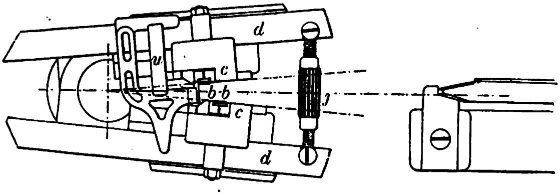


Fig. 266.

The gear blank A (Fig. 260) is now mounted, selecting as stiff an arbor as possible. The blank is set in its proper position by means of the cone distance gauge J (Fig. 267). This is done by setting the slide of the gauge at zero, and moving both the head D (Figs. 260 and 267) and the gauge till the pointer registers zero, when the arms are set at the face angle. To move the head and mandrel D lengthwise, the bolts *v*, *v* (Fig. 260) are loosened, and the binding bolt *w*, and the adjustment is made by the crank K (Fig. 258), after which the bolts *v*, *v* (Fig. 260) must be tightened. The gauge is set on the arm as shown in Fig. 267.

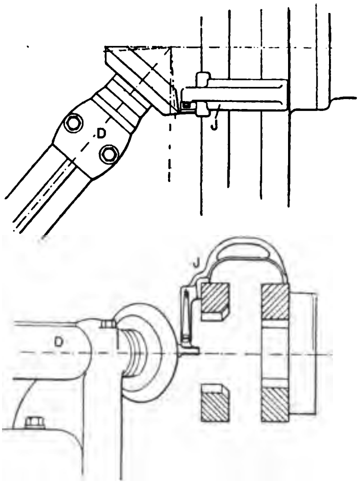


Fig. 267.

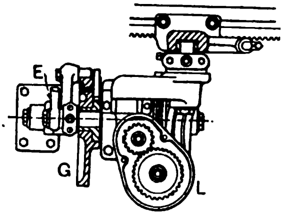


Fig. 268.

A cam key, seen in Fig. 268, is pulled out and locked in position by turning a small plate on the side of the key. This disconnects the generating cam, and has to be done while the arm F is on the jack. The finishing cam G, is taken off, and the roughing cam put on and fastened. To take off the finishing cam it must be reversed until the slot in the cam is opposite the roller. The stroke of the tool is now set according to the length of the tool face to be cut, allowing ¾in. for the clearance of the tools. This is done, as stated, by the graduations on the crank plate (Fig. 262).

The speed change gears, *k*, *l* (Fig. 262), must be selected to suit the length of face and amount of stock to be cut. A table gives the number of strokes per minute which each change makes. The feed change gears enclosed at L (Figs. 258 and 268) must also be selected to suit the varying conditions of size of gear, pitch, and amount of material to be removed, and nature of material, based on a table furnished with the machine. The machine may now be started. The roughing-out is first done with the stocking tools (Fig. 269), after which the finishing tools are put in; the roughing cam being removed and the finishing cam put in its place. The arms being closed to the proper tooth angle, the machine is then ready for the finishing cut.

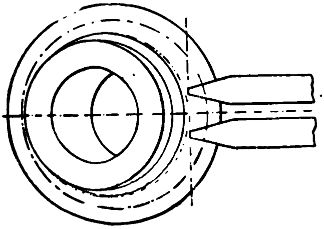


Fig. 269.

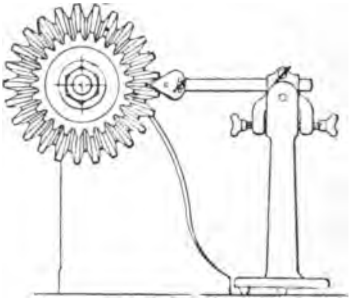


Fig. 270.

Generating.—The generating process commences with the yoke in the lowest position. This then moves upwards to the highest position (Figs. 258 and 265), and the gear blank rolls with it at the same rate, measured on the pitch lines. The cutting tools reciprocate simultaneously, and plane each side of the tooth nearly to size. The first movement from the lowest to the highest position takes a roughing cut over both sides of one tooth. When the tools are at the highest position they are fed inwards slightly, which feed serves on the downward movement to take a light scraping cut, bringing the tooth to size and ensuring a smooth surface. When the slides are returned to the lowest position the turret automatically moves the tools out of the plane of the tooth just formed, and the gear is indexed one space. The tools are returned to the cutting position, and the cycle of operations can be repeated until the gear is completed. One tooth is finished first. Then when the machine has indexed to the next tooth it is stopped, and the stock-dividing gauge (Fig. 270) is set. Each succeeding gear can then be set with this gauge without the use of tools.

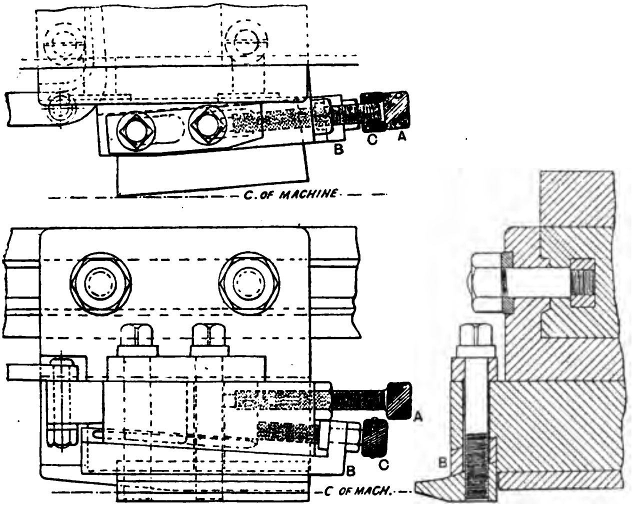


Fig. 271.

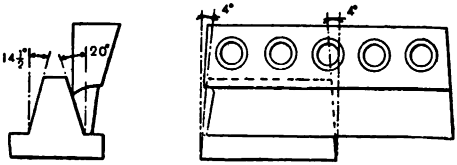


Fig. 272.

Fig. 271 shows the newest style of tool-block used on these machines. The longitudinal adjustment of the tool is obtained by means of the screw A set with a lock-nut. Its vertical adjustment is provided for by the wedge B operated by the screw C, which, as will be seen, affords extremely fine adjustments. The cutting angles are checked against a grinding block (Fig. 272), which contains the two angles of 14½° and 20°.

THE GLEASON TOOLS.

THESE ARE SHOWN FOR THE 11″ AND 18″ GENERATING MACHINES.

To select tools for cutting a pair of bevel gears:—

1st. Note that the length of point of tool, "F," Fig. 273, is not less than the depth of tooth at the outside "T," Fig. 274. It can be longer.

2nd. Note that the width of point of tool, "B," Fig. 273, is at least ·015 less than the width at bottom of space at inside of tooth "S," Fig. 274.

The stocking tool point "B," Fig. 275, is made same width as 20° finishing tools of the same pitch. The sides "D" and "E" are made 14½°.

The sizes of tools given below are figured for 20-tooth mitre gears with face $\frac{1}{3}$ of cone distance.

For gears with greater length of face the tools should be ground on side "E" only.

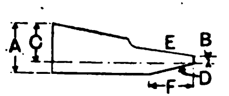


Fig. 273.—Finishing Tool.

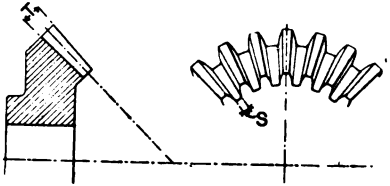


Fig. 274.

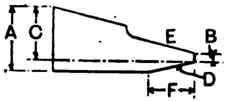


Fig. 275.—Stocking Tool.

14½° FINISHING TOOLS.		
C=·588″	D=14½°	E=8°
Pitch.	A.	B.
3	·787″	·200″
4	·742″	·147″
5	·711″	·115″
6	·691″	·093″
8	·665″	·061″
10	·650″	·050″
12	·640″	·039″
7	·680″	·078″

20° FINISHING TOOLS.		
C=·533″	D=20°	E=8°
Pitch.	A.	B.
3	·813″	·145″
4	·750″	·107″
5	·706″	·082″
6	·678″	·061″
8	·642″	·045″
10	·620″	·033″
12	·606″	·025″
7	·660″	·054″

STOCKING TOOLS.		
C=·625″	D=14½°	E=14½°
Pitch.	A.	B.
3	·824″	·145″
4	·779″	·107″
5	·748″	·082″
6	·728″	·061″
8	·702″	·045″
10	·687″	·033″
12	·677″	·025″
7	·715″	·054″

Gauging Tooth Thickness.—The method of gauging the tooth thickness is either by a solid gauge or by an adjustable vernier tooth caliper, the first named being more expeditious and easier to use than the second. The tooth thickness is the chord thickness, based on a gear about half-way between a pinion and a rack tooth. This thickness would be very slightly different for gears of different numbers of teeth, and theoretically it would not be absolutely correct to use the same for all gears of the same pitch. But practically this difference may be ignored, and a solid gauge would make the pinion teeth very slightly thicker than standard, and its wheel slightly thinner, due to the differences in curvature. If a solid gauge made from a rack tooth were used it would make the teeth of wheels and pinions very slightly thicker than standard, sufficient to throw a pair of bevels out of square. If a vernier is set for the thickness to the chord of half the circular pitch, and for depth to the addendum plus the versed sine, the exact thickness of tooth at the pitch line would be obtained. For methods of calculation see p. 28 in connection with Fig. 20.

Pressure Angle.—With regard to the influence of the pressure angle of the teeth on the amount of roll, the Gleason Works give the following as the practical limit to which the 14½° pressure angle may be followed on the generator:—

Ratios.	Teeth in Pinion.	Ratios.	Teeth in Pinion.
1 to 1	14	4 to 1	21
1½ to 1	18	5 to 1	21
2 to 1	19	6 to 1	21
3 to 1	21		

Any pinion within the limits named in this list can be cut of 14½° pressure angle, regardless of the pitch, up to 3 diametral pitch, as that is the limit of this particular machine, provided the blanks are turned to standard proportions—that is, with the usual addenda. But the 14½° pressure angle may be followed down to twelve-toothed pinions or less if the blanks are turned to long addenda, and short flanks, and *vice versa* on their mating gears. Or by using a 20° pressure angle with the blanks turned to standard proportions, gears of any practical limit may be cut. Fig. 31, p. 42, gives the profiles of three twelve-toothed pinions of a 4 to 1 ratio. A, of standard proportions, has too much undercut; B, of similar proportions but of

20° pressure angle, has less undercut, and is preferable to A. C has a long addendum, and gives the greatest bearing surface and is strong in the flank. The generating machine produces the true curve in each case.

The machines are now being equipped with cams which operate with one generating cut on the upward roll, using the downward roll for swinging out and indexing. If preferred, the machine can be arranged with cams to give the roughing cut up, and the finishing cut down. The latter style is recommended where extreme accuracy and a smooth finishing cut is desired, as in automobile work. Modifications are being frequently made in the Gleason machines, but the operating instructions as given are substantially those which underlie the designs.

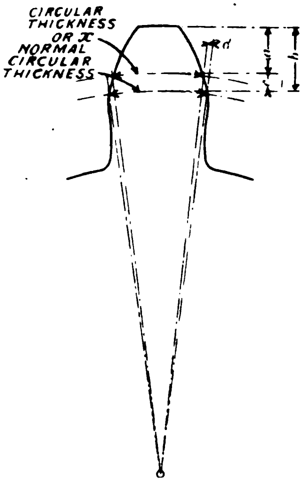


Fig. 276.

The Gleason Works give the following rule for corrected circular thickness of teeth with reduced addenda (Fig. 276):—

Let

a=Given pitch depth
and b=Normal pitch depth.

Then $c=b-a$.

$d=c \times$ tangent pressure angle.

$x=\frac{1}{2}$ circular pitch $-(2 \times d)$.

CHAPTER XV

GENERATING BY HOBBING

THE foregoing selections of spur-gear cutting machines illustrate the best and highest practice of leading firms in these designs. There are many other examples, of course, but they would present no striking departures from the machines which have been selected, for a casual study of the examples chosen will show that certain leading principles of design run through the whole, and that it is the details chiefly that vary. The value of details is cumulative, so that the best machines to-day are those which, though similar in outline to rival machines, embody the results of much care in the working out of details, which contribute to precision and accuracy of operation, and which are so manufactured as to combine great accuracy with durability.

The group which is designed to produce spur gears with single-disc rotary cutters represent the latest evolution from earlier machines, the position of which has been retained undisputed until the hobbing machines of recent gears came into rivalry with them. These we now consider.

The Wallwork Hobbing Machine.—The essential features of the design of most hobbing machines are shown by the figures which illustrate one by Messrs Henry Wallwork & Co., Limited, of Manchester. It is of the complete or universal type, that is, it will cut spurs, spirals, and worm gears. Some machines are designed for cutting spurs only. The principle of action of hobs has been described on p. 99. The views of the machine given are external, Fig. 277 being the side or front elevation—that is, the side on which the operator stands; Fig. 278 is a plan; and Fig. 279 is an end elevation. The essential parts are as follows:—

Framing.—The framework consists of two castings, the bed A and the head B, which is keyed and bolted to the bed. On the latter slides the carriage C, actuated by a screw which may be turned by hand or power. This saddle is fitted to the bed with a long narrow guiding strip and adjustable taper gib, and the keep plates prevent any tendency to lift. Bolts are provided for locking this saddle in

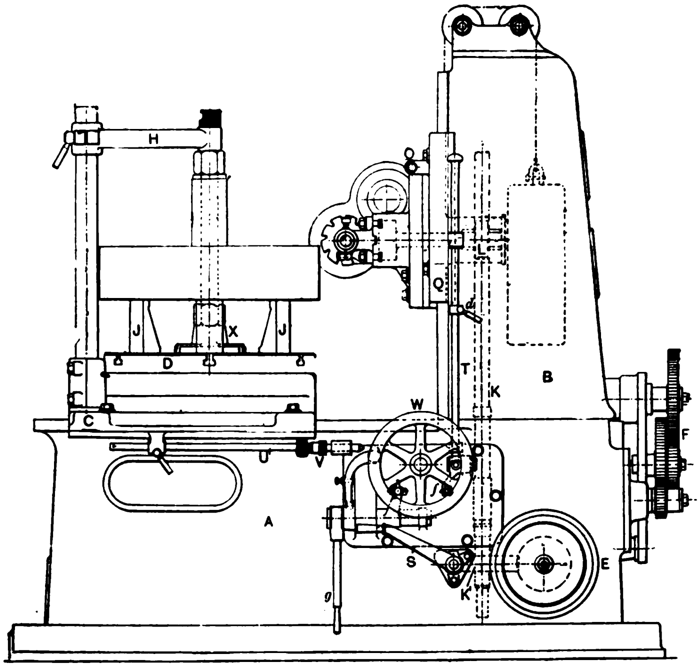


Fig. 277.

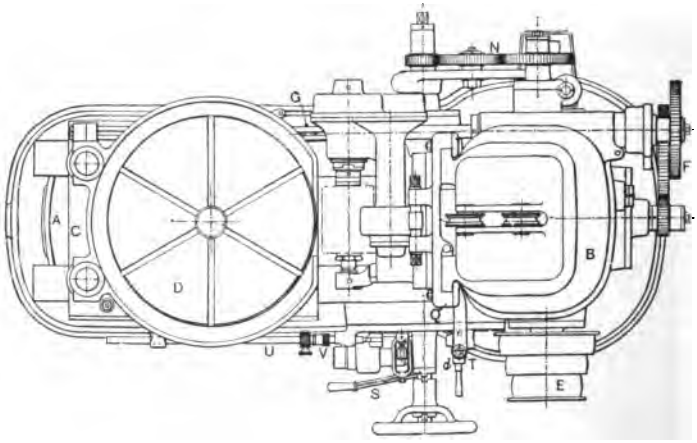


Fig. 278.

any position, by holding it securely to the face of the bed. The work-table D revolves in this saddle, on a large annular bearing, and is provided with a tail-pin and lock-nuts to prevent any lifting tendency; it is driven by worm gear (not indicated). The steel driving worm is hardened and ground, and is adjustable with its wheel for wear, while ball washers for end thrust are provided.

Hob Mandrel.—The hob is adjustable on its mandrel, and is carried in a head which can be swivelled through a complete circle by means of a worm O and a wheel P, and its edge is divided by a vernier scale. This swivel head is bolted to a saddle Q made to slide on the head B of the machine. The saddle Q is fitted with a long narrow guiding strip and adjustable taper gib, keep plates preventing any tendency to chatter when the saddle is sliding, and bolts are provided to lock the saddle in any position, by holding it securely against the face of the head B. The saddle is moved by a screw, turned either by hand or power.

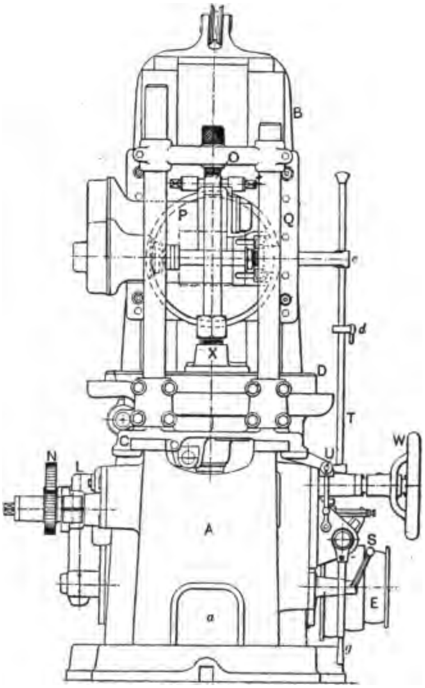


Fig. 279.

Method of Driving. — The machine is driven by a three-speed cone pulley E, which in conjunction with a two-speed counter-shaft gives six changes of speed arranged in geometric progression. From this shaft the drive is taken through a worm and wheel to a horizontal shaft which drives through the change gears F to the table worm shaft G. This last is key-grooved to permit of the travel of the table, the driving worm, and saddle. From the shaft of the cone pulley E the drive is also taken through a worm and wheel to the vertical shaft K, outlined in Fig. 277, then through worm gears indicated at L to the swivel head, and through helical gears to the hob. To the end of the pulley shaft also are connected two worm reduction gears K; these drive the change gears N which give the power feed, and the change gears N vary the feed to the vertical screw (not indicated) which travels the hob saddle, and to the horizontal screw which travels the table saddle.

When cutting spur or spiral gears the hob is fed across the face of the wheel blank. The power feed is controlled by the handle *g* operating a sliding clutch; connected to this handle is a trip-rod T passing through a lug *e* cast on the saddle Q. When the saddle has travelled down to any desired point, the lug *e* strikes a collar *d* and throws the power feed out of gear. This actuates a lever *f* operating the reversing mechanism. The collar *d* may be moved and secured to the trip-rod T at any point. A micrometer V is provided for measuring the depth of tooth. This may be secured to the trip-rod U, and the trip-rod may be secured to the table saddle to trip out the feed when the correct depth of tooth has been cut. When cutting worm wheels the face of the blank is fed into the hob. An oil pump supplies lubricant to the hob.

The hob saddle is provided with a quick elevating and lowering motion controlled by the lever S. This motion is independent of the above feeds, and is used for rapidly setting the hob when commencing to cut a gear, or returning the hob after a gear has been cut. The hand wheel W gives motion either to the table saddle or to the hob saddle. By drawing the wheel forward (*see* Fig. 278) it is clutched up with the spindle, and by rotating it to the right or left the table saddle is moved in or out, while by pushing the hand wheel in it is clutched up with the sleeve which gives motion to the hob saddle.

The driving and feed gears, which are spurs, helicals, or worms and wheels (bevels not being used), are contained in the bed, and run in a bath of oil. The remainder of the bed is utilized as a receptacle for cuttings and cutting lubricant, the hole *a* being used to take out the cuttings through. A trough runs around the bed to catch any drippings.

From three to four work arbors are supplied, according to the size of machine. These are for use when cutting a gang of gears, and are provided with taper ends, which by a right and left-hand nut are forced into the bracket X bolted to the table. The other end of the arbor is supported by a movable stay H. This stay is secured to two uprights, also movable. The large gears when being cut are supported on blocks J, J. The machine is self-contained and is made in four sizes.

Setting Up.—For setting up work on this machine principles common to all machines of this class are applied. The following details are summarized from the shop operating sheets of the firm. The basis operations are those for spur gears, those for spiral and worm gears following naturally. By the help of the diagrams the elements of setting up will be understood.

For spur gears two sets of formulæ are used, the indexing or pitching, and the feed. The latter is the distance which the hob travels down the face of the gear being cut when the gear has made one revolution. The formula for the index gears is:—

$$\text{Index gears} = \frac{15}{\text{Number of teeth to be cut in blank}} = \frac{\text{driver}}{\text{driven}}$$

A full table of these gears is supplied, ranging from pinions of 9 teeth to wheels of 142 teeth.

$$\text{Feed gears} = \frac{\text{Feed in fractions of an inch} \times 600}{\text{Number of teeth to be cut in blank}} = \frac{\text{driver}}{\text{driven}}$$

The feed is reckoned in fractions of an inch, and is the distance the hob travels down the face of the wheel being cut, when that wheel has made one revolution.

Example:—

Spur wheel, 115 teeth, ½in. circular pitch, feed say $\frac{1}{20}$th in.

$$\text{Index gears} = \frac{15}{\text{Number of teeth to be cut in blank}}$$

$$= \frac{15}{115} = \frac{30}{90} \times \frac{45}{115} = \frac{\text{drivers}}{\text{driven}}$$

$$\text{Feed gears} = \frac{\text{Feed} \times 600}{\text{Number of teeth to be cut in blank}}$$

$$= \frac{1}{20} \times \frac{600}{115} = \frac{24 \times 20}{23 \times 80} = \frac{\text{drivers}}{\text{driven}}$$

Right and Left Hand Hobs.—Although the majority of gears are cut with right-hand hobs, yet a considerable proportion are cut with left-hand ones, and then the machine has to be set up differently. There is no difference in the working out of the index gears, but when using a left-hand hob an intermediate gear has to be interposed as in ordinary screw-cutting for left-hand threads, also the direction of rotation of the work-table shaft must be reversed. The feed gears remain the same in both cases because the hob saddle is always fed downwards.

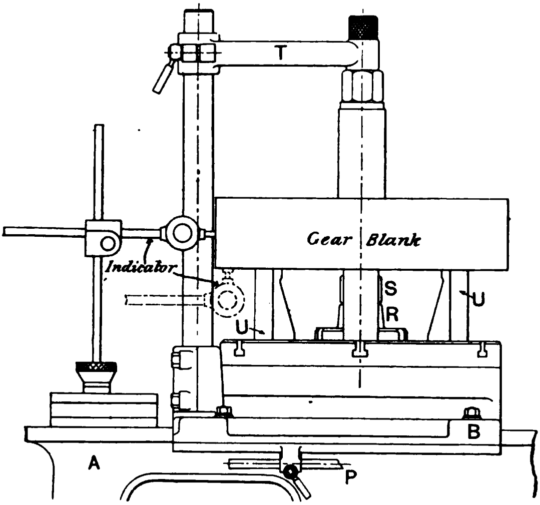


Fig. 280.

Setting the Blank.—When setting a blank on the table it is advisable when possible to use the work mandrel to centre by, employing a bush if necessary. After a blank is secured by a nut on the work mandrel and supported on the parallel packings (Fig. 277), its truth is checked by an indicator (Fig. 280). This being set against the rim as the blank revolves shows if the rim is out of centre, and if set against the edge it shows how much it has been sprung by the bolting down.

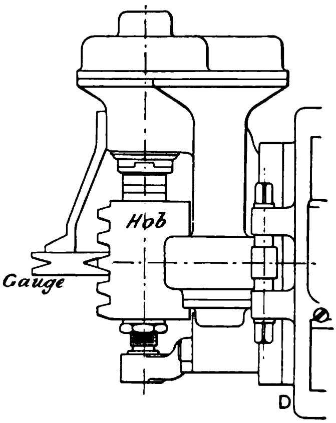


Fig. 281.

Setting the Hob.—The hob is next set by means of a vee-gauge, and also for angle. One tooth of the hob should always lie in the centre of the vee of the gauge (Fig. 281); also, as the hob is adjustable along its arbor, different teeth should be centred on the different occasions when the hob is used, in order to equalize the wear. The angle of the hob being stamped on it, the head is swivelled and locked to this angle so as to bring the threads perpendicular when cutting spur gears. This is effected by the worm gear, degrees being marked on the flange of the saddle, with a zero mark on the flange of the head.

Depth of Cut.—The depth of cut is measured by lowering the hob saddle until the hob is in line with the centre of the face of the wheel blank. Then the table carriage is moved towards the hob until the face of the gear blank just touches the points of the hob teeth. The micrometer O (Fig. 282) is now screwed out to its full extent until it reads zero, when it is locked to the trip-rod. After pressing it firmly against the boss on the apron, the trip-rod is locked to the carriage, the depth of the tooth to be cut being measured off on the micrometer. The hob is now raised clear of the wheel blank and the table carriage is moved inwards until the micrometer again abuts against the boss on the apron. Finally the table carriage is locked to the bed, the collar on the trip-rod is set to trip out the feed when the hob has passed through the wheel face, and the machine is then set up ready to work.

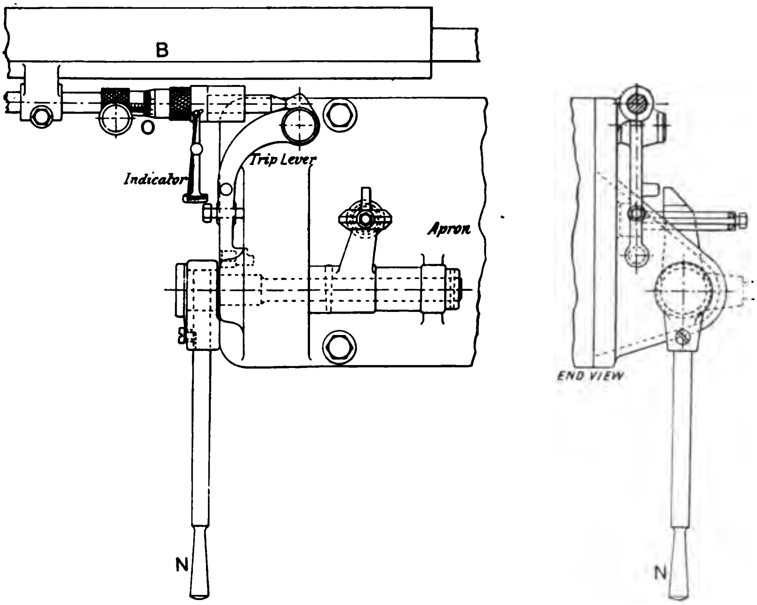


Fig. 282.

Setting Up for Spiral Gears.—In hobbing spur gears the hob feeds perpendicularly, and the ratio between it and the work-table remains constant. In spiral gears (Fig. 283) the feed depends both on the downward travel of the headstock and the angle of the tooth. The larger this angle is the greater is the distance which the hob must travel in relation to the travel of the headstock. In a 45° angle spiral the hob cuts 1.414in. per 1in. of down-travel of the headstock. To produce this, a differential motion is introduced which imparts a constant acceleration or retardation, to combine with the down feed of the hob in producing the angle of spiral. The table movement is the base of a triangle, the vertical movement of the hob is the perpendicular, and the angle of spiral produced is the hypotenuse.

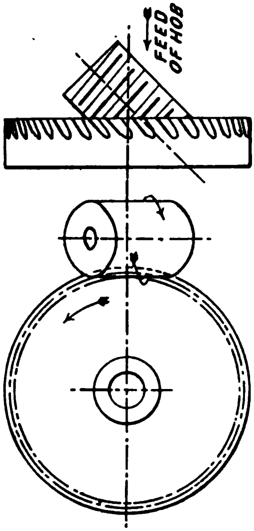


Fig. 283.

Setting the Headstock.—The hob headstock is set for spiral gears to the angle of the spiral gears, plus or minus the angle of the threads of the hob, the latter depending on whether the spiral is cut with a hob of the same hand or with one of opposite hand. The formulæ for indexing and feeds, with the assistance of the diagrams, will render this clear.

$$\text{Index gears: }\left\{\begin{array}{llll}\text{Right-hand spiral cut with right-hand hob}\\ \text{Left} & ,, & ,, \quad \text{left} & ,,\end{array}\right\}$$

$$=\frac{15}{\text{Number of teeth to be cut}}\left(1+\frac{\text{feed}}{\text{lead}}\right)=\frac{\text{drivers}}{\text{driven}}$$

$$\left\{\begin{array}{llll}\text{Left-hand spiral cut with right-hand hob}\\ \text{Right} & ,, & ,, \quad \text{left} & ,,\end{array}\right\}$$

$$=\frac{15}{\text{Number of teeth to be cut}}\left(1-\frac{\text{feed}}{\text{lead}}\right)=\frac{\text{drivers}}{\text{driven}}$$

$$\text{Feed gears}=\frac{\text{feed}}{\text{lead}}\times\frac{600\times\text{lead}}{\text{Number of teeth to be cut}}=\frac{\text{drivers}}{\text{driven}}$$

Example:—

Number of teeth to be cut in blank,—72.
Lead 68″ Feed say $\frac{1}{45}$″.
Right-hand spiral cut with a right-hand hob.

$$\textit{Index Gear Formula}=\frac{15}{\text{Number of teeth}}\left(1+\frac{\text{feed}}{\text{lead}}\right)$$

$$=\frac{15}{72}\left(1+\frac{1}{45}\times\frac{1}{68}\right)$$

$$=\frac{15}{72}\left(1+\frac{1}{3060}\right)$$

$$=\frac{15}{72}\times\frac{3061}{3060}$$

Look in the table of prime factors for two consecutive numbers which will factor out easily as near to 3060 as possible, say $\frac{2976}{2975}$ Then by substitution the fraction reads:—

$$\frac{15}{72}\times\frac{2976}{2975}$$

$$=\frac{40\times31}{85\times70}=\text{Index gears}=\frac{\text{drivers}}{\text{driven}}$$

As altering 3060 to 2975 alters the feed, work back to find what the feed is.

$$\frac{15}{72}\times\frac{2976}{2975}$$

$$=\frac{15}{72}\left(1+\frac{1}{2975}\right)$$

The fraction $\frac{\text{feed}}{\text{lead}}=\frac{1}{2975}$

The feed $\frac{1}{2975}\times\frac{68}{1}=\frac{1''}{44}$ approx.

Feed Gear Formula.

$$\frac{\text{Feed}}{\text{Lead}}\times\frac{600\times\text{lead}}{\text{Number of teeth to be cut.}}$$

$$=\frac{1}{2975}\times\frac{600\times68}{72}$$

$$=\frac{30\times28}{90\times49}=\frac{\text{drivers}}{\text{driven}}$$

Example:—

Gears for cutting spiral wheel.

Spiral wheel. Left hand.

44 teeth, 19.84 pitch dia. 1.001 normal pitch 62.30 lead.

Assumed feed $\frac{1}{30}''$, using right-hand hob.

Formula. $$\frac{15}{N}\left(1-\frac{F}{L}\right)=\frac{15}{44}\left(1-\frac{\frac{1}{30}}{62.3}\right)$$

$$=\frac{15}{44}\left(1-\frac{1}{30}\times\frac{1}{62.3}\right)$$

$$=\frac{15}{44}\left(1-\frac{1}{1869}\right)$$

$$=\frac{15}{44}\left(\frac{1868}{1869}\right)$$

From a table of prime factors it is found that $\frac{1749}{1750}$ are two consecutive numbers, which will factor out easily.

The fraction now reads $\frac{15}{44}=\frac{1749}{1750}=\frac{15}{44}\times\frac{11\times3\times53}{5\times5\times5\times2\times7}$

$$=\frac{9\times53}{14\times100}=\frac{45\times53}{70\times100}=\frac{\text{drivers}}{\text{driven}}\quad\text{Index gears.}$$

$$\text{Feed gears}\quad\frac{600\times F\times A\ P}{L}$$

$$1-\frac{F}{L}=\frac{1749}{1750}=1-\frac{1}{1750}\ ;\ \text{then, }\frac{F}{L}=\frac{1}{1750}$$

$$\frac{600}{1}\times\frac{1}{1750}\times\frac{62.3}{44}=\frac{\overset{3}{\cancel{60}}\times\overset{89}{\cancel{623}}}{\underset{25\cancel{0}}{\cancel{1750}}\times\underset{22}{\cancel{44}}}=\frac{3\times89}{5\times110}=\frac{60\times89}{50\times22}$$

$$\frac{F}{L}=\frac{1}{1750}\qquad F=\frac{62.3}{1750}=\frac{1}{28}{}''\ \text{approx.}$$

LIST OF CHANGE GEAR WHEELS SUPPLIED WITH GEAR HOBBING MACHINE.

20	21	22	23	24	24	25	26	27	28	29	30
30	31	32	33	34	35	36	37	38	40	41	43
44	45	47	48	49	50	53	55	58	59	60	61
62	65	67	70	71	73	74	75	79	80	82	83
85	86	89	90	94	95	97	100	105	110	115	120

Settings and Directions for Rotation.—Figs. 284 to 287 show settings for angles, minus or plus, and the directions of rotation of the table to cut the spirals with left and right hand hobs. Thus in Fig. 284 a right-hand spiral is being cut with a right-hand hob, and the swivel head is set to the angle of spiral minus the angle of the hob. If the angle of the hob is 3°, and the angle of spiral 45°, the swivel head is set to 42°, and the table rotates as shown by the arrow, while the hob is being fed downwards. In Fig. 285 a right-hand spiral is being cut with a left-hand hob, and the swivel head is set to the angle of spiral plus the angle of the hob. Thus with the same hob and angle of spiral as before, the swivel head is set to 48°, but the table is rotated in the opposite direction. Similarly, Fig. 286 shows a left-hand spiral being cut with a left-hand hob with the minus setting of the swivel head, and Fig. 287 that of a left-hand spiral being cut with a right-

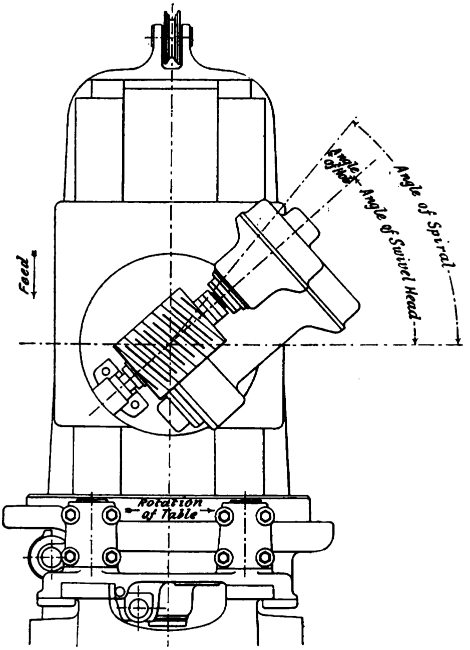


Fig. 284.

hand hob, with a plus setting of the swivel head, and the table is rotated in opposite directions in each case.

The photograph, Fig. 288, illustrates the method of producing the threads of a multiple worm by hobbing on this machine—a rapid mode of production where quantities are required. No very great degree of accuracy is wanted in this instance, because the worms are subsequently ground true after the hardening which follows this hobbing. The sides of the hob teeth are laid out with a suitable curve to produce straight flanks to the worm, instead of the involute form which an ordinary hob would generate. This laying out is done by Messrs Wallwork on the drawing board, and the hob is made accordingly.

The Gould & Eberhardt Machine.—Fig. 289 shows the Gould & Eberhardt hobbing machine for spur, helical, and worm gears,

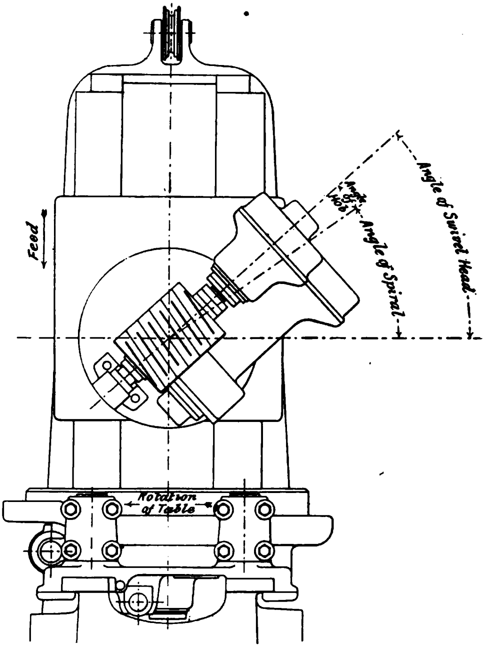


Fig. 285.

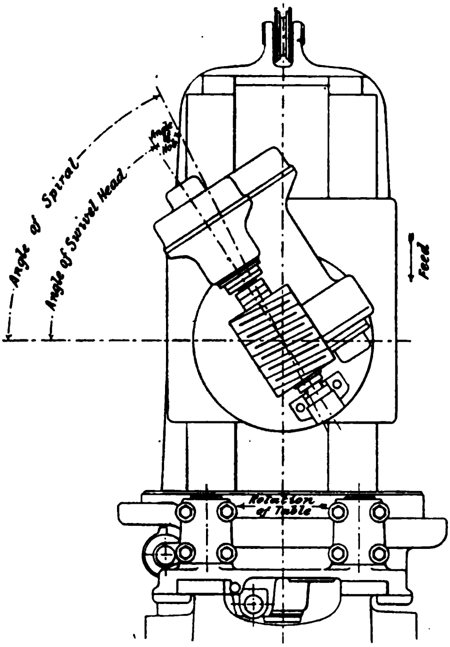


Fig. 286.

the 36in. by 14in. size being represented, working up to 2½ pitch in cast iron and 3 pitch in steel. Its principal features are as follows:—

The Work-table.—This is made in one with the index worm wheel, and is carried by one conical and two flat bearings. The travel of the table is arrested when the desired depth has been reached, while a scale and vernier are provided for setting the correct distance, as when hobbing worm wheels. The top of the work arbor is steadied by arms clamped to two pillars held in an adjustable bracket, which permits of gears of the full swing being accommodated. In the smaller-sized machines the arms are clamped to pillars held in the saddle of the work-table, instead of in a separate slide.

The Dividing Gear.—The indexing worm wheel is of the split type, hobbed in place, and the worm is of hardened and ground steel, with provision for taking up backlash when desired.

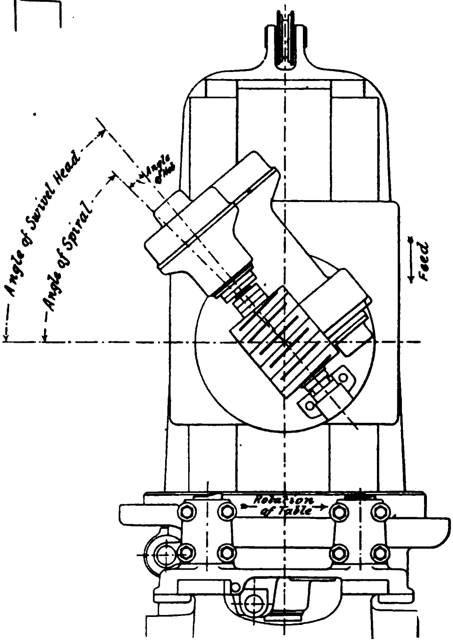


Fig. 287.

There is a micrometer-adjusting device upon the worm by means of which the work can be adjusted circumferentially without disconnecting any part of the mechanism, a useful feature for resetting a spur gear which has been already cut, or when cutting helical gears. A resetting device is incorporated for use when it is required to take two cuts in helical gears, as for obtaining correct thickness of tooth, comprising a zero setting to which the machine may be returned to take a recut through the same helices as at first.

Cutter Spindle.—The cutter spindle—of chrome-nickel steel—is arranged in a supplementary slide which swivels on the vertical moving slide upon the face of the standard, and the swivel slide always swings above the horizontal when cutting right or left hand gears, the driving gears being so arranged that the spindle revolves in the proper direction at either setting. A gauge, always in position on the slide, is used for setting the hob centrally, by the fitting of the tooth of the gauge into a space in the hob teeth.

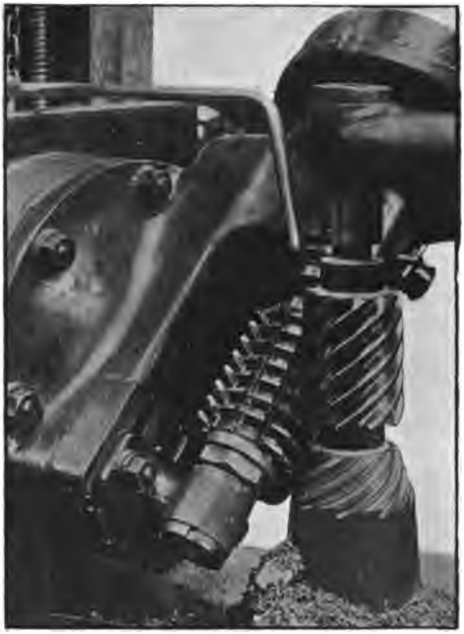


Fig. 288.—Hobbing a Multiple Thread Worm on a Wallwork Machine.

Feed.—The feed, which is downward, is on the draw principle, and the feed gears are located at the rear; the advance of the feed is in accordance with the rotation of the work, regardless of the diameter being cut, or the number of teeth. The indexing gears do not need any differential gear connection when cutting helical gears, owing to a special arrangement patented by the firm. The automatic stop or throw-out which is set up when hobbing spur or helical gears does not merely stop the feed, but the whole mechanism with the exception of the driving pulley.

The Schuchardt & Schutte Machine.—The two views, Figs. 290, 291, represent the hobbing of a spiral pinion and wheel for turbine reducing gears of fine pitch with a large number of teeth, on one of the Schuchardt & Schutte machines. A special fixture is used on the table for holding the stem of the pinion shaft, the clamping being performed with bolts.

Leading Features.—The principal features of these hobbing machines are the following:—The principle of action, identical in all sizes, comprises the primary drive through the cone-pulley, belted from the countershaft, and back-geared, thence passing the drive to an internal bevel train. The bevel gear on the end of the cone-pulley shaft drives a wheel which is attached to the lower end of a vertical shaft that drives the cutter spindle through intermediate mitre wheels on the centre of the swivelling hob headstock. Another bevel wheel gears into the bevel gear on the cone shaft, and this wheel is keyed to the end of a split shaft, carrying a set of differential bevel gears and a worm wheel, passing through the

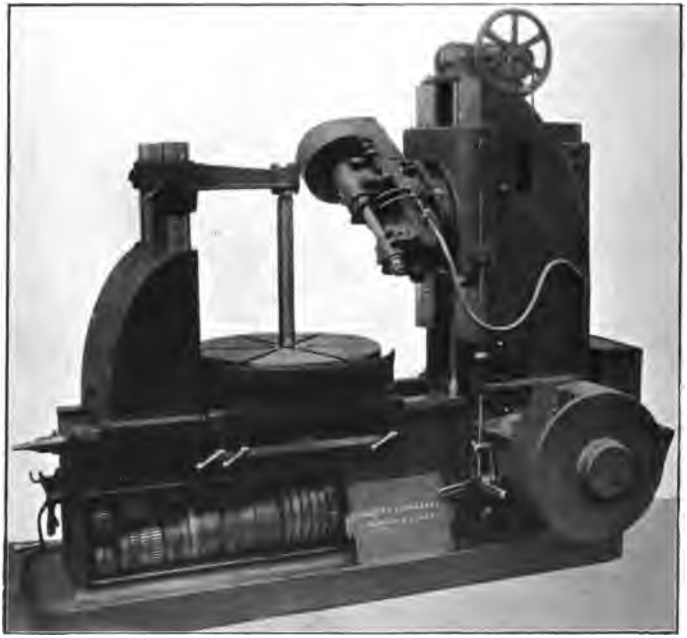


Fig. 289.—Gould & Eberhardt Hobber.

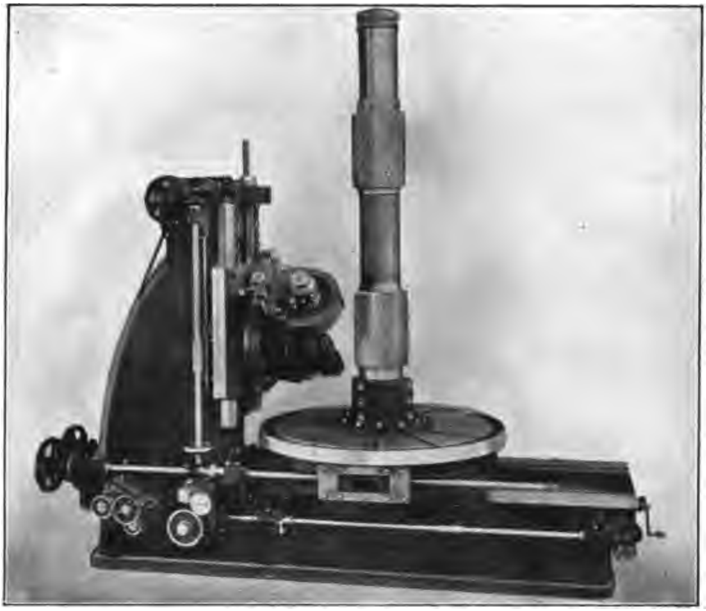


Fig. 290.—Hobbing a Spiral Pinion on a Schuchardt & Schutte Machine.

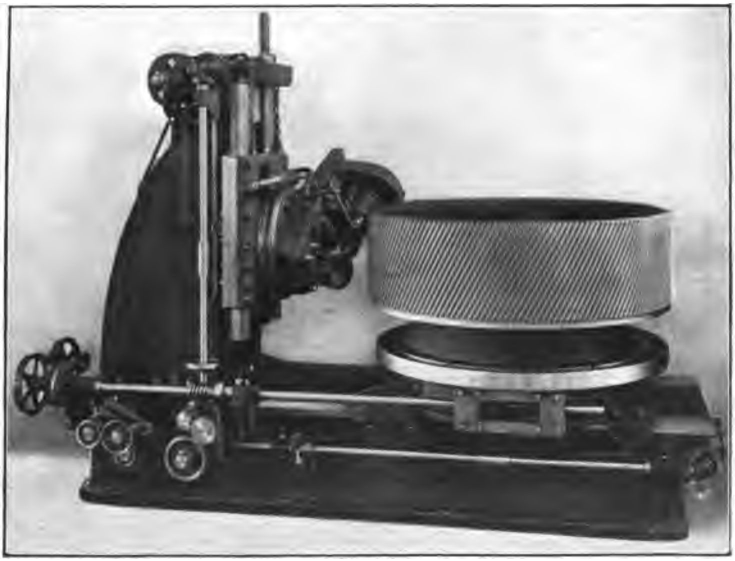


Fig. 291.—Hobbing a Spiral Gear on a Schuchardt & Schutte Machine.

T

back of the body, and carrying the first driving wheel of the primary driving train.

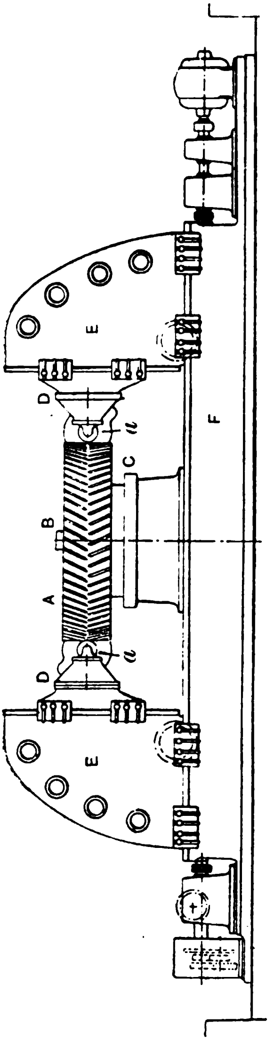


Fig. 292.

Feed.—The vertical feed to the cutter saddle is derived from a feed bracket through a chain to a worm and worm wheel, which operates a nut working on the vertical screw of the hob saddle. The feed changes are obtained by setting up different ratios of change gears. The horizontal feed to the table—used when hobbing worm wheels—is obtained from the same bracket through chain, worm and worm wheel, with a revolving horizontal screw and fixed nut. Each of these feed motions is stopped or started as required by means of a drop-out worm.

The differential motion is only required when hobbing spiral gears, and has the effect of accelerating or of retarding the revolution of the blank at a rate which in conjunction with the downward feed of the hob will produce the desired angle. The acceleration or retardation is produced by the differential motion which is placed on the driving shaft.

When worm wheels are hobbed the hob saddle does not feed downwards, but the table automatically feeds inwards and is tripped when the correct depth of teeth is reached. If spur gears of unusual pitch have to be cut an ordinary form cutter can be employed, and the table is indexed for each space like that of an ordinary gear-cutting machine, the indexing arrangement being an extra fitting.

Fig. 292 illustrates the elements of the Wüst machine used in cutting double-helical gears with hobs. The right and left hand teeth are cut simultaneously. In this, the blank A is mounted on the mandrel B driven by the face plate C from the work spindle below, through worm gear. The hobs *a, a* are carried in slides D, D, which are moved vertically on the standards E, E. The standards can be adjusted horizontally along the bed F of the machine, and they have micrometer screws for adjusting the depth of cut. The slides D, D are fed up and down. The dividing wheel is driven through change gears. Another set of change gears imparts the spiral lead to the teeth. Fig. 293 illustrates the machine in the works of the Power Plant Co., Ltd.

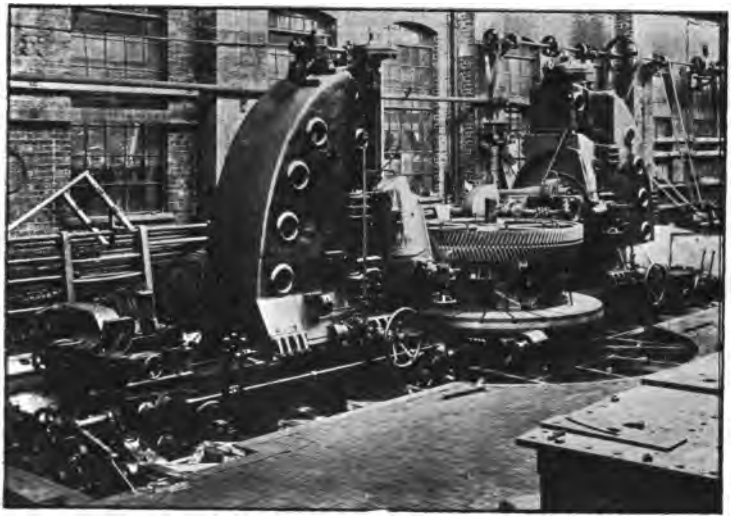


Fig. 293.—Hobbing a Double-helical Spur Gear in the Shops of the Power Plant Co., Ltd.

Worm Gear Generation.—The cutting of worm wheels with single disc rotary cutters, as in Fig. 294, is, except for light dividing wheels, passing out of practice. So, too, is that of finishing the teeth thus cut with hobs which drive the wheel as they correct the tooth shapes. The generating machines are preferable in which the hob (or the single fly cutter) and the blank are moved positively in relation to each other, precisely as the actual worm and finished wheel are when in operation. Each movement is produced by separate sets of change gears. The worms may be

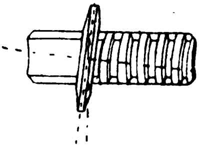


Fig. 294.

cut either in lathes or in one of the worm-milling machines now numerous (*see* p. 147).

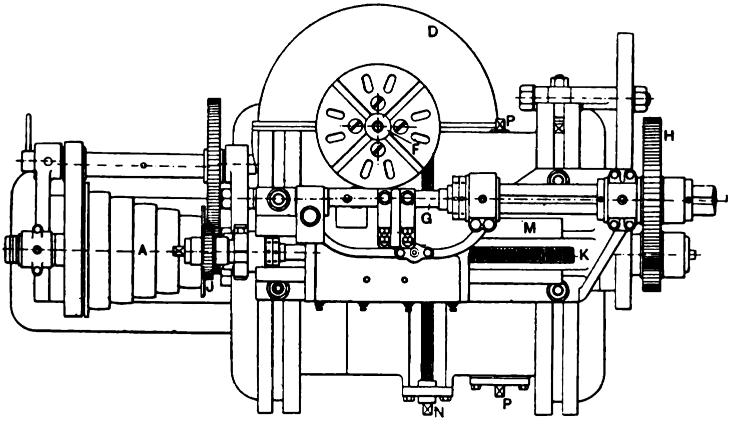


Fig. 295.

The Wallwork Machine.—Figs. 295 to 297 illustrate a worm-wheel generating machine by Messrs Henry Wallwork & Co., Limited, of Manchester. In this design the wheel blank is carried on top of a vertical arbor, and the worm hob on a horizontal shaft, the axis of which occupies the same position in relation to the blank as the axis of the worm will bear to the wheel. Fig. 295 gives

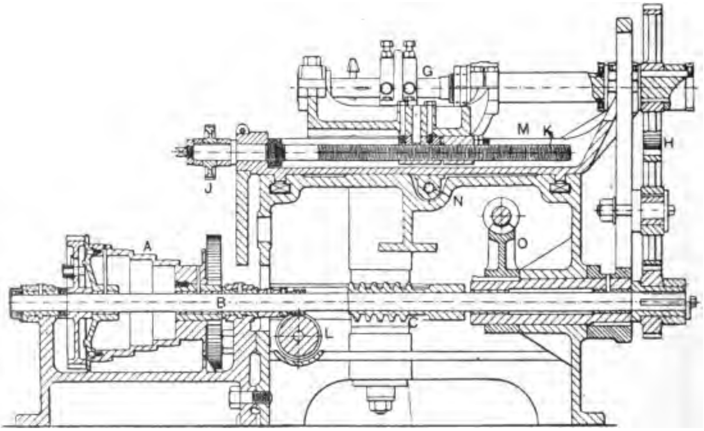


Fig. 296.

a plan view of the machine, Fig. 296 a longitudinal section taken in a vertical plane, and Fig. 297 a transverse vertical section through the work arbor and its dividing worm wheel.

Method of Action.—The movements are all derived from the stepped cones A, which are provided with back gear and drive to the shaft B, on which the worm C, which is adjustable for wear drives the dividing wheel D, keyed on the work spindle E, made of high-carbon steel, within which the work arbor F is drawn and tightened by the nut and tapered neck. There are five steps on the cone, and two counter-shaft speeds of 240 and 500 revs. per min., so giving ten rates of revolution.

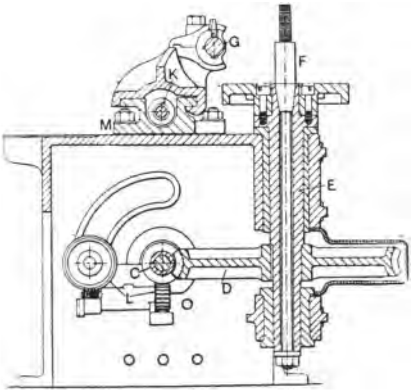


Fig. 297.

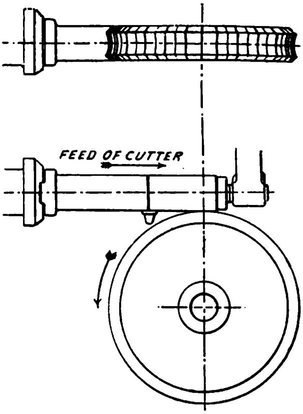


Fig. 298.

The cutter arbor G is driven by two sets of change gears, one being the change gears proper, at H, for effecting the rotation of the arbor with its hob; the other the change-feed gears J, for imparting the proper rate of endlong motion to the hob, which is set into correct centres before cutting commences. The latter is effected through the screw K which traverses the cutter slide. The motion is taken through the worm gears L seen in Figs. 296 and 297, driving through the gears on the quadrant plate to the last wheel J on the end of the screw K. The quadrant slot seen in that end of the bed permits of the swivel motion of the quadrant. This in brief states the essentials of the machine.

Details.—With regard to these, the shapes and proportions of the main castings are apparent from the drawings. The cutter slide has a long bearing on its saddle M, and the feed screw K is located centrally to avoid twisting action. The nut on the feed screw has adjustment for wear. The cutter spindle G is of high-carbon steel, and runs in bearings of phosphor bronze. The nose is bored to a Brown & Sharpe taper, and the arbor is held in place with a draw-bolt. The saddle M is moved towards or from the work spindle for adjustment of centres by the screw N, and it is steadied by tongues in the tee-groves which carry the clamping bolts. Measurement for centres is provided for by a steel scale which is let into the bed. The driving worm C is hardened and ground, and runs in oil. A quadrant worm wheel O, the worm of which is actuated from the ends P, P of the shaft, is used when making adjustments of gears on the quadrant plate of the gears H.

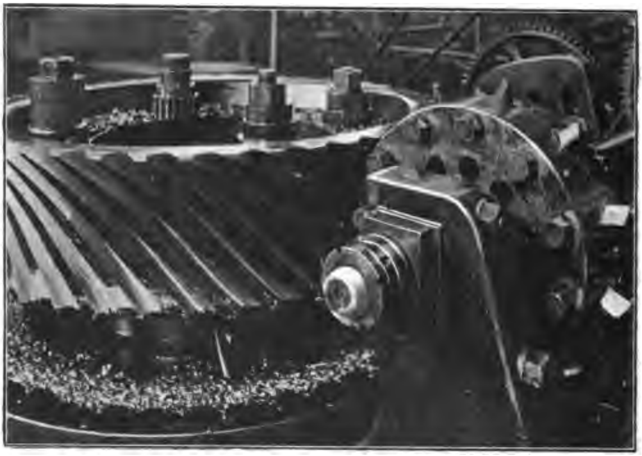


Fig. 299.—Generating a Hollow-faced Spiral Gear with Fly Cutters on a Wallwork Machine.

Fig. 298 illustrates diagrammatically the action of a single tooth cutter which feeds across the face of the blank. The movements being controlled by the gearing, the results are as perfect as those produced by hobbing, while the cost of the cutter is very much less than that of a hob, but the operation is slower.

An extension of the fly-cutter principle is seen in Fig. 299, where a hollow-faced spiral gear is being generated on the Wallwork worm-wheel generating machine. A row of fly cutters is bolted up around a head, and the feed is correspondingly increased by comparison with that given in the case of a single tooth.

A Holroyd Machine.—A large worm-wheel hobbing machine, working on the sliding hob principle, is shown in Fig. 300, this being manufactured by Messrs J. Holroyd & Co., Limited, and dealing with wheels up to 72in. diameter. It cuts by means of a sliding taper hob (Fig. 301), or a single fly cutter (Fig. 298), which travels tangentially across the wheel. The latter is held on the arbor of the table, which is bolted to a large spindle carried in a parallel bearing. A large worm wheel is attached to the other end of this spindle, and is driven by a hardened steel worm which is connected to the driving cone by means of chain gears. The drive to the spindle is by a phosphor-bronze worm and cast-iron wheel, from a three-speed cone. The whole cutter-head is moved bodily upon the bed for setting the depth of cut, by means of a handle actuating mitre gears and a screw and nut. Mechanism is incorporated to

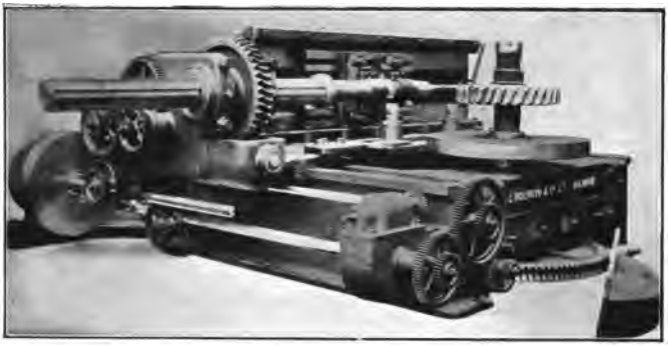


Fig. 300.—Hobbing a Worm Wheel on a Holroyd Machine.

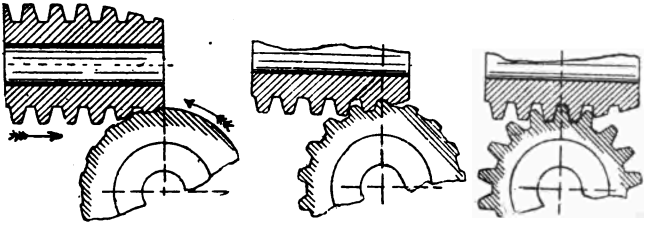


Fig. 301.

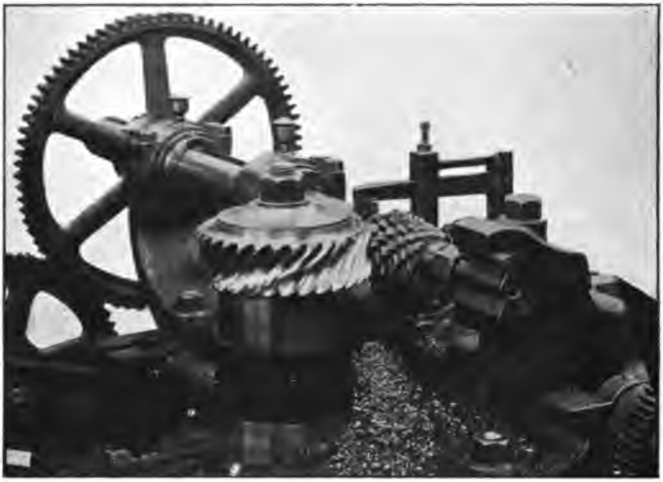


Fig. 302.—Hobbing a Worm Wheel on a Wallwork Machine.

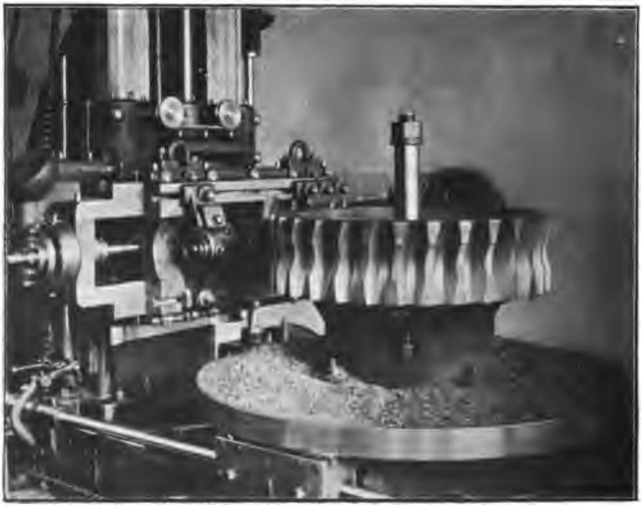


Fig. 303.—Generating a Worm Wheel with a Fly Cutter on a Schuchardt & Schutte Machine.

impart to the wheel the necessary rotative motion to compensate for the sliding of the hob. This is done by employing a differential gear box which is connected to the feed screw of the hob slide through change gears, and so alters the speed of the work to suit the travel of the hob. The feed to the hob slide is obtained from the table-driving shaft, to which is fitted an eccentric disc, coupled by a lever to a silent ratchet feed, which through bevels and change gears operates the feed screw in the hob slide. Smaller machines working in the same manner are built, but with the difference that the work is carried on a sliding table by which adjustment to or from the hob, for diameter and depth of cut, is made in place of adjusting the cutter-head.

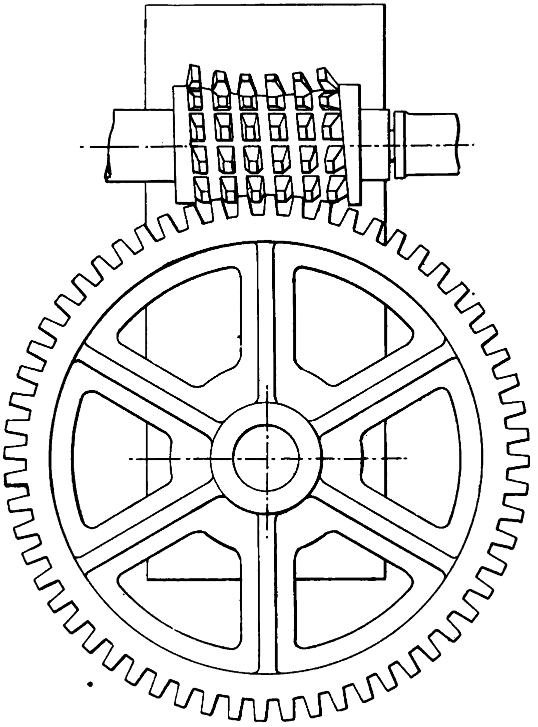


Fig. 304.

The setting up for generating a worm wheel with a sliding tapered hob on the Wallwork machine is shown in Fig. 302. It will be observed that the cutter arbor is steadied with a small bearing block maintained in contact with its top side by the short horizontal pressure arm and screw pillar, this arrangement acting in much the same manner as a lathe work steady.

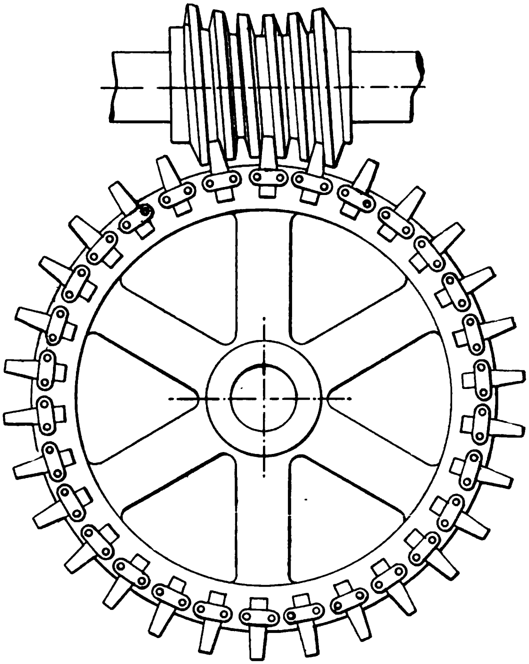


Fig. 305.

A worm wheel being hobbed with a fly cutter is shown in Fig. 303, this being done on a Schuchardt & Schutte hobbing machine. This happens to be a case where the work would not pay for a proper hob.

The two illustrations, Figs. 304 and 305, represent the mode of generating the Hindley wheel and worm on one of the Newton Machine Tool Works worm-wheel hobbing machines. The wheel is hobbed with a worm hob as seen in the first figure, and the worm is generated with a special type of cutter-head.

CHAPTER XVI

GENERATING BY MILLING

Milling Spiral Gears.—The Brown & Sharpe Manufacturing Company, of Providence, R.I., cut small single-helical gears on an automatic screw machine by a moulding generating process, using a special attachment for the purpose, in place of cutting such gears in a separate machine. The cutter in this case is a helical gear of tool steel, hardened and ground on the front or advancing face. It is rotated in unison with the blank which it cuts. The blank and the cutter revolve just as though they were a pair of gears in mutual engagement, which movements are produced by a very ingenious geared mechanism. The following is a very brief description of the mechanism, which will be clear from the group of illustrations (Fig. 306). The general rig up is shown by the upper figure in the group, and details in subsequent ones. It is mounted in the tool carrier of the screw machine, and the blank which is itself rotated by the work spindle imparts the motion to the cutter, thus: A is the rotating work spindle, and B the bar which is held and fed through, and from which the finished blanks are cut. C is the body of the attachment which carries the operating mechanism, and terminates in a shank D, by which it is held in the tool carrier of the machine. The cutter and its gears are carried in a frame E within the body C. Within this frame is a spindle *a*, and long screw *b*. The latter engages with a worm gear *c*, which is secured to a short shaft *d*. At the other end of this shaft a gear F meshes with an intermediate gear G, which engages with a gear H on a shaft *e*, on the other end of which the cutter J is fixed.

These gears and the cutter derive their motion from the hole in the end of the stock bar B, previously drilled and tapered. Into this hole the polygonal end of the spindle is pushed and is thus revolved. When cutting steel gears additional driving power is obtained by a flat milled (from the cross slide) on one portion of the blank. The tool carrier advances after the spindle *a* takes on the work of driving, with which object the spindle *a* can be moved lengthwise in its frame E. To permit of this the spindle is mounted on a bearing K secured in the hollow shank D. Within the latter a hollow sleeve *f* receives a spring-pressed plunger *g*, bearing against the rear end of the spindle *a* with a hardened ball pivot.

Clearly now, if the tool carrier is fed forward, the rotation of the

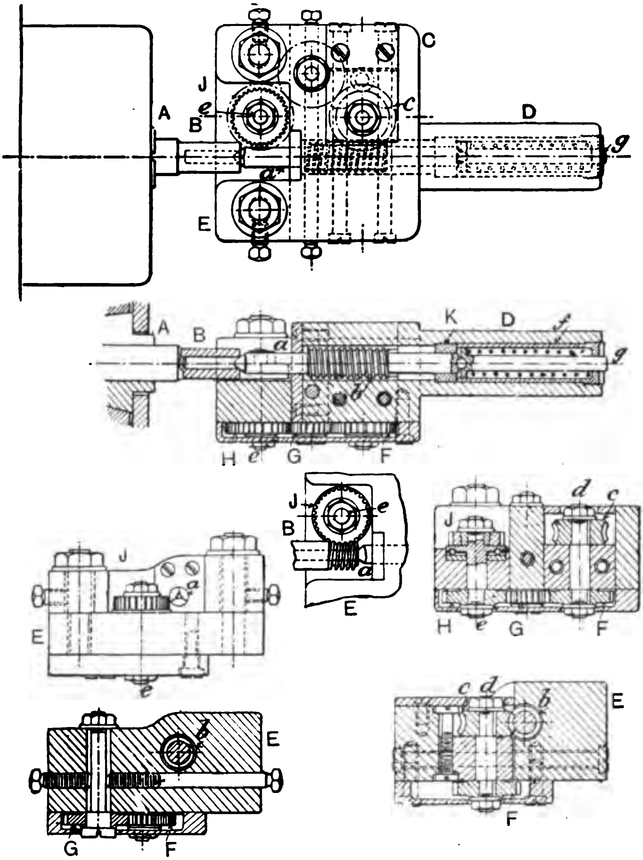


Fig. 306.

spindle *a* effected by its pressure into the tapered hole in the stock bar, will rotate the screw thread *b*, and with it the worm gear *c*, and thence the train of gears to the cutter J. The movement of the cutter slide rolls the cutter in correct relation to the blank precisely as in a pair of finished gears. The precision of these movements is not affected by movements of the cutter slide, or by its stoppage, because the gear ratios remain unaffected. Hence reciprocations of the cutter slide go on as the cutter is forced into the work, until, when

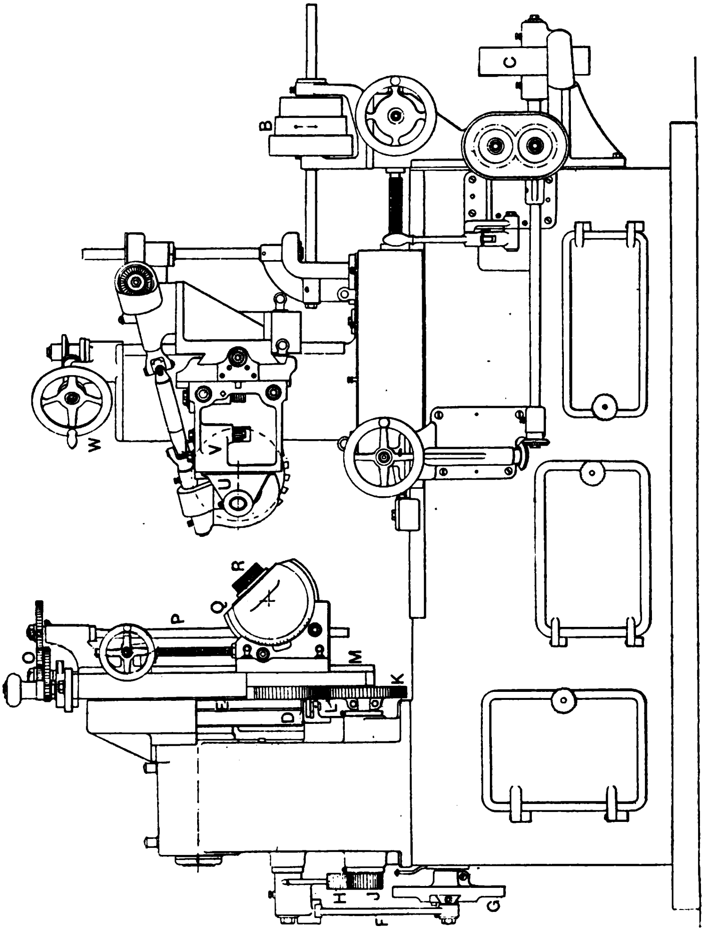


Fig. 307.

the cutter has completed its traverse, the cutting of the gear will be completed.

Milling Bevel Gears: The Beale Machine.—The illustrations, Figs. 307 to 310, are those of a machine by the late Mr O. J. Beale, built by the Brown & Sharpe Manufacturing Company. It will cut teeth of different pressure angles by simple angular adjustments of the cutters. One pair of cutters will cover a moderate range of pitches, but not all, because as the pair have to occupy the tooth spaces, a pair which would fill a small tooth space would be too far apart for a large space. Fig. 307 is a front elevation of the machine, Fig. 308 a partial rear elevation, Fig. 309 a plan view of the cutter-carrying arrangements, Fig. 310 a vertical section through the indexing and generating head, and Fig. 311 a detail of a cutter. The following are the principal elements:—

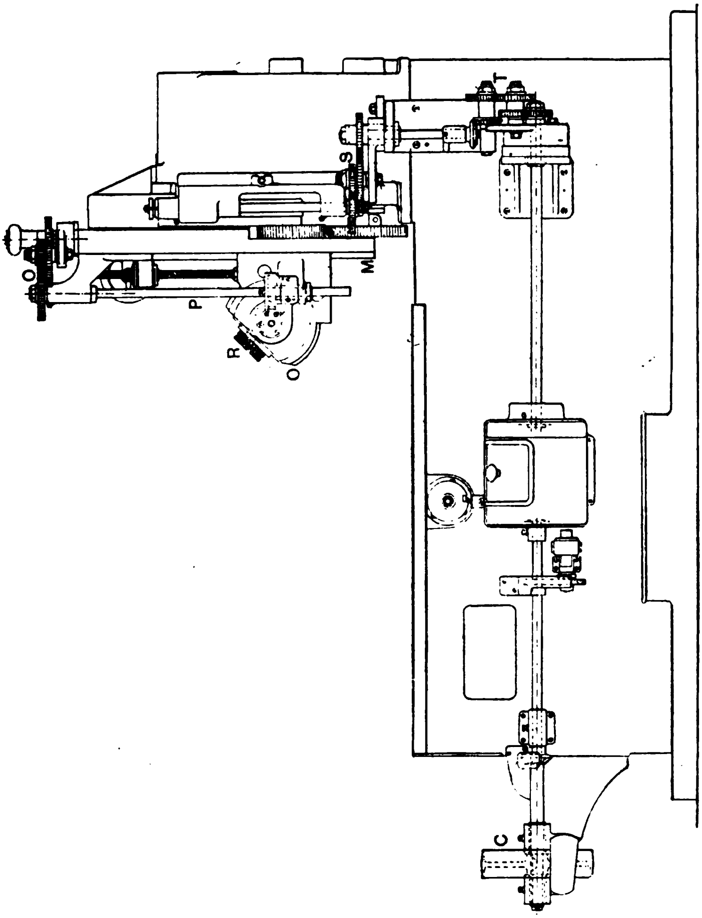


Fig. 308.

The motions are derived from two pulleys. The cone pulley B drives the cutters, the plain pulley C the rolling and the indexing movements, and the advance and withdrawal of the cutters. In Figs. 307 and 310 at D are shown the dividing or index worm wheel. Cast with it is a flat bevel gear E, which through gears rolls the blank past the cutters. The indexing and rolling are therefore

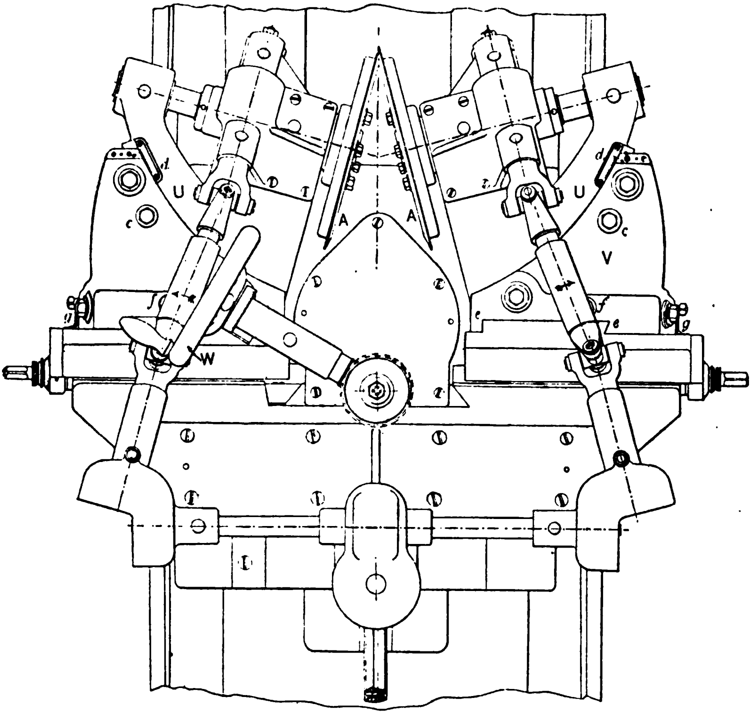


Fig. 309.

inseparable, which, as we shall see later, entails the insertion of a compensating train of gears.

The Blank and its Movements.—The rolling motion of the blank derived from the pulley C is transmitted through a connecting rod F in a slotted crank arm G to a toothed sector H. The sector drives the pinion J, and thence K at the opposite end (compare with Fig. 310), K engaging with a segmental gear L oscillates the face plate M. A bevel pinion N above engages with the bevel wheel E, solid with the index worm gear. As the gears D and E are stationary during the rolling motion of the face plate, the bevel gear N rolls in gears E, and thus through the set of gears O, shaft P, and gears within the block Q, which carries the spindle R of the work arbor; the arbor and blank are rolled. The angle of the block Q is adjustable by means of its curved seating, and the graduations indicated in Fig. 307.

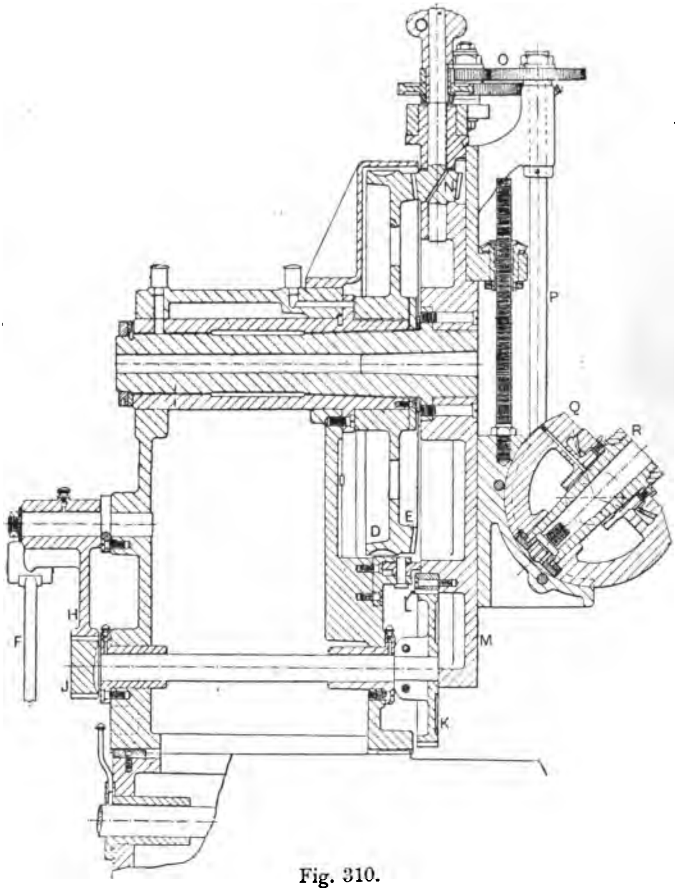


Fig. 310.

The gears O are change gears, altered to suit the sizes of blanks being cut. But as the indexing gear and the gear D is an integral portion of the bevel gear E, a change in the gears O would affect the indexing. Hence, a compensating train S (Fig. 308) is inserted between the indexing train T and the change gears O. The gears S are the same as the gears O, but are arranged in reverse order. Those at O are calculated first, and then a similar set reversed are inserted at

S. So that whatever rate of movement is imparted by the gears T is transmitted to the gear blank through the shaft P. The gears S are stationary, except when actual indexing is taking place, and therefore they do not affect the rolling action of the blank imparted by the gears O.

Adjustment of Cutters.—The methods of adjustment of the cutters are outlined in the plan view, Fig. 309, with which comparison may be made with Fig. 307. The supports for the cutter arbors are mounted on guides U, U, having curved edges, along which they are adjusted by a wrench at *c, c*, and set by the graduations at *d, d.* This adjustment is only changed when the angle of obliquity and pressure of the teeth are changed, as, say, from 14½° to 20°, or 22°. The adjustments for the cone angle, which vary with every different size of gear, are effected by virtue of the arc shape given to the guides V at *e, e*, adjustments being made by a wrench applied at *f, f*. The angles are read by graduations at *g, g*. Provision for vertical adjustments of the cutter-head is made at the hand wheel W. This is necessary in order to bring the intersection of the planes of the cutters to meet in the horizontal centre line of the index wheel D, which is the apex of all pitch cones. The precise measurement is taken with a caliper gauge and test arbor, the lower caliper jaw resting on the major diameter of the blank at the pitch line, and the upper jaw on the horizontal centre of the index wheel and apex of the pitch cone of the gear to be cut.

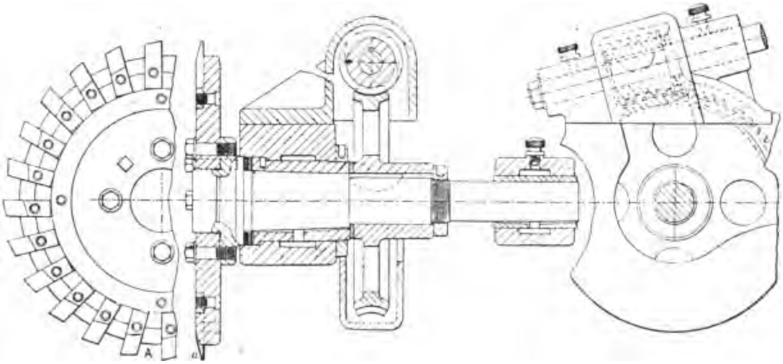


Fig. 311.

The Chambon Machine.—Reference was made on p. 105 to the Chambon hob for bevel gears, and the photograph, Fig. 312, illustrates the machine in which it is employed, made by Messrs J. E. Reinecker, Akt. Ges. It is especially adapted for automatically roughing-out the gears by the self-generating process. In comparison with the ordinary milling method the hobbing process offers—in addition to increased rate of cutting—the advantage that the thickness of tooth is reduced towards the apex of the gear in an almost theoretically correct manner, thus leaving a finishing cut of uniform thickness for the bevel-gear planing machine to take off the tooth flank.

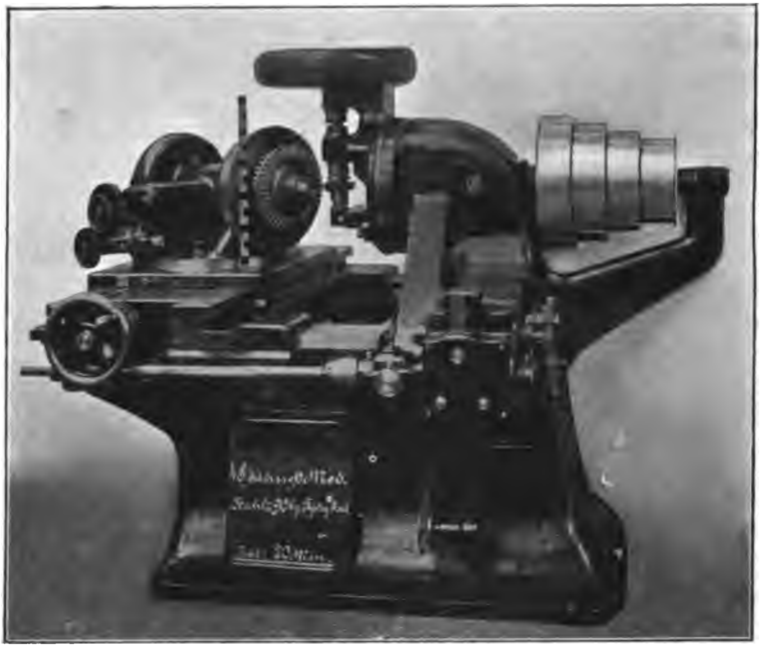


Fig. 312.—The Chambon Bevel-gear Generator.

The number of necessary hobs is very small, as these are independent of the number of teeth in a gear; thus, a hob carrying for instance the mark Large Modul 6, Small Modul 4, can cut all gears with outside and inside pitch lying between Modul 6 and Modul 4. In general these hobs are arranged for gears with a length of tooth not exceeding one-third of the height of the pitch cone. For greater lengths this method is not suitable.

The drive for the machine is through a cone pulley with positive gear box permitting changes of speed during work. The self-generating movement is also positive, and is adjusted through spacing wheels according to the number of teeth. A gauge is supplied to enable the hob to be quickly set so that the teeth point exactly to the apex of the pitch cone. The end thrust of cutting is received against a face plate. If necessary, a set of gears can be employed to enable the machine to cut bevel or spur gear teeth in the ordinary fashion with formed cutters.

SECTION IV.—MATERIALS, MANUFACTURE, AND STRENGTH

CHAPTER XVII

THE MANUFACTURE OF GEARS

VASTLY more care is exercised in the manufacture of gear blanks than formerly. The demands of the machine-tool and automobile builders have met with a response in these directions:—Increased accuracy in the teeth and the suppression of noise, the employment of harder, tougher, more durable materials, and in the practice of case-hardening and of grinding. The large numbers required have given rise not only to a great increase in the number of machine tools, but also to the adoption of many labour-saving economies in the preparation of the blanks. The following is a summary of the leading features of the existing practice.

Materials.—These include cast iron, gun metal, phosphor bronze and other copper alloys, cast steel, forged steel, alloy steels, as nickel, chrome-vanadium, and nickel-chrome steels; rawhide and paper also are used for certain kinds of gears. Each metal or alloy differs from the others in toughness, wearing qualities, method of treatment and cost. For all ordinary purposes good cast iron, gun metal and carbon steel are suitable enough. But for the smallest gears which are worked hardest at the highest speeds one of the alloy steels must be used if durability is required. The majority of the small gears are made now of steel, either of carbon, or of the alloy kinds. The slow-running gears are still of cast iron, which calls for no special observation except that it should be of good, tough, highly-mottled quality to be strong enough, and to endure wear. Constant vigilance is necessary in dealing with the steel gears, from the selection of the material to the final hardening, if good results are to be secured. Nickel steel, which is chiefly used, contains from 3.5 to 5 per cent. nickel, from 0.15 to 0.20 per cent. of carbon, and small percentages of manganese and silicon, and traces only of phosphorus and sulphur.

Manufacture.—The gear blanks are either used as cut from the bar, or drop-forged in dies, the latter lessening the waste of material and cost of machining, against which the expense of the dies has to be set. Two pairs of dies are usually necessary, the first for roughing, the second for finishing, followed by trimming dies to remove the fin. Before any machining is done the blank is annealed. This is done in a muffle or box, so that flame does not come into contact with the metal. The blanks are packed in powdered charcoal or in charred leather (not in bone) and heated to a temperature of about 1600° Fahr. and allowed to remain at that for two or three hours, the period depending on the dimensions. The box is then removed and allowed to cool before its enclosed contents are thrown out. The steel is thus softened, and some of the initial internal strains are removed. The rough machining follows. This removes such surface strains as remain, and afterwards another annealing is done before the finish-tooling, which is of a very light character.

Tooling.—The method of tooling adopted is very much that of the general practice of the shops. Very light blanks sometimes have their rims milled, but generally the turret lathe, using a chuck with jaws specially made to suit the blank and a special set of tools fixed upon the turret, is employed. If the bores have to fit over castellated shafts, the hole is broached out on a Lapointe machine or similar type, or on a key-grooving machine. The teeth are cut on any of the machines described in the previous pages. The cheaper or the more expensive methods are selected according to the standard of accuracy which is sought. But when gears are generated it is very usually the practice to gash out the teeth with rotary cutters or with a planing cut previous to finishing them by generating. It is becoming increasingly common to harden gears after cutting. As all the internal strains have been practically removed by previous annealing the risk of distortion is but slight. The gears are first washed in hot soda water to free them from oil and chips, and dried. They are packed in boxes with carbonizing material and placed in a furnace raised to a temperature of about 1700° Fahr., at which they remain from ten to twelve hours. The boxes are then taken out and allowed to cool before the gears are removed.

So exacting are the demands now made on strong, durable and quiet gears for automobiles that the whole practice relating to small fast-running gears is being modified. Referring here to the materials used, the various alloy steels are employed in preference to the carbon steels, not only for automobile gears but often also for those put into the feed and speed gear boxes of machine tools. There are hundreds of varieties of alloy steels made. Of nickel steels alone, Mr Mathews says there are at least fifty-seven varieties. Such being the case, it appears that the user has to rely on the metallurgist to supply just what is wanted. And as in the cast irons and the carbon steels, so the behaviour of a small test piece may be misleading, if applied too slavishly to the behaviour of a larger mass, whether casting or forging, subjected to heat treatment, annealing, hardening, or case-hardening.

The alloy steels embrace the nickel steels, the nickel-vanadium, the chrome-vanadium, and the chrome-nickel steels. A vast number of experiments have been put on record of the tensile strength, elastic strength, and elongation of each of these in variations of percentage composition. There is little advantage in quoting these because the purchaser generally goes to the metallurgist and buys a steel of some recognized brand. A few typical results are, however, tabulated as a general guide (p. 311).

These results, which were given among others by Mr John A. Mathews, in a paper read before the Franklin Institute, indicate, if they show nothing else, that the generic terms nickel steel, nickel-vanadium steel, etc., tell nothing whatever of the physical qualities. Even chemical analysis is misleading, since results depend as much on treatment as on that. This explains the reason why steel-makers attach such vital importance to the exact methods of treatment to be given to different brands. The case is paralleled with that of the high-speed tool steels.

Carbon plays an important part in the alloy steels because that element affects the hardening. The higher carbon is used for oil-hardened gears, the lower for case-hardened gears. The first may contain from about 0.45 to 0.60 per cent. of carbon, the second from 0.20 to 0.25. The choice between oil-hardened and case-hardened gears has been a matter of controversy. In quenching in oil from a bright red a thin surface hardening is imparted, leaving the interior unaffected. In case-hardened gears the hardening is deeper. Experiments have shown that the latter are more durable than the other. The objection to case-hardening is the uncertainty of the depth and the risks of distortion. Dr Guillet says, with reference to the proper procedure in case-hardening:—

NICKEL STEELS.

Carbon.	Manganese.	Nickel.	Chromium.	Vanadium.	Elastic Limit.	Tensile Strength.	Elongation in 2 ins.	Reduction in Area.	Treatment.
·18-·28	·60-·90	3·5	...	...	68	103	12·9	54	1550 F. oil 600 F.
·18-·28	·60-·90	3·5	...	...	84	116	12·4	48	1600 F. brine.
·14	·63	3·64	...	...	30	41	33·5	72·4	1400 F. oil.
·35	·45	3·39	...	...	130	137	10·0	36·3	1500 F. water 430 F.
·25	·86	3·45	...	...	31	45	31	60	Natural as rolled.
				NICKEL-VANADIUM STEELS.					
·33	·16	3·40	...	·24	49	66	17·8	40	Natural as rolled.
·33	·16	3·72	...	·12	70	76	14·5	56	1500 F. oil 600 F.
·24	·72	3·33	...	·12	92	117	14·5	52	1600 F. water.
·24	·72	3·33	...	·12	91	116	15·2	57	1600 F. brine 400 F.
				CHROME-VANADIUM STEELS.					
·26	·39	...	·78	·17	44	68	20	64	Natural as rolled.
·26	·39	...	·78	·17	103	139	3	8·2	1570 F. oil 400 F.
·45	·58	...	2·37	·30	138	146	6·0	16	1600 F. oil 430 F.
·36	·21	...	2·78	·24	98	104	13	45	1500 F. oil 600 F.
				CHROME-NICKEL STEELS.					
·37	...	2·9	1·04	...	60	78	18·5	61	1475 F. oil 1200 F.
·25	...	3·0	1·5	...	110	116	6·0	40	Hardened-oil.
·30	...	2·0	1·0	...	68	98	9·0	37	Tempered.
·40	·18	2·1	·80	...	106	118	10·0	34	1500 F. oil 600 F.

In the column marked "Treatment" is given the temperature to which the steel has been heated, followed by the liquid in which it is quenched, and then by the temperature to which it was reheated to draw the temper or anneal.

" As is well known, the operation of case-hardening consists of placing a steel in some medium capable of imparting carbon to it, at a suitable temperature; after a sufficient interval a steel is obtained, the interior of which possesses the same percentage of carbon as before case-hardening, but the exterior of which is much higher (0.80 to 1.00 per cent). On quenching such a steel an extremely high degree of mineralogical hardness is obtained on the surface, while the centre of the piece is non-brittle, if, that is, the operation has been properly carried out. One generally uses steels containing .10 to .25 per cent. of carbon for case-hardening. In order to avoid all brittleness in the interior and exterior of the piece, and, at the same time, to obtain a high degree of superficial hardness and a very regular degree of carburization (in carbon steels), it is necessary:—

1. " To use a steel containing less than 0.12 per cent. of carbon, and with a low percentage of manganese (less than 0.30 per cent.), and

2. " To case-harden with a chemically definite material such as a mixture of 60 parts charcoal and 40 parts of barium carbonate at a temperature between 850°C. and 1050°C.—the higher the temperature the more rapid the case-hardening—and allow it, after the operation, to cool down just below the transformation point (about 600°C.).

3. " To reheat the piece and quench it at 900°C. (just above the transformation point of the centre), this operation having the effect of rendering the centre fibrous."

This operation has the effect of toughening the centre, but the outside will be coarse-grained and brittle. The next operation, according to Guillet, is:—

4. " To quench for a second time at 800°C. (above the transformation point of the exterior), to render the skin non-brittle.

" In any case, the methods by which it is possible to obtain, with low-carbon steel, case-hardened pieces which shall not be brittle, are exceedingly delicate."

Mr Mathews says that an ideal way to make a nickel-steel gear is to first anneal the blank, then rough-machine approximately to size, then give a light re-annealing before taking the last finishing cut. Then pack in a suitable mixture and carbonize to a depth of $\frac{1}{64}$in. to $\frac{1}{32}$in. according to requirements, at a temperature of about 1625° to 1650°F. Cool in the pots, heat to about 1500°F., and quench in a hot brine or calcium chloride solution. This will put the core into excellent physical condition. Finally, re-heat to

1375° to 1400°F., and quench in oil. This last operation will refine the grain of the case and harden it with best results. The temper need not be drawn. It will be noticed that the temperatures for treating nickel steels are considerably lower than those advocated by Guillet for carbon steels.

The S.A.E. Standard Specifications for Steel.—The following is a selection of standard alloy steels suitable for transmission gears, all of which must be used when they have been subjected to heat treatment and not in a natural, or annealed condition.

Nickel Steels.—.15 carbon, 3½ per cent. nickel steel.

The following composition is desired:—

Carbon	·10 per cent. to	·20 per cent.	(·15 per cent. desired).
Manganese	·50 ,, to	·80 ,,	(·65 ,, ,,).
Phosphorus, not to exceed		·04 ,,	
Sulphur, not to exceed		·04 ,,	
Nickel	3·25 per cent. to	3·75 ,,	(3·50 ,, ,,).

Steel of this kind, if properly carbonized and heat-treated, will produce a gear having a very tough and strong core, together with the desired high carbon exterior. The best heat treatment is as follows:—

After forging or machining:—

(1) Carbonize at a temperature between 1600° and 1750°F. (1650°-1700°F. desired).
(2) Cool slowly in the carbonizing material.
(3) Reheat to 1450°-1525°F.
(4) Quench.
(5) Reheat to 1300°-1400°F.
(6) Quench.
(7) Reheat to a temperature of from 250°-500°F. (in accordance with the necessities of the case) and cool slowly.

The second quench (No. 6) must be conducted at the lowest possible temperature at which the material will harden. It will be found that sometimes this is lower than 1300°F.

A nickel steel with a slightly higher carbon content (.25) is also used.

Nickel-Chromium Steels.—These include low nickel, medium nickel, and high nickel-chromium steels. A sample specification for a low nickel-chromium steel is given:—

Carbon	·45 per cent. to	·55 per cent.	(·50 per cent. desired).
Manganese	·50 ,, to	·80 ,,	(·65 ,, ,,).
Phosphorus, not to exceed		·04 ,,	
Sulphur, not to exceed		·04 ,,	
Nickel	1·00 per cent. to	1·50 ,,	
Chromium	·30 ,, to	·75 ,,	

The heat treatment, after forging or machining, is:—

(1) Heat to 1500°-1550°F.
(2) Quench.
(3) Reheat to 1300°-1400°F.
(4) Quench.
(5) Reheat to 600°-1200°F. and cool slowly.

A sample specification for a high nickel-chromium steel is given:—

Carbon	·40 per cent.	to ·50 per cent.	(·45 per cent. desired).
Manganese	·30 ,,	to ·60 ,,	(·45 ,, ,,).
Phosphorus, not to exceed		·04 per cent.	
Nickel	3·25 per cent.	to 3·75 ,,	(3·50 ,, ,,).
Chromium	1·25 ,,	to 1·75 ,,	(1·50 ,, ,,).

This steel is largely employed for gears in which extreme strength and hardness are necessary. The carbon is sufficiently high to cause the material, in the presence of chromium and nickel, to become hard enough to make a good gear when quenched, without case-hardening.

It is difficult to forge, and during the forging should be kept at a thoroughly plastic heat, and not be hammered or worked after falling to ordinary forging temperatures, otherwise cracking is liable to ensue. The steel also becomes so very hard as to forge with great difficulty. On the other, too high a temperature is inadvisable, because the steel becomes red-short and breaks. The forging temperature limits are thus narrow, and the consequence is that the steel must be reheated more frequently than is necessary with other steels.

The heat treatment for gears is:—

After forging:—

(1) Reheat to 1475°-1525°F. (Hold at this temperature half an hour to ensure thorough heating.)
(2) Cool slowly.
(3) Reheat to 1450°-1500°F.
(4) Quench.
(5) Reheat to 250°-550°F. and cool slowly.

This steel should be thoroughly annealed before machining (Operations 1 and 2).

(The medium nickel-chromium of course occupies an intermediate position.)

The physical characteristics of the high nickel-chromium steel just given are:—

	Annealed.	Heat Treated.
Elastic limit, lb. per square inch	50,000-60,000	150,000-250,000
Reduction of area	45-50 per cent.	15-25 per cent.
Elongation in 2in.	15-25 ,,	2-15 ,,

Chrome-vanadium Steel.—A sample specification for this is given:—

Carbon	·15 per cent. to	·25 per cent.	(·20 per cent. desired).
Manganese	·50 ,, to	·80 ,,	(·65 ,, ,,).
Phosphorus, not to exceed		·04 ,,	
Sulphur, not to exceed		·04 ,,	
Chromium	·70 per cent. to	1·10 ,,	(·90 ,, ,,).
Vanadium, not less than		·12 ,,	(·18 ,, ,,).

Heat Treatment.—After forging or machining:—

(1) Carbonize at a temperature between 1600° and 1750°F. (1650°-1700°F. desired).
(2) Cool slowly in the carbonizing mixture.
(3) Reheat to 1600°-1700°F.
(4) Quench.
(5) Reheat to 1475°-1550°F.
(6) Quench.
(7) Reheat to 250°-550°F. and cool slowly.

Brayshaw Hardening Furnace.—The Brayshaw furnaces are suitable for the carbonizing of gears. Recent designs are illustrated by Figs. 313 to 317. The carburizing furnaces are used for the first heating and the salt-bath furnaces for the reheating. Exact temperatures are secured by the use of pyrometers.

The furnace consists of a single chamber with a balanced sliding door, the doorway being of the same size as the chamber itself. The interior is lined with firebrick and is provided with gas fittings and burners, a damper and flue. The whole is self-contained and stands on the floor. The inlets, which are close to the sides of the chamber, discharge vertically downwards from the arched roof and heat the chamber uniformly. Any gas can be used, coal or producer gas at ordinary pressures, or high-pressure gas with modifications in the burners (Fig. 318). Compressed air at a pressure of 1 or 2 lbs. per square inch is required. The regulation of the supply of gas A and air B, mingling in C (Fig. 316), is provided for by quadrant taps. A patent door has recently been fitted, the leading features of which are an apparatus for locking the vertically-sliding door by means of a transverse bar mounted in brackets across the front of the furnace, a bar loosely connected thereto to engage the door and capable of moving vertically, a lug on the door to raise the vertical bar, a hand lever on the transverse bar, and a catch lever pivoted at the side of the furnace to engage the hand lever and lock the apparatus. In the illustration, Fig. 317, A is the furnace, B the door, suspended by chains C, passing over pulleys and counterbalanced by weights; D is the transverse bar across the furnace front, which can be rocked in its brackets *a*. At about the centre of the transverse rocking bar D, a vertical bar E hanging down projects in front of the furnace door, but being loosely mounted in its bracket it can be lifted clear of the door. A flange *b*, projecting from the front of the door B and at the bottom engaging with the bar E, lifts it when the door is raised, which falls again when the door is lowered. A hand lever F is attached to the end of the bar D. A stud *c* in the lower

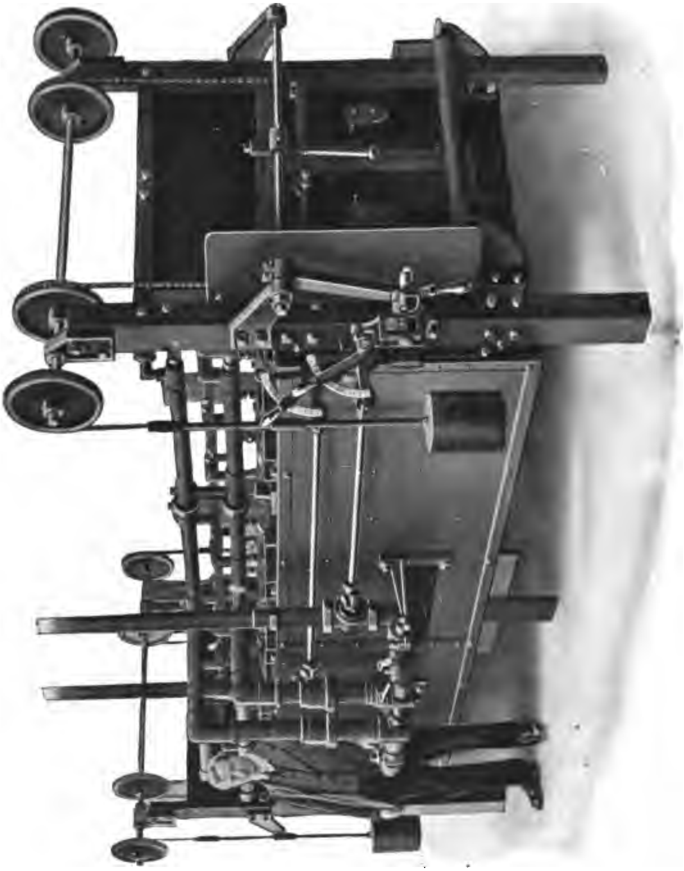


Fig. 313.—A Brayshaw Carburizing Furnace.

end of the lever engages with a slot in a catch lever G counterweighted, which catch lever on being raised thus releases the hand lever F. It is pivoted in its bracket to be readily adjustable. After the door B has been lowered it is forced against the furnace mouth by moving the hand lever F backwards and rocking the bar D to a slight extent, thus bringing the lower end of the bar E against the door near its centre. The stud *c* engages the notch in the catch lever G, so locking the door in position. By raising the catch lever and freeing the stud *c* the door is released, and hanging free, can be raised.

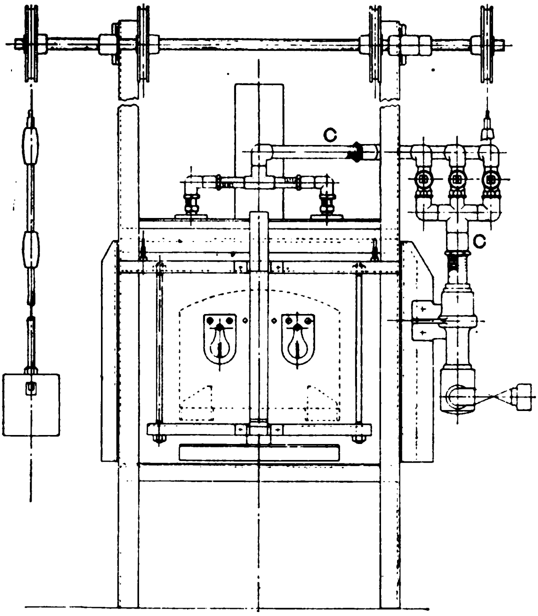


Fig. 314.

The commonest method of case-hardening is, in general terms, to heat the pieces in boxes with suitable materials for a sufficient length of time at a temperature of about 950°C. (1472°F.) and then to quench at once in cold water. The best case-hardening is, however, done in two or more separate processes, which vary according to circumstances. Briefly, the first process consists of carburizing only; that is, the articles are heated as above, but are then allowed to cool slowly, or are quenched in oil. The pieces so carburized are

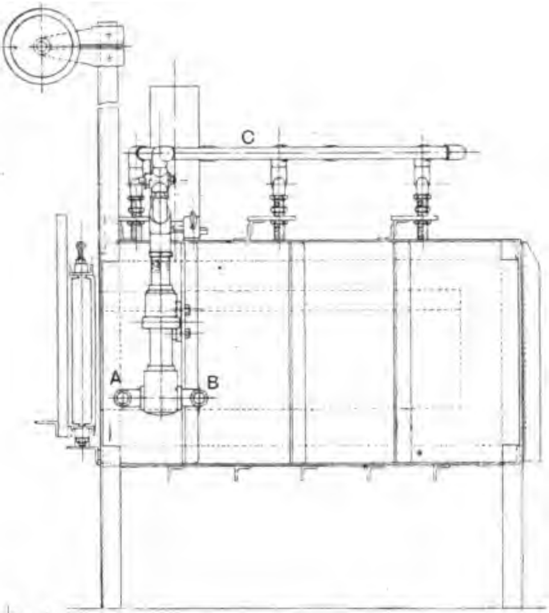


Fig. 315.

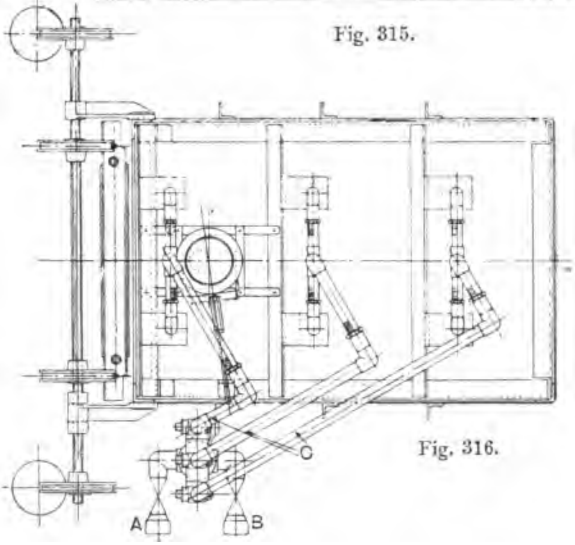


Fig. 316.

next heated to a temperature of about 760°C. (1400°F.) and quenched in water, after which they may be tempered or further treated in other ways. The carburizing furnace is specially adapted to the first of these processes, and is also suitable for the second heating. But a further improvement is effected by using a salt-bath furnace in the second process for heating the carburized pieces, as the salt bath ensures absolute uniformity of heating. The temperature is known with certainty and all the elements of chance are removed.

Salt-bath Furnace. — This is illustrated by two vertical sections (Figs. 319, 320), taken at right angles. The name is derived from the fact that the heating medium is a liquid composed of a mixture of fused salts, the "melt," consisting mainly of the chlorides of sodium and potassium. This is fluid at a temperature of 680°C. (1256°F.), which is a lower temperature than is ever required for heating carbon steel. The objections to heating in a bath of lead or lead alloy, and the unsatisfactory nature of heating in air, explain the reason for the adoption of the salt-bath. Lead does not circulate readily, and consequently temperature may be unequal in various parts of a bath, while another objection is that steel floats on the surface and must be held under forcibly. There are other objections to the employment of lead, such as the formation of dross, the presence of impurities, etc.

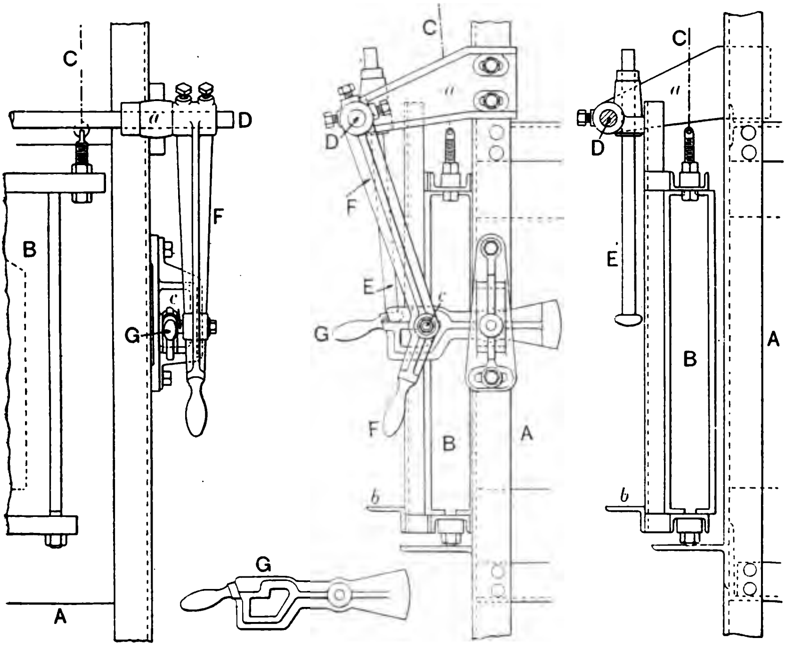


Fig. 317.

The pot A in the figures rests on a firebrick stand, and is surrounded by a firebrick-lined iron casing. The "melt" contained in the pot stands 2in. or 3in. below the level of the lip. The opening which gives access to the furnace is closed by a vertically-sliding door. The lip of the pot fits closely to the brickwork around the doorway (Fig. 320), so as to prevent the splashing of the salts around the outside of the pot. A perforated tray C, suspended by two rods D, D, passing through the cover of the furnace, receives the work to be heated; the rods afford a means of lifting the tray up opposite the doorway to receive the work, and to then lower it into the bath. Grids with a fork are employed for small pieces. The pyrometer E, protected with a tube, is inserted, and its readings are obtained on a Whipple indicator, or a Callendar recorder. Various instruments are supplied in connection if desired, including thermo-couple indicators and thread recorders.

The salt-bath furnace is heated by coal gas and is provided with two sets of burners, one set for air blast, the other a Bunsen burner. The gas is thus burnt efficiently. Gas-taps each with a large graduated quadrant allow of accurate adjustment for regulating the supply. The furnace heated by producer gas is similar to the coal-gas type, but some necessary modifications are made to suit the different gases. The crude-oil furnace is provided with one or more special burners, which are worked with compressed air, the oil and air being regulated separately. The furnace heated by refined oil has burners comprising each a vaporizer, in which the oil is continuously converted into vapour at a high temperature. The vapour is discharged in a jet, suitably arranged to draw and mix sufficient air for complete combustion. A closed oil tank is provided with an air-pump, pressure-gauge and all fittings complete, and flexible hose for leading the oil under pressure from the tank to the burners. No air-blast is required. These furnaces are made in six sizes, ranging from 7in. to 24in., inside diameter of the pot, and in depths from 10in. to 24in. of the pot.

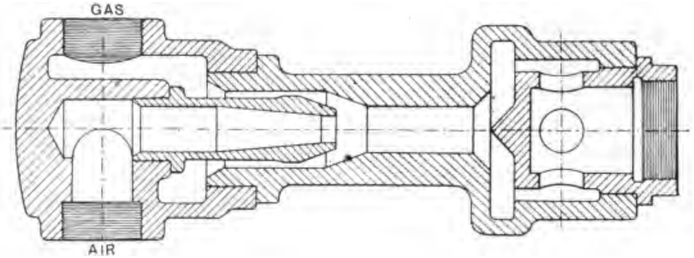


Fig. 318.

The Preparation of Blanks.—Gang-mills for circular milling on gear blanks are shown in Fig. 321. In the case of the spur A, and worm gears B, the two side cutters can be adjusted to cut wider or narrower by the interposition of washers of suitable thickness. The teeth of the two cutters for the bevel-gear blank C are interlocked so as to mill right up to the sharp corner of the blank.

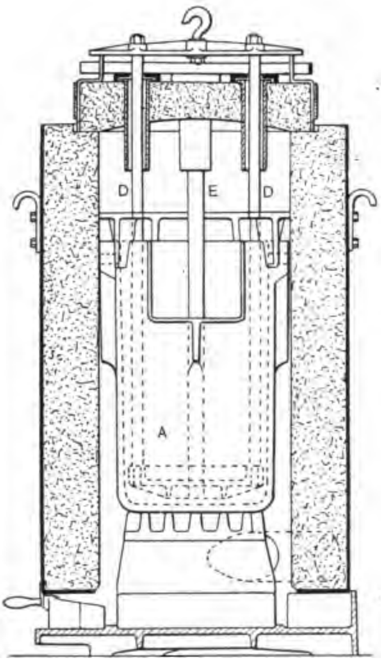


Fig. 319.

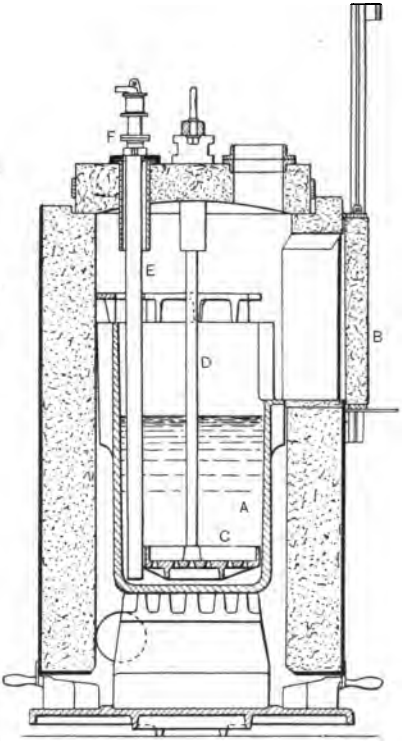


Fig. 320.

Circular Milling Machines.—One of these, by Messrs John Holroyd & Co., Limited (Fig. 322), is designed to produce by milling, gear-wheel blanks and other circular work of so light a character that gripping in chucks would be an extremely delicate operation. These can be done with the exercise of care in ordinary lathes, but this is a machine built specially for the work. Two headstocks are mounted on a bed, comprising two bases at right angles with each other. The mechanism of one headstock rotates the milling cutter. The circular feed imparted to the work may be varied between ¾in. and 6in. per minute. The work-head is carried on compound slides with adjustable stops. Each slide has a large hand wheel for making rapid adjustments. A trip gear arrests the circular movement of the work at either a trifle over one turn, or, if desired, just over two turns. The work is driven by adjustable pin dogs, which take a bearing against arms on opposite sides of the centre. The work-head has a cast-iron spindle, with necks 6in. and 5¾in. in diameter, running in capped bearings. The drive is from a worm wheel bolted directly to the driving plate. Thence the drive is connected through a drop worm and wheel to the operating change gears at the back end of the cutter spindle. A bracket which is bolted to the cutter head carries an adjustable bush which supports the outer end of the work mandrel. The cutter head is similar to a lathe head. Its spindle, which runs in parallel gun-metal bearings, is driven through back gears with a ratio of 6¼ to 1, from a three-speed cone pulley taking a 4in. belt. The cutter spindle makes 21, 40 and 74 revolutions per minute. The spindle is bored for the cutter arbors, which are supported at their outer ends by an adjustable stay bar and bush.

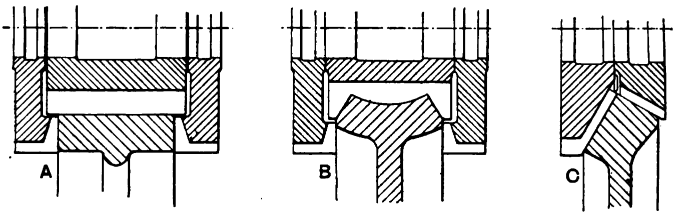


Fig. 321.

Bevel-gear Blanks.—Fig. 323 illustrates a method of turning the bevel face of a blank on one of the turret lathes by Messrs Alfred Herbert, Limited. A special attachment is employed, comprising a slide moving on a bracket bolted to the turret; this slide is tee-slotted and has bolted to it a holder receiving a stem which carries a roughing and a finishing tool, either being brought into use by a half turn of the stem, which is then locked by the handle. As the turret moves along, the slide is also moved along the bracket by the coercion of a hardened steel roller seen in contact with a case-hardened former plate, which is gripped at a suitable angle in the cross-slide turret holder. A spring within the slide maintains proper contact. The square turret also carries turning tools—not seen—for the various operations on the blank which cannot be done from the turret. By the substitution of a suitably-curved former plate, other profiles than straight ones may be tooled if gear blanks are not being dealt with.

Mitre-gear Blanks.—An outfit for tooling mitre-gear blanks in one of the turret lathes of Messrs John Lang & Sons is shown in Fig. 324. The operations are as follows:—

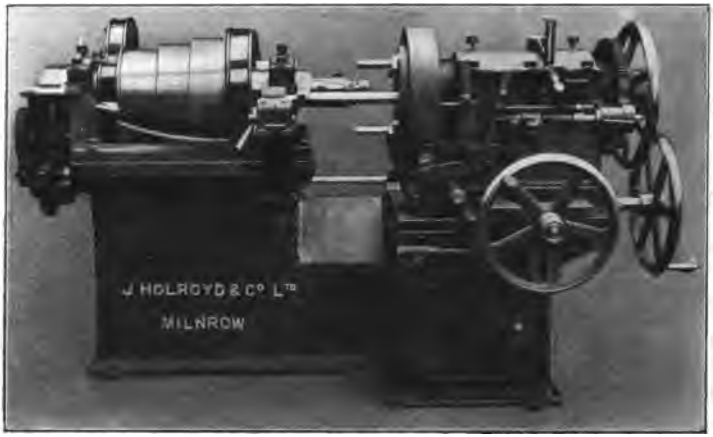


Fig. 322.—Circular Milling Machine for Blanks.

(1) Rough turn A and B, and size mitre.
(2) Rough bore, and recess.
(3) Rough turn C.
(4) Scrape A.
(5) Scrape C and B; recess F.
(6) Finish bore to size, and recess.
(7) Ream hole.

The blank is then reversed and held in the split chuck seen at the top right-hand corner, for operations:—

(8) Rough turn D.
(9) Rough turn E.
(10) Scrape E.
(11) Scrape D.

The various angles are tested by the sheet-metal gauge shown on the drawing.

Protractors.—By means of bevel protractors angles of bevel-gear blanks can be laid out and checked. The vernier fitting is desirable here. As usually applied to the protractors it reads to $5'$ or $\frac{1}{12}$ of a degree. Fig. 325 shows the vernier fitting as applied to the Brown & Sharpe protractors. Each division on the vernier is $5'$ shorter than two spaces on the main scale. When the 0 line on the vernier corresponds with the 0 line on the main scale the edges of the base and of the blade are parallel. If the head is swivelled until the line on the vernier next to the 0 mark corresponds with the line that is next but one to the 0 mark on the main scale, the angle which is included between the base and the blade is 5′. To read an angle, therefore, read off directly on the main scale the number of whole

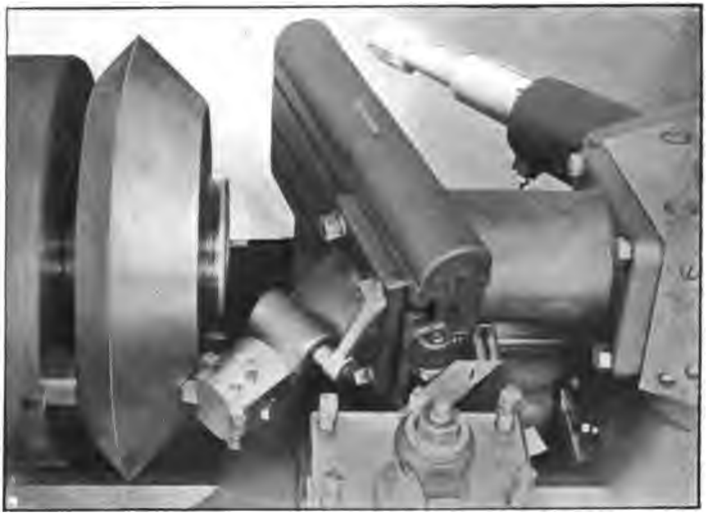


Fig. 323.—Turning a Bevel-gear Blank in a Herbert Turret Lathe.

degrees between 0 on the main scale and 0 on the vernier scale. Then count in the same direction the number of spaces from 0 of the vernier scale to the line which corresponds with a line on the main scale, and multiply the number counted by 5. That will give the number of minutes to be added to the number of whole degrees. In Fig. 325 the vernier has moved 12° to the right of the 0 mark on the main scale, and the eighth line on the vernier coincides with a line on the main scale. Hence $8 \times 5 = 40$ minutes, and the reading is 12° 40′.

Figs. 326 to 332 show various methods of mounting bevel-gear blanks on the Oerlikon machines. Arbors fit into the spindle of the work-head and are tightened with a screw passing through the centre. Fig. 326 shows a blank centred by its hole against the face of a bush

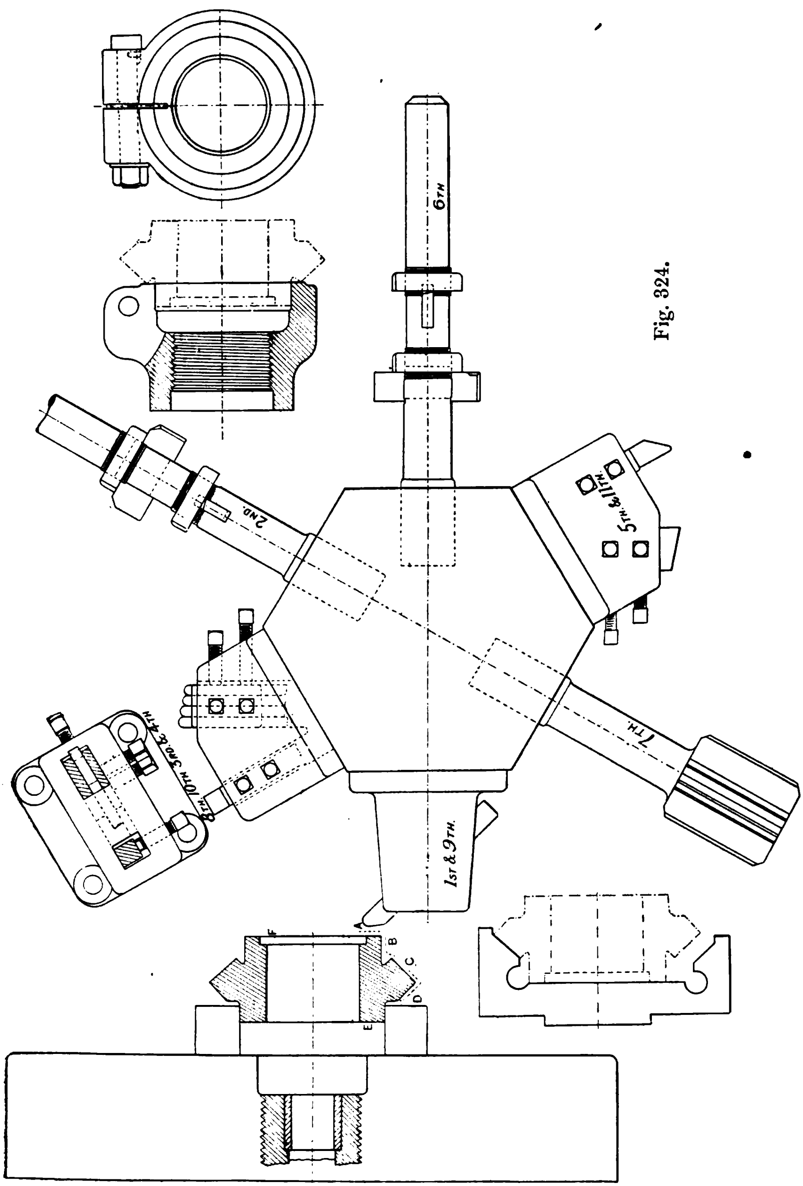


Fig. 324.

fitting in the taper. Fig. 327 is an example of a gear with a very long boss, the overhang of which is taken by an adjustable steady. Figs. 328 and 329 show gears with solid stems which are carried in special bushes with a pull-up screw. Small gears with large holes are shown in Figs. 330 and 331, requiring special bushes. Fig. 332 is a disc fitting for gripping bevel gears, such as are used on automobiles.

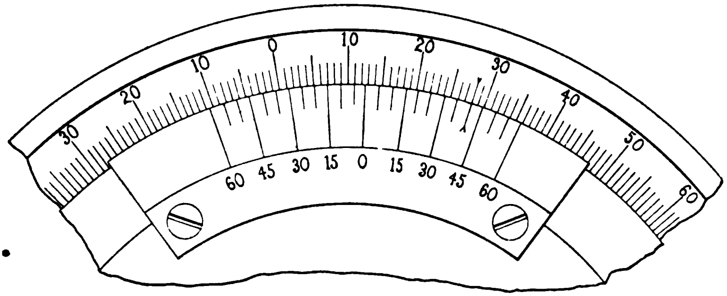


Fig. 325.

Gashing or Stocking.—Messrs Gould & Eberhardt, of Newark, N.J., build a modification of their regular automatic gear-cutting machine (Fig. 158, *see* p. 177) with special cutter and work slides, for the purpose of cutting two or three gears at one time, either spur or bevel gears, worm wheels, or clutches. In the photographs, Figs. 333 and 334, a 24in. by 8in. machine with two spindles is seen roughing out three gears, of 5 pitch, $1\frac{5}{16}$in. face, preparatory to their being finished on a bevel-gear planer. The cutter slide is arranged to accommodate one, two, or three cutters, and the slide bearing is so constructed that each cutter has a separate bearing on each side of it, and located close to it. This kind of bearing is employed on account of the extra number of cutters used, which increases the torsion, and duty required of the arbor. The slide is actuated on the draw-cut principle, that is the feed screw is anchored in the base of the upright, so putting the feed screw into tension and overcoming the

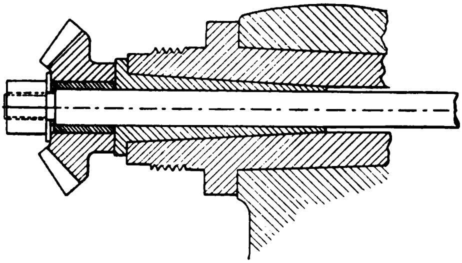


Fig. 326.

vibration that would occur if the screw was under compression, by being anchored the other way. All the strains of the cutter are thus transmitted directly to the base of the machine. The feeding and indexing of the cutter slide is controlled by mechanism operated by two levers that enable the operator to return the cutter slide by power without the indexing mechanism being unlocked, an important feature in sizing a blank. The mechanism also includes a safety device that holds the cutter slide from feeding while the indexing of the work is taking place, so that it cannot commence to feed until the indexing is completed.

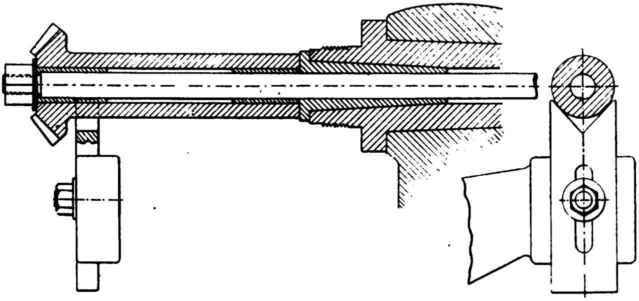


Fig. 327.

There are three work spindles on the work slide, which are controlled by a positive indexing device, and are indexed in unison by means of helical gears directly under the blanks. When the three spindles are used, bevel gears up to 5⅝in. diameter can be cut. But if only the two outer spindles are utilized, bevel gears up to 10¼in. diameter can be cut, while if only one spindle is in service a gear up to the capacity of the machine can be dealt with, which is 24in. diameter in this case.

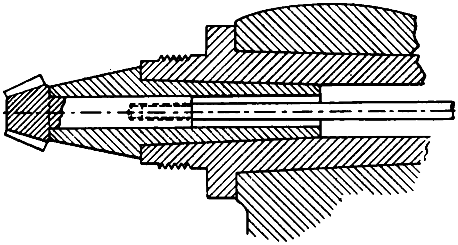


Fig. 328.

In addition to the regular segmental support to the table, which is controlled by a worm and worm-wheel sector located in a central position under the work-table, and held by an extra clamping device, there is an adjustable brace to support the periphery of each blank being cut; this also serves as a chip chute and directs the oil into the pocket located in the base of the machine where it is separated and strained from the chips before it reaches the oil-pump again. This machine should be of especial value in automobile factories, where bevel gears are produced largely. It has all-hardened steel driving gears and ball bearings.

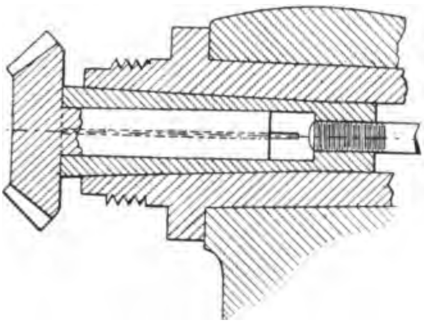


Fig. 329.

Roughing Shaper.—An arrangement of the Gould & Eberhardt shaping machine (Fig. 335) has been designed to rough-out bevel gears quickly, in place of doing all the cutting on the more expensive gear planer which must be employed for finishing the teeth. The shaper is particularly intended for roughing-out automobile differential steel bevel gears which have projecting bosses on the small end that prevent them from being roughed-out in the ordinary disc-cutter type of machine (Fig. 333), described on p. 326, because the rotary cutter would not be able to cut across the entire face owing to the interference of the boss. The shaper is modified so as to allow one or two bevel-gear blanks to be held at the proper angle and indexed at every return stroke of the ram, while the shaper head is automatically fed down at each stroke until the proper depth is reached, and then the feed is thrown out automatically and the blanks removed. The machine makes 100 strokes per minute.

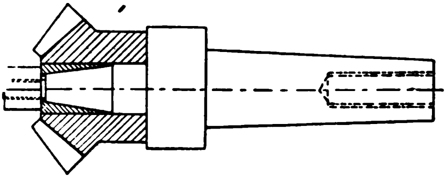


Fig. 330.

The regular crosshead slide and table are removed and a special work-head with an indexing mechanism are substituted. This work-head may be arranged so as to cut one or two gears at a time up to the capacity of the machine with suitable means for holding the blanks and lubricating the tools. In the photograph two nickel-steel gears of 20 teeth, $\frac{5}{7}$in. pitch, are roughed-out at one time in six minutes, or three minutes each. They have projecting bosses.

Two hook tools are provided to remove the chips that accumulate in the groove between the projecting boss and the inside face of the gear. The hook tools are under spring tension, and can be easily thrown out of position when taking off the gear.

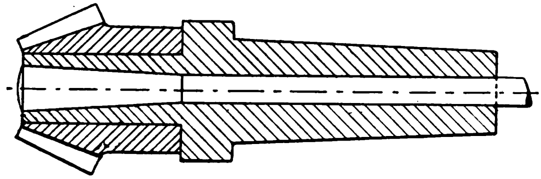


Fig. 331.

The indexing of the blank from tooth to tooth takes place when the ram has been drawn back, and is connected positively to the reciprocations. After the indexing has occurred, and before the tool starts to cut, the work spindles are locked so that the blank is securely held while being cut.

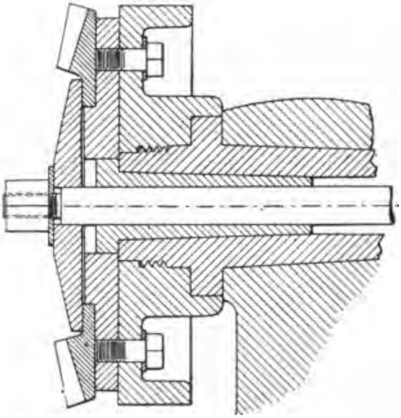


Fig. 332.

As the ram returns and the tool is out of the cut, the automatic vertical feed to the head takes place and so continues to operate until a small trip-dog located on one side of the head, and which must be previously set to feed the tool to the proper depth, disengages the feed. A foot-brake is arranged to act on the driving pulley, and it stops the machine instantly when the operator desires.

Fig. 336 illustrates a rapid method of gashing large bevel gears, by the use of a special fixture with indexing plate, being done on a Herbert plain milling machine.

Speeds and Feeds.—The speeds and feeds which can be used in gear cutting cannot be stated definitely, because so many variable elements enter into the problem. The Brown & Sharpe firm gives some useful data. A good range of speeds and feeds is embodied in gear-cutting machines, and the most suitable results must be obtained by experiment. Cast iron can be cut more rapidly than wrought iron, and the steels and brass can

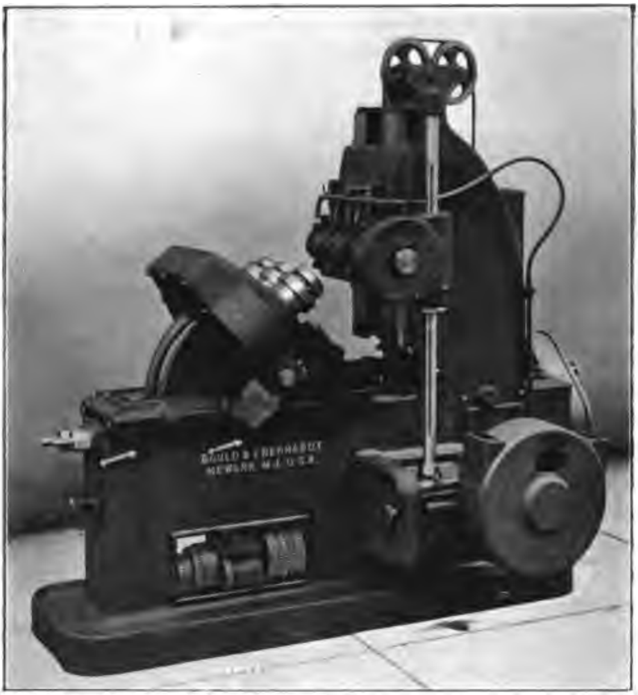


Fig. 333.—Stocking Bevel Gears on a Gould & Eberhard Machine.

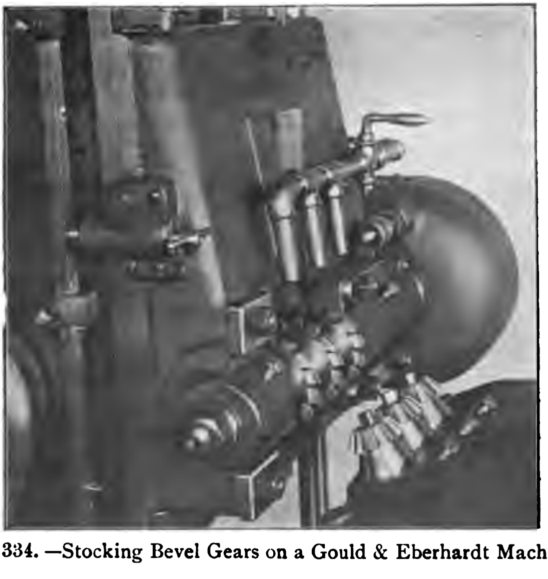


Fig. 334. —Stocking Bevel Gears on a Gould & Eberhardt Machine.

be cut twice as fast as cast iron. The differences in cast irons, in steels, and in the brasses and bronzes, somewhat invalidate the utility of any set of figures. As a general guide the following is by the Brown & Sharpe Manufacturing Company:—

AVERAGE SPEEDS FOR GEAR CUTTERS.

Diametral Pitch of Cutter.	Diameter of Cutter.	Turns per Minute Cutting Cast Iron.	Turns per Minute Cutting Wrought Iron and Steel.	Feed to One Turn of Cutter in Cast Iron.	Feed to One Turn of Cutter in Wrought Iron and Steel.	Feed per Minute in Cast Iron.	Feed per Minute in Wrought Iron and Steel.
	In.			In.	In.	In.	In.
2	5	24	18	0·025	0·011	0·60	0·20
$2\frac{1}{2}$	$4\frac{1}{4}$	30	24	0·028	0·013	0·84	0·31
3	$3\frac{13}{16}$	36	28	0·031	0·015	1·12	0·42
4	$3\frac{3}{8}$	42	32	0·034	0·017	1·43	0·54
5	$3\frac{1}{16}$	50	40	0·037	0·019	1·85	0·76
6	$2\frac{11}{16}$	75	55	0·030	0·016	2·25	0·88
7	$2\frac{9}{16}$	85	65	0·032	0·018	2·72	1·17
8	$2\frac{1}{2}$	95	75	0·034	0·020	3·23	1·50
10	$2\frac{1}{8}$	125	90	0·026	0·014	3·25	1·26
12	2	135	100	0·027	0·017	3·64	1·70
20	$1\frac{7}{8}$	145	115	0·029	0·021	4·20	2·41
32	$1\frac{3}{4}$	160	135	0·031	0·025	4·96	3·37

In the endeavour to quicken the cutting of gears the employment of multiple cutters is adopted. In this the cutters are set side by side, two or more forming a gang, the outer ones being modified in shape to suit the curve on the wheel blank (Fig. 68, p. 85). The teeth of adjacent cutters are set staggered to enable smoother cutting to be done. Their use, however, seems rather limited by comparison with single cutters, probably on account of the liability to inaccuracy. The blanks must be turned very accurately to diameter, and the depth of cut has an important influence on the thickness of teeth, so that the cutters are chiefly suitable where a large quantity of duplicate gears have to be cut. This ensures a more even distribution of heat around the blank, and avoids distortion induced by local heating. But by working at moderate speeds and feeds, heating may be avoided, if very accurate results are desired. The Brown & Sharpe formulæ provide for these conditions. If, as is usual, 120=the number of revolutions of the index worm to one of the wheel:—

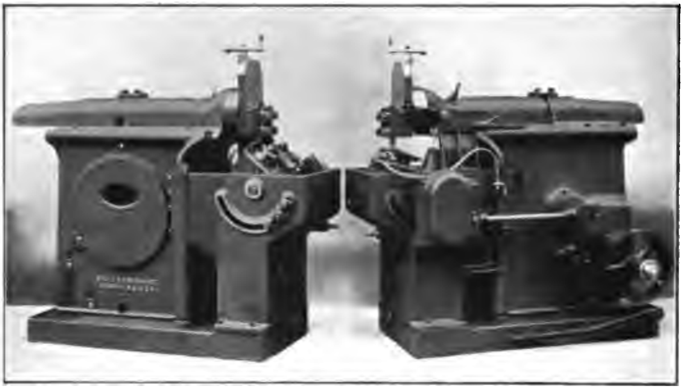


Fig. 335.—Stocking Gears on a Gould & Eberhardt Shaper.

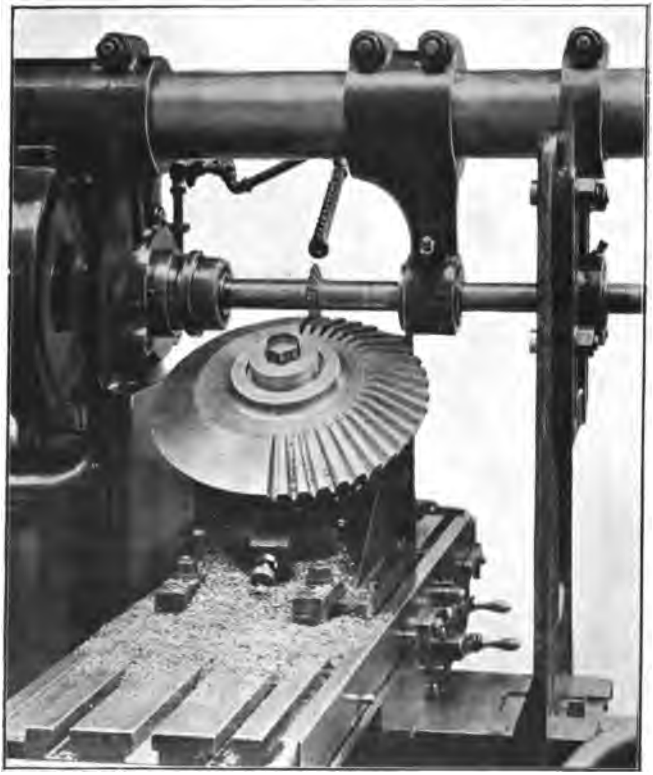


Fig. 336.—Stocking a Bevel Gear on a Herbert Milling Machine.

N = Number of teeth to be cut.
L = Number of turns of disc.
M = Number of teeth at each indexing.
D = 1st driver.
F = 1st follower.
d = 2nd driver.
f = 2nd follower.

Indexing one tooth at a time	Indexing more than one tooth at a time
$\frac{D \times d}{F \times f} = \frac{120}{N \times L}$	$\frac{D \times d}{F \times f} = \frac{120 \times M}{N \times L}$

Hobbing Speeds and Feeds.—These vary with the materials and, to a considerable extent, on the build of the machine used, a rigid machine as in other kinds of tooling being capable of much larger output than a weak one. The feed is the distance traversed across the face of the blank during one revolution of the blank. The coarser the pitch the finer must the feed be, because of the larger area of metal which has to be removed. In fine pitches a 0.1in. feed may be adopted easily, in coarse pitches from 0.03 to 0.06in. feed per revolution.

Multiplying the number of revolutions of the blank in a minute by the feed per revolution gives the feed per minute, and the revolutions per minute of the hob is to the revolutions per minute of the gear blank as the number of teeth in the blank is to the number of threads in the hob. So that, while a single-threaded hob must make as many revolutions to one of the blank as there are teeth in the latter, a double-threaded one will make only half the number of revolutions.

Thus, let:—

N = Number of teeth in blank.
f = Feed per revolution of blank.
r = Revolutions per minute of hob.
t = Number of threads on hob.
F = Feed per minute.

Then—

$$F = \frac{f.r.t}{N}$$

Some examples of speeds and feeds used on the Farwell gear hobber are as follows:—

Pitch.	Material.	R.P.M.	Feed.	Max. No. Threads.
3	Cast Iron . . .	65	·03	1
4	,, . . .	65	·032	1
5	,, . . .	85	·035	1
6	Nickel Steel . . .	75	·02	1
6	Carbon Steel . . .	90	·035	1
6	Cast Iron . . .	100	·05	1
8	,, . . .	114	·068	1
10	Mild Steel . . .	95	·05	1
16	Bronze	175	·1	2
16	Cast Iron . . .	114	·1	2
32	Bronze	225	·1	4 or 5
50	,, ...	250	·1	9

Tooth Rounding.—The quantity of gears which require rounding of the teeth on one or both ends, in automobile work and in machine tools, to permit of their sliding easily into gear, has resulted in the use of tooth-rounding machines, which accomplish neatly and rapidly what was formerly done laboriously with chisel and file. In a machine, by the Selson Engineering Co., Limited, the gear to be rounded is mounted on an arbor and is rotated by a worm above. The reciprocating movement is imparted by a cam. The gear is set at a suitable angle for rounding by means of slot-holes in the base of its bracket. The worm and the cam revolve at the same rate, so that the slide and gear make one reciprocating movement while the gear is rotated through a space equal to one tooth, during which the cutter imparts the rounding, or the chamfer as the case may be. This difference in shape is controlled by the shape and relationship of the cutter. The mechanism is driven by a single belt-pulley. A separate worm is required for every pitch, but one worm will serve for all numbers of teeth of the same pitch. Gears from 2in. to 9½in. pitch diameter and from 4 to 10 diametral pitch can be dealt with in this machine, about 6 teeth per minute being operated on.

The Lorenz automatic tooth-rounding machine is seen in Fig. 337. The wheel or wheels to be rounded are mounted on an arbor which is set between centres, and driven by a dog from the spindle of the driving head, which is rotated by worm gear through the change gears seen. A slight degree of circular adjustment is provided in connection with the worm wheel, to bring the teeth in exact relation to the cutter. The table carrying the heads is arranged to move lengthwise by means of a screw, so that several gears can be mounted on one arbor, and their teeth be rounded off successively. The cutting is continuous. The cutter spindle is arranged to swivel to either side of the vertical, from 0 to 20°, so that teeth can be rounded to their base without diminishing their width of face. The spindle

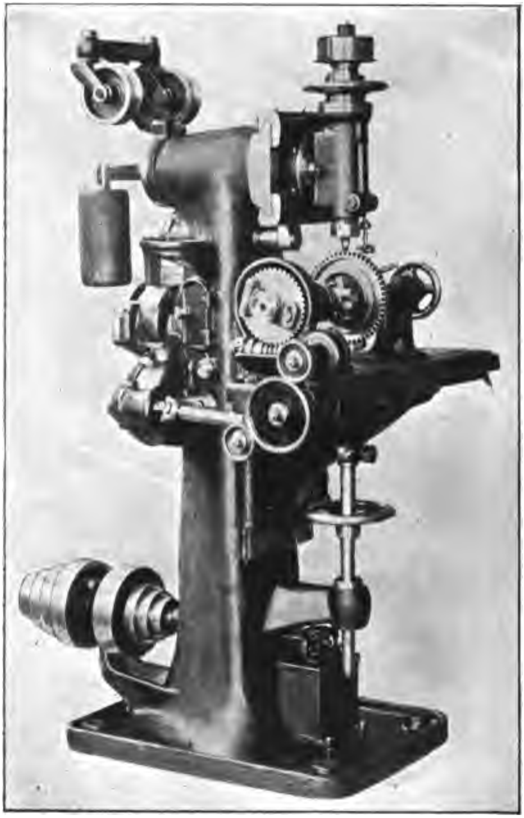


Fig. 337.—Lorenz Tooth-rounding Machine.

has a vertical adjustment for setting the depth of cut. Roundings of various kinds can be produced without the use of formers, such as circular arcs, symmetrical or one-sided, ellipses, and straight-ended points. A suitable shape and size of cutter is required for each pitch of tooth.

The Reinecker tooth-rounding machine (Fig. 338) has a reciprocating movement imparted to the cutter slide, instead of to the work-table, the reciprocation to and fro occurring while the tooth under treatment is travelling round. The gear is then pitched round for dealing with another tooth, the whole movement being automatic. A steady bracket is attached to the table as seen to support the shaft of the gear; this is especially useful when the smaller gears formed in the solid are being dealt with.

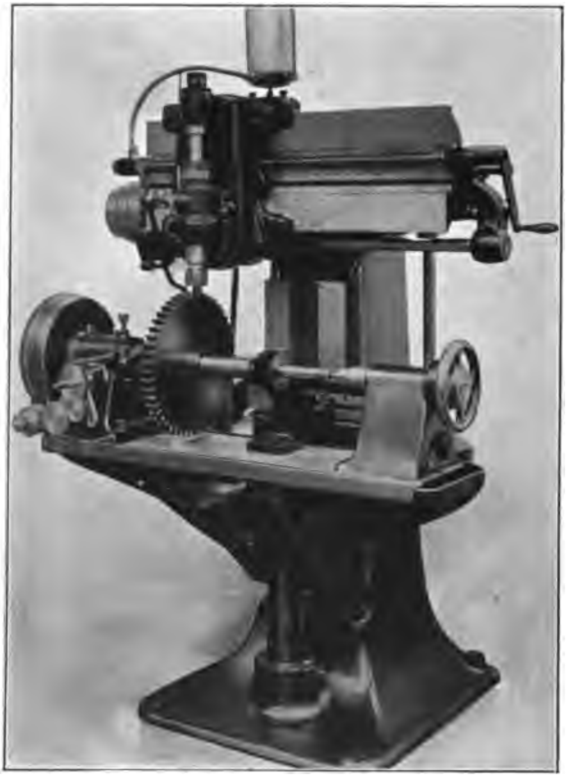


Fig. 338.—Reinecker Tooth-rounding Machine.

Castellated Shafts.—The demand for these has resulted in special machines being constructed for their production. In one of these, of French origin, the shaft is mounted in the spindle of a headstock, the other end being supported by a poppet. A pair of tools, set on opposite sides of the shaft, cuts the grooves with a planing action, leaving the ridges or keys standing up. As the tool slide makes its backward stroke, the shaft is partly rotated and locked by a plunger entering one of a series of holes in a disc mounted on the end of the headstock spindle. The disc thus acts as a division plate, having its number of holes corresponding to the number of ridges on the shaft.

Building Up.—As the cost of material put into gears increases, the practice also grows of building them up as rings on solid centres of cheaper material. These occur in the worm gears of electric cranes and hoisting machinery (Figs. 339 and 340), in the larger gears in speed and feed change boxes, and in those of automobile transmission gears. Worm gears for dividing purposes are made in halves and hobbed twice, once nearly down to the finished size, followed by a fine finishing corrective cut. Fig. 341 shows such a gear, also mounted on a cast-iron centre. Examples of automobile gears are shown in Figs. 342 to 345. Sometimes two rings of teeth are attached to one common centre; sometimes a ring is bolted to the face of a smaller solid gear. In other cases two rings are attached to flanges on a common sleeve. Many of the small pinions are forged or cast in one with long bosses or sleeves, or with sleeves and clutches. Again, in the fast-running gears the practice of trusting to a single key is abandoned in favour of two or four keys. In the sliding gears, too, instead of using separately fitting loose keys in the shafts they are made solid with their shafts as in the four-keyed or castellated system, an alternative to which is the square shaft. These give rise to a special design of machine for planing the shafts, unless they are milled with form cutters. The holes are also cut on one of the broaching machines. These are complications which render the preparation of blanks a less simple matter than it was a few years ago. They are done in the lathe chiefly, working by fixed calipers, gauges and templets. A large volume is done in the turret lathe when the number of identical pieces is sufficient to justify the cost of the

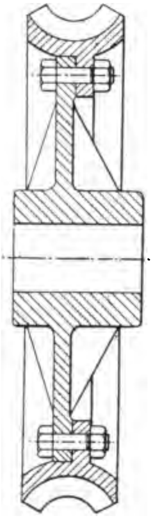


Fig. 339.

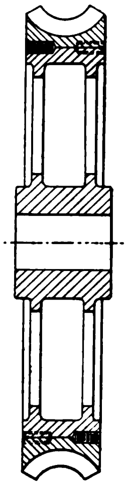


Fig. 340.

rig up. Some of the blanks are milled with straddle mills, especially those of light type.

Rawhide gears are employed in situations where high speeds prevail, and where the noise of metal gears would be objectionable, while they also possess the advantage of resisting shocks in a better manner than metal gears, with the result that vibration is much reduced. The action of the rawhide upon the teeth of the mating wheel produces a fine polished surface on all the teeth, and the friction of driving is thereby reduced. A rawhide gear may be run in mesh with a gear of any of the usual metals or alloys; a small amount of lubrication—preferably with graphite lubricant—is necessary. The strength and capacity for power transmission is approximately equal to that of cast-iron gears.

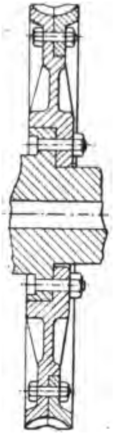


Fig. 341.

These gears are necessarily shrouded with brass or other plates clamping the laminations of hide, but the shrouding must never touch the mating metal gear, and it is essential to have the hide portion slightly wider than the companion gear. The details of the shrouding depend upon the size of gear and its function. The simplest method is to rivet the plates through (Figs. 346, 347), but for large gears bolts are preferable, being a more convenient method, and affording opportunity for subsequent tightening up with facility should shrinkage occur. Lock-nuts may be employed if necessary. When there is insufficient space to allow of the heads projecting to their full amount, the plates can be countersunk (Fig. 348), or

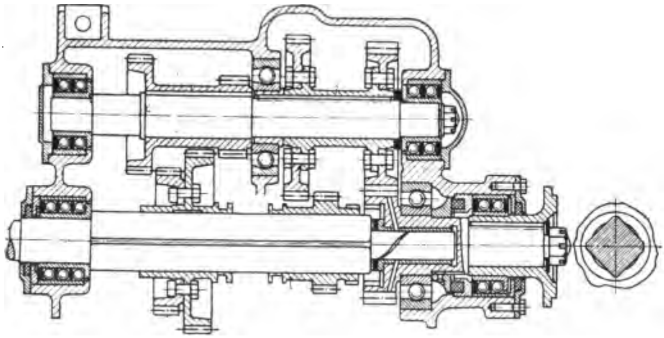


Fig. 342.

if a face is demanded, countersunk nuts with slots for the employment of a special spanner can be used. The shrouds may be cut through at each tooth space, as in all the examples illustrated, or be left intact as discs. A bushing (Fig. 349) strengthens a pinion, and is alternative to the practice in Fig. 346; it is a necessity when a pinion or wheel has to run loose instead of being keyed to its shaft. The metal centre in the case of Fig. 348 is employed on account of the large size of the gear, to reduce the amount of rawhide required.

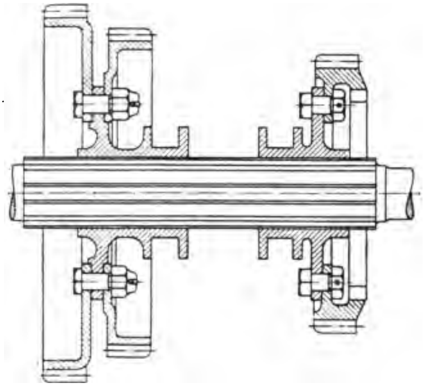


Fig. 343.

Thorough seasoning, usually for a period of many months, is desirable to produce a good rawhide gear. A properly made gear will last in good condition for years. When faults develop they are usually traceable to the fact that poor hides have been used, or inferior parts or trimmings, which are worked up and lined to give them an appearance of solidity and hardness.

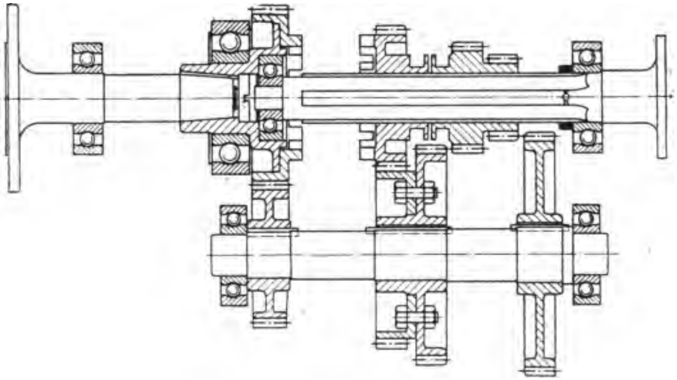


Fig. 344.

The principal objection to rawhide is that it is affected by heat and oils, and for this reason compressed paper and cotton are utilized for gears which have to run in situations where heat, moisture, or oil cannot be effectually guarded against. These materials are built up under great pressure, being subjected thereto in hydraulic presses, but in other respects the construction of the gears resembles that of rawhide. Graphite lubricant is applied to paper gears. The necessity for such lubrication is avoided in cotton gears by impregnating them with oil during manufacture, a method which also renders them impervious to moisture or atmospheric conditions.

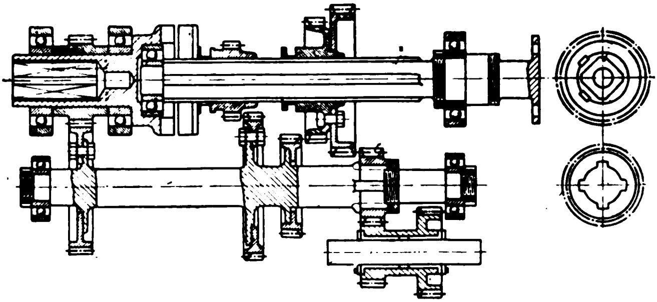


Fig. 345.

Examples of Manufacture.—There is no difficulty in obtaining either approximately accurate or absolutely accurate teeth, regarded from the practical point of view, by the systems of cutting employed. The principal difficulties arise from the fact that the teeth have to be hardened on the wearing surfaces in order to ensure their durability. Hardening without some resulting distortion is impossible, and there is also a risk of cracking. Not only has care to be exercised in hardening, but subsequent grinding may be necessary. These processes are rendered more difficult because the gears are made of highly tensile steels, which are more liable to suffer distortion than the milder qualities. Looking at one of these slender gears the marvel is that so high a combination of durability and accuracy has been attained. As an illustration of the care required in the manufacture of gear wheels, a few examples are given:—

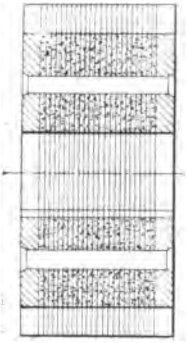


Fig. 346.

In one system the gear wheels are made from chrome-vanadium case-hardening steel. The blanks are drop-forged, with a sufficient amount left on for cutting down to size. They are then annealed before being cut. Careful inspection follows. They are turned in a turret lathe by means of tools and fixtures which are designed specially for repetition work, and by which more than one operation is performed simultaneously.

A blank, having been gripped in a three-jaw concentric chuck, is centred with a centring tool, and a drill, of smaller size than the finished hole required, is started. While this is running through, a roughing cut is taken across the face of the blank to remove the hard scale, the two operations being completed almost simultaneously. A fixture which carries a flat broad cutter shaped to the correct profile of the face of the gear is next brought into action. It is bolted to the turret of the machine, and, being fed up to the face of the gear blank, cuts it accurately to shape and dimensions at one operation. A cutter bar is next put through the hole, boring it to within 1-100in. of its finished size, which amount is ground out after the gear has been case-hardened. The other side being then finished in a similar manner to the first, the blank is ready for the cutting of the teeth, done with a single disc cutter, in any automatic machine.

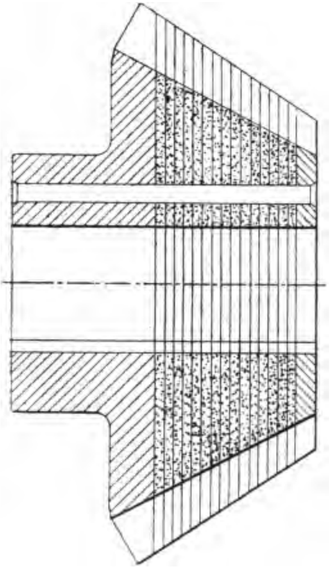


Fig. 347.

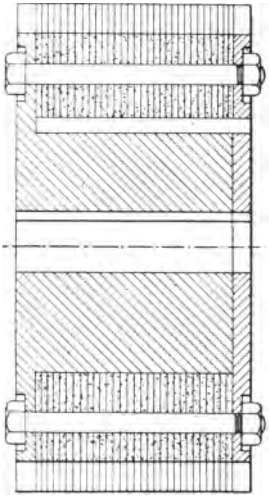


Fig. 348.

The case-hardening of the gears is done with care, since a few degrees of temperature more or less has a serious effect on their wearing properties and strength. A coke or gas heated muffle furnace is used, which is carefully maintained at a constant temperature by means of properly arranged dampers and air flues, and a pyrometer is employed for reading the temperature of the furnace.

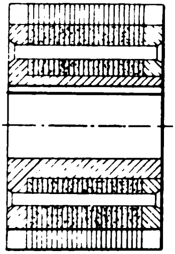


Fig. 349.

The gear wheels, after having been cleaned from oil or grease, are placed in boxes with the case-hardening medium, and are hermetically luted with a fire-clay mixture. Test pieces of similar material to the gears and shafts are placed in the boxes, which are then put in the furnace, where a temperature of 800° to 1000°C. is maintained for from six to twelve hours, depending on the depth of hardening required. The furnace is then allowed to cool down, the boxes are withdrawn, and the pieces removed for reheating and quenching in oil or water for hardening. The test pieces having been hardened and broken, in order to ascertain if the carbonizing is satisfactory, the gears are carefully reheated to the proper temperature and quenched in oil or water. The gears are ground and inspected for assembling.

For service in motor work, as used by Messrs John I. Thornycroft & Co., Ltd., the steel has a tensile strength of from 37 to 40 tons per square inch, with an elastic limit of from 27 to 30 tons. The blanks are roughed-out and annealed, and then finished in a lathe. The teeth are cut on a Fellows gear shaper. Case-hardening follows, the gears being packed in charred leather, heated for four hours in a coke-fired furnace, and quenched in oil. They are ground in a machine of special pattern designed by the firm for this work, and inspection for flaws and inaccuracies precedes the assembling.

The Wolseley Tool and Motor Car Co., Ltd., of Birmingham, use steels which are manufactured by Messrs Vickers, Sons & Maxim, Ltd., of a special case-hardening brand. The parts are machined, and the teeth cut, after which they are hardened and ground so carefully that interchangeability is secured in the parts when fitted in the gear box. They are then tested for hardness and strength before being assembled.

The firm has introduced a refinement in a box designed for a large car in which the teeth are first roughly cut, then hardened, and finally shaped after hardening. This is an expensive process, requiring a special brand of steel for the wheel, and also for the cutters used for the final shaping.

Messrs David Brown & Sons, Ltd., of Huddersfield, carbonize the gears in cast-iron or steel boxes, the sizes of which boxes vary with the dimensions of the work. A layer of case-hardening compound is placed at the bottom of the box to a depth of about 1in. On this the pieces are laid, leaving a space of at least ½in. to ¾in. between each one, and between the sides of the box. The compound is now thoroughly packed between the articles, and if the dimensions of the pieces permit, another layer or more is put in until the box is filled. Care is taken that the compound is thoroughly packed between the pieces. The cover is luted with fireclay and the boxes are placed in a furnace which has been heated to a temperature of 850° to 900°C. This is maintained for three to four hours, according to the size of the articles and the depth of casing required. Test bars are employed to ascertain the depth, several being used in the larger boxes. When the proper depth is reached the boxes are removed and allowed to cool, and the test pieces are treated and fractured, after which they are subjected to careful examination before dealing with the articles from the same box. If the depth of hardening and fracture are satisfactory, the articles are reheated to about 875°C., or some approximate temperature which is suitable for a particular grade of steel, and are then dipped into water or oil, whichever is better suited to the steel. An installation of pyrometers automatically registers the variations of temperature in the furnace, which variations are reduced to a minimum by the means of regulation of the furnace which is employed. A generator is used in connection with these, which is coke fired, and a system of flues through which the hot gases are distributed, so that a very uniform temperature is maintained in the muffles. No flame is allowed to come into contact with the work.

Grinding : The Heald Chucks.—Gear wheels are held for grinding the bores by the substitution of draw-in collets for the common chucks, a valuable method when the gears have been slightly distorted by hardening. A chuck body is screwed to the nose of the spindle and encloses a tapered sleeve split in three or four places which sleeve is fitted with false jaws held in place by screws, the jaws being made of various diameters. The latest Heald collet-chuck is shown by Figs. 350 to 355; it has adjustable jaws made in two parts to hold work of any size from ⅝in. in diameter up to 8in.

in diameter. In Fig. 350, A is the body, B the tapering split sleeve, C the false jaws, D the adjustable jaws. In this, gears are chucked by three methods. The usual one is that in which they are held by

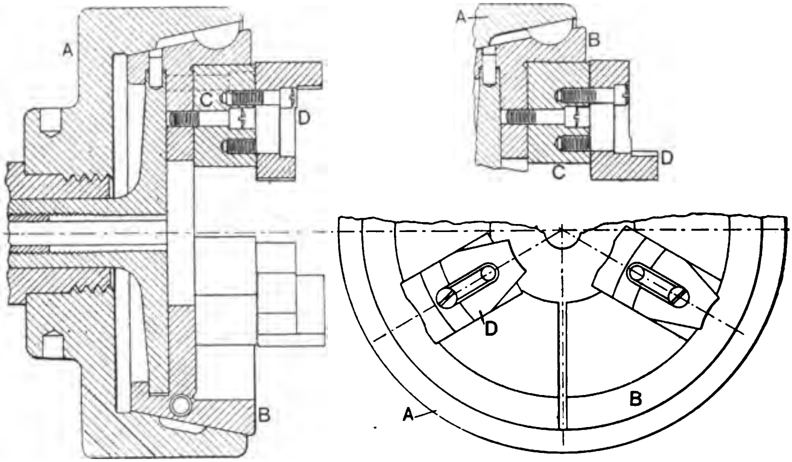


Fig. 350.

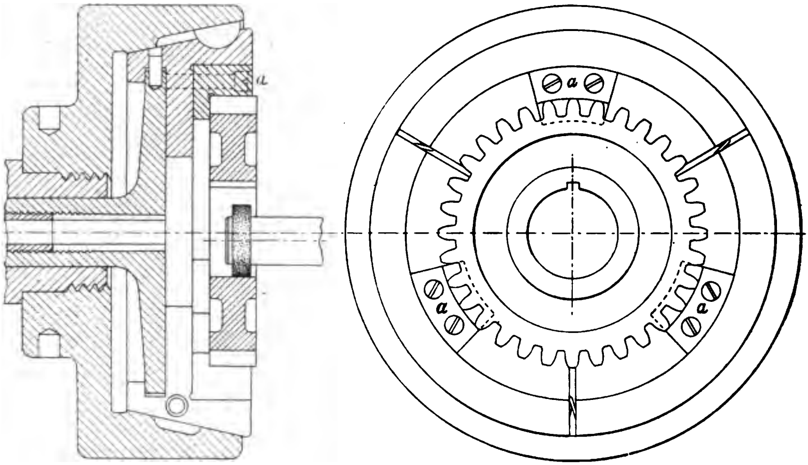


Fig. 351.

the tops of the teeth, as in Fig. 351, *a, a, a*. The objection to this is the possible want of truth in the teeth, due to inaccurate cutting, or to distortion in hardening. Another is that of centring by three rolls *a, a, a,* fitting between the teeth at the pitch circle, as in Fig.

352, adopted very much in consequence of the practice of hardening teeth. The objection to this is the inaccuracy developed by hardening, by reason of which one roll may fit too tightly and another too freely, since tooth spacings and pitchings and the pitchings of blocks of teeth may vary slightly. Fig. 353 shows an improvement on this method, two pins with self-adjusting ends being substituted for a single rigid one. The third method is that in

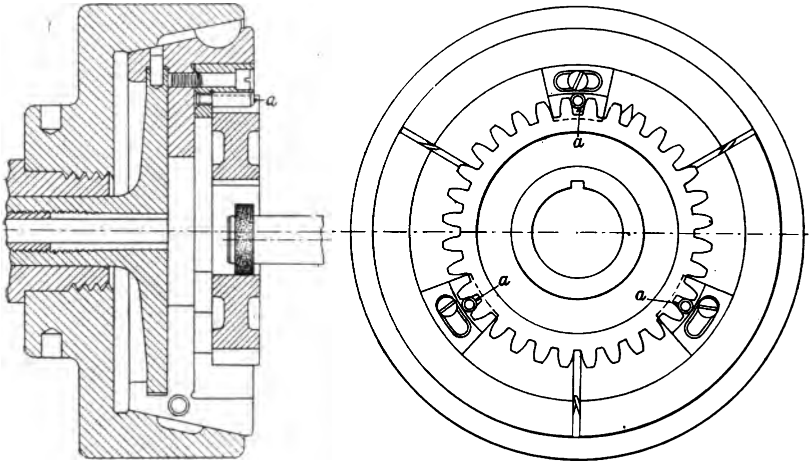


Fig. 352.

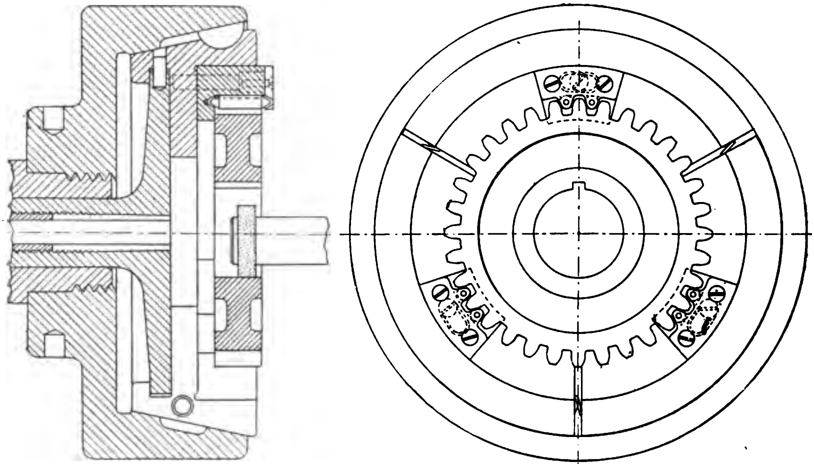


Fig. 353.

which the jaws make contact with the roots of the teeth. This is not open to either of the foregoing objections, since if the gear is not true on the tops of the teeth or at the pitch line, it is set by the roots, from which, as a base, corrections may be made. The contact of pins in the roots may be that of single pins, or of two or three in adjacent tooth spaces. Adjustment can be made by securing the pins in a plate with a slot hole as in Fig. 354, *a, a, a.* This is necessary

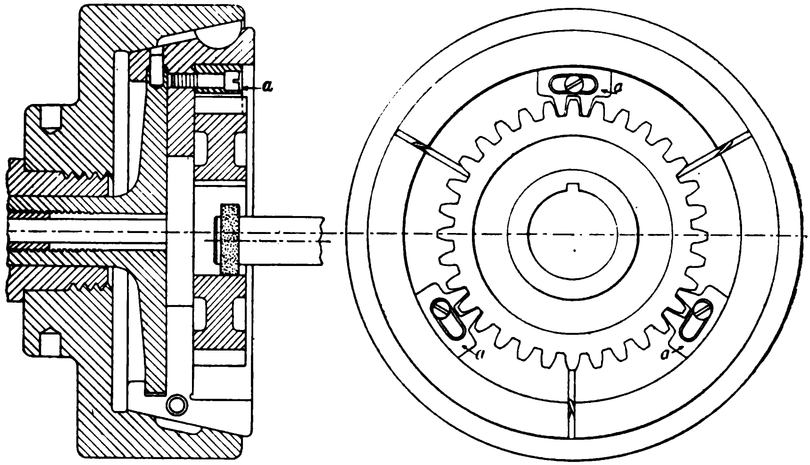


Fig. 354.

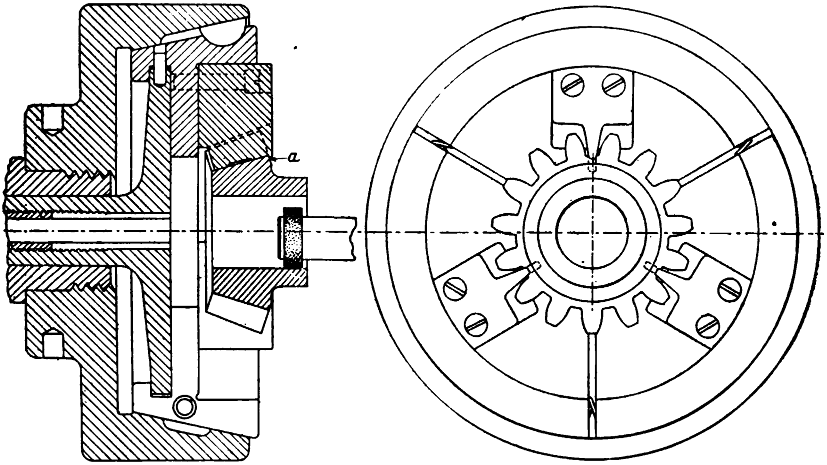


Fig. 355.

when the teeth do not occur in multiples of three, in which case the slot hole is a necessity. The slot must be a curved one, as shown, to maintain the radial distance of the pins correct. Fig. 355 shows the chucking of bevel gears by root contact. Here the jaws of the collet have their contact faces bevelled to fit the roots of the gear. By making the outer edges of the jaws as shown at *a*, they will prevent the gear from slipping out on the closing of the collet.

Fig. 356 illustrates the method adopted by the Heald Company for chucking the large bevel gears in quantity as used in automobile work. An unhardened gear B is sunk slightly into the face plate A, and secured with screws. The gear C to be ground is set with its face to the unhardened gear, and held by three clamps, D. In the illustration the face portions of the teeth of the chucking gear are seen cut away, leaving teeth of full length at only three parts of the circumference, on which the gear to be ground will be supported without rocking, when the hole can be ground truly.

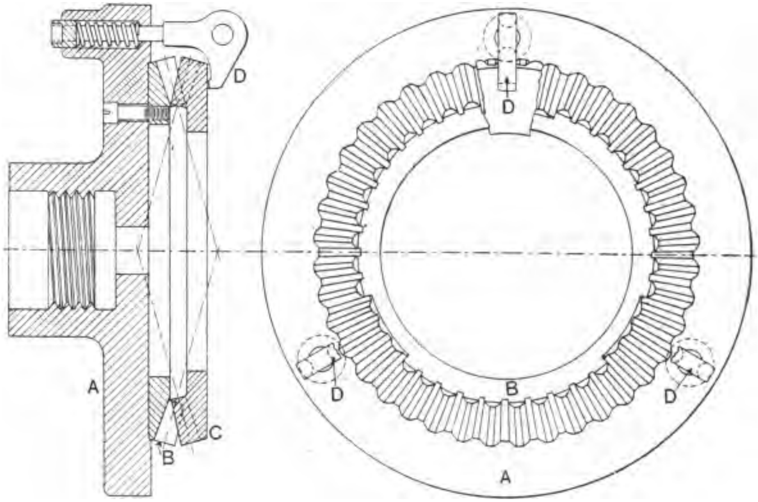


Fig. 356.

Hob Sharpening and Grinding Machine.—A machine designed for hob sharpening and worm grinding is seen in Fig. 357, with a worm in place being ground. Messrs J. Holroyd & Co., Ltd., are the makers of the machine, and have supplied a large number to firms whose practice demands rectification of worm threads after hardening. The worm is carried between centres, and a twisting motion is imparted by change wheels. Multiple indexing is arranged for if required. The table carrying the work is fed by belt, with a quick return in either direction.

The grinding wheel is carried on a universal head, permitting it to be adjusted to the desired angle. In the first instance the machine was arranged for grinding the sides of the threads by taking successive cuts, feeding the wheel down at an angle after each stroke of the table, the angle being set according to the pressure angle of the gear. This method produces work of the highest accuracy, but is not used very much because it takes longer than the mode of grinding the whole depth of face at once. When following the latter method a difficulty is met with in obtaining the correct shape of grinding wheel to suit the twist of the thread. A special attachment is therefore used for generating the shape of wheel, by employing a diamond held and moved in a path corresponding to that described by the face of the worm, thus turning up the face of the wheel automatically and correctly. One side of the thread of the worm is ground at one time.

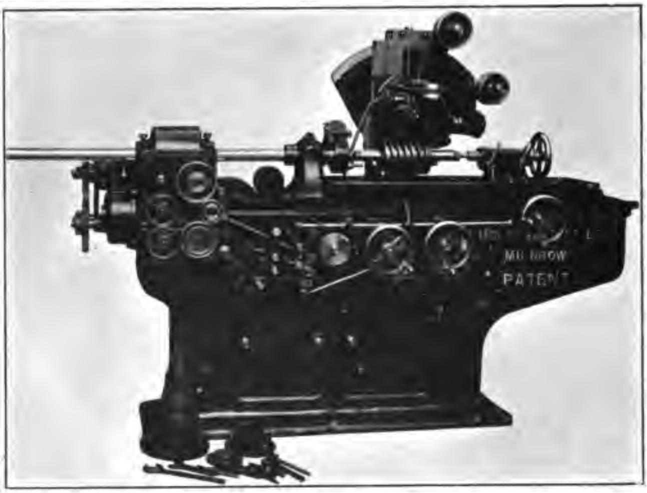


Fig. 357.—Holroyd Machine for Grinding Worms.

Testing.—The general method of testing the accuracy or otherwise of toothed wheels is to run them round on their shafts in place. In comparatively few shops is any other method available, excepting that of trying them in gear apart from their shafts, by hand, and without any rig up. This latter crude method shows the contact of the teeth, and, in an approximate manner, the depth of their contact and bottom clearance; but it tells nothing of their truth—whether square across the face in spur wheels, or running to the apex in bevel wheels; neither does it permit of very exact measurement for centres.

Another objection to these rough methods is that when high-class work is in question trouble is often liable to arise at the last moment, when mechanism is being put together. Fixed centres and no flank clearance to the teeth demand very perfect gears—much more exact than is required of cast wheels, the teeth of which have flank clearance. So that at the final fitting the chances are that teeth may fit too easily, or may, on the other hand, require some correction. Such slight differences in the diameters of the blanks and the depthing of the teeth, and in their thickness, and slight inaccuracies also in setting on the machine, produce these results; so that the average shop methods are not good enough to ensure reliable results. In every shop, therefore, there should be a simple method of testing wheels as they leave the machine, to become a check on the work of the machine, and to prevent possible waste of time subsequently. It is easy to fit up adjustable arbors by which to test gears. Briefly, the method adopted by the Brown & Sharpe Manufacturing Company is, as follows:—

For spur wheels a horizontal bed is fitted with two arbors or studs —one fixed, the other movable on a slide. The bed is divided out into inches and parts for the location of exact centres, and the slide which carries the movable arbor can be locked in any position. The difference in the bores of different wheels is met by having a set of bushings to fit those various bores, and all to slip over the two arbors, the device being similar to that in which bushings are fitted to the holes in the tables of drilling machines to receive boring bars of different diameters.

When pairs of wheels are on their bushes they are tried round by hand. A single wheel is tested with another wheel of the same pitch, a standard set of which is kept in a cupboard under the table of the machine.

For testing small bevel wheels and worms, a machine is used having a horizontal bed and a vertical standard at its end. The arbors are carried in slides which are adjusted on bed and standard with screws. Worms and their wheels are tested by carrying the worm in a special sliding bracket, the wheel being run on the vertical slide. Large bevel wheels and large spurs are tested on a machine having two horizontal beds, situated at right angles. To accommodate bevels, the axes of which are at other than right angles, the arbor bearings are made to swivel on the slides.

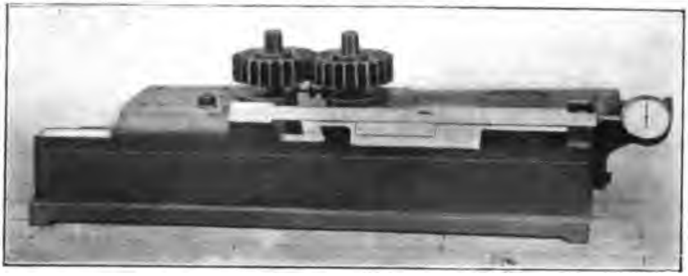


Fig. 358.—Parkinson Testing Machine for Spur Gears.

The Parkinson testing jigs for spur and spiral gears are illustrated by Figs. 358 and 359. They take gears ranging from 1½in. to 9 in. centres. Wheels are tested in pairs as they have to work, or one wheel can be compared with a master gear. In either case, one of the pair is mounted in a fixed position on a carriage which may be adjusted along the bed, the other on a floating carriage which rests on three steel balls, and carries a spring plunger at one end. The spring inside the plunger may be compressed by an adjusting screw fixed at the end of the bed. Another adjusting screw acts against the same end of the floating carriage, so that the distance between the centres of the gears may be controlled to a definite and minute

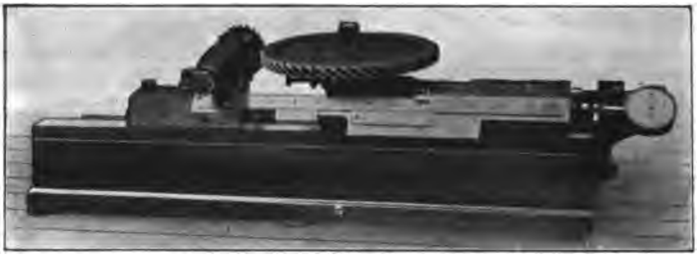


Fig. 359.—Parkinson Testing Machine for Spiral Gears.

amount. An adjusting screw at the other end of the carriage limits or prevents motion. Two methods of holding the gears are provided: one is to ascertain how the gears run when the centres are fixed, the other to judge of the effect produced by eccentricity of gears, errors in the sizing or spacing of teeth, and so on. Either of these will cause movements of the carriage when the gears are rotated in close engagement, and the movements are read off on a sensitive dial indicator seen to the right. The centres of gears are read on a vernier scale.

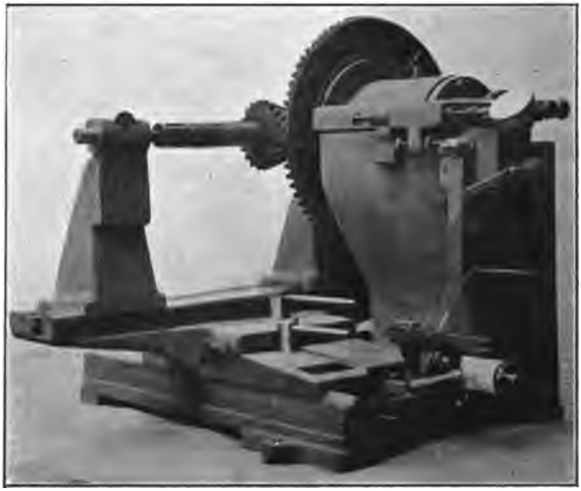
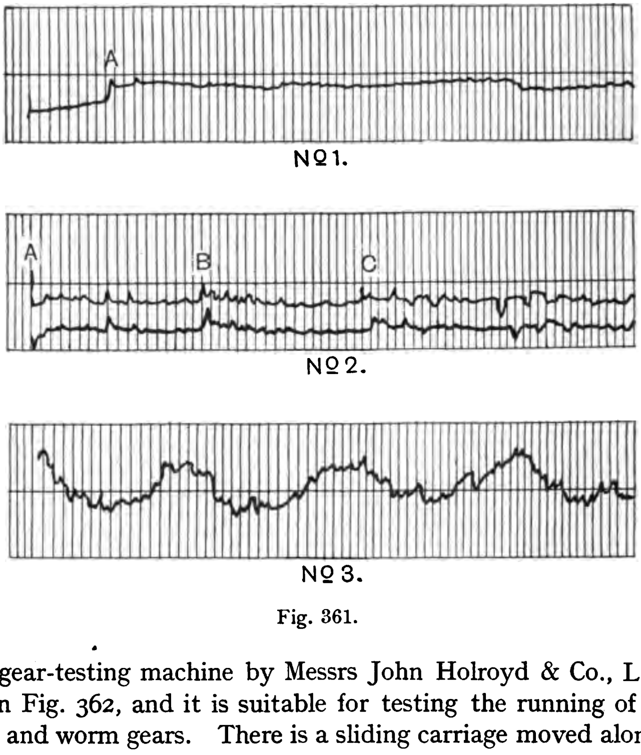


Fig. 360.—Parkinson Testing Machine for Bevel Gears.

The jig for bevel gears (Fig. 360) is constructed similarly to the spur-gear jig in all its essentials, modified only by the difference in the shapes of the wheels, and by the addition of an indicator, optional, which traces a permanent record on a chart of the action of the pair of gears under test. Three of these charts are shown in Fig. 361. Chart No. 1 is good, only showing a discrepancy at the point A, which indicates a jump in the thickness of the teeth of 0.0045in. Elsewhere the straight-line course shows that the gears were rolling perfectly. Chart No. 2 is a good one generally, but at the points A, B, C, occurring at regular intervals, thick teeth are indicated. Owing to the gear ratio, it was found that these points coincided with the revolutions of the pinion, and hence these thick teeth were found on the pinion. Chart No. 3 is a very bad one, as indicated by the wavy line. This was taken from gears having a 4 to 1 ratio, and there are four waves. These show that the pinion was out of truth to the extent of about 0.008in. And, further, the jump in the curve between various teeth shows that the gears were rolling badly, which was also borne out by the fact that they ran noisily. A large size is made to take spur, spiral, and bevel gears.



Nº1.

Nº2.

Nº3.

Fig. 361.

A gear-testing machine by Messrs John Holroyd & Co., Ltd., is seen in Fig. 362, and it is suitable for testing the running of spur, bevel, and worm gears. There is a sliding carriage moved along the bed by screw and handle, with micrometer adjustment, and a scale on the bed is read with the aid of a vernier plate attached to the carriage. The cross-slide seen at the right-hand end of the bed carries a couple of brackets to receive the mandrel of a worm, shown in mesh with a wheel. When spur gears are being tried, a mandrel is stuck vertically in this cross-slide to hold one gear, and the other gear is carried on a mandrel inserted in the main carriage. The upright at the left-hand end is utilized when testing bevel or mitre

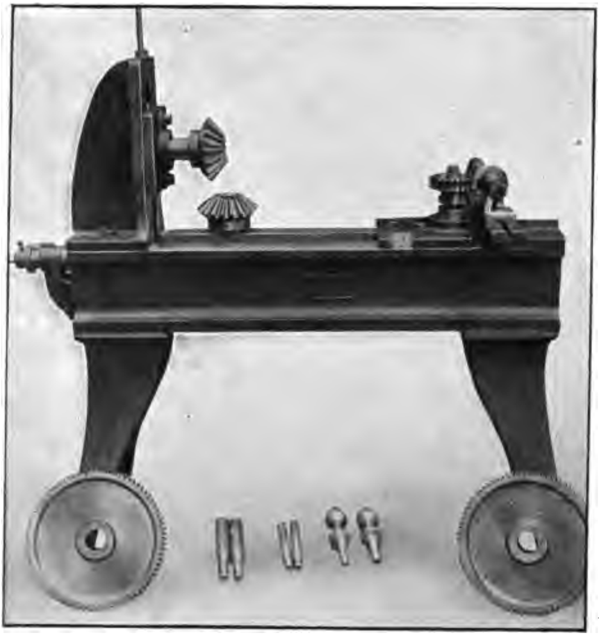


Fig. 362.—Holroyd Gear-testing Machine.

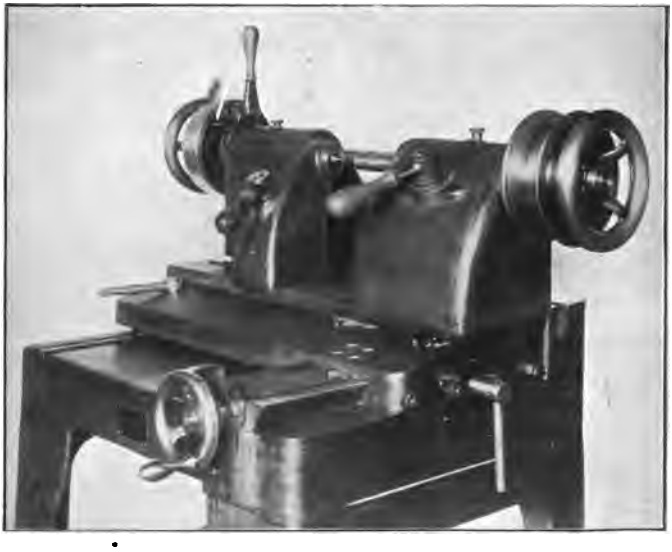


Fig. 363.—Gleason Testing Machine for Spur Gears.

gears. Spur gears up to 2ft. 8in. diameter, bevels up to 18in., worm wheels up to 4ft., and worms up to 6in. diameter, can be tested on this machine.

The Gleason Gear-testing Machines.—These are shown in the photographs, Figs. 363 and 364, for spur and for bevel gears. The same general features are present in each type, with necessary modifications introduced by the setting of the heads parallel, or at right angles to one another. In the spur-gear machine the heads at the front are carried upon a cross-traversing slide, by which the pair of gears may be set at exact centres, the hand wheel of the adjusting screw being graduated. The outboard support on this slide is adjustable to suit different widths of gears. The heads which carry the arbors have their spindles bored to standard tapers, so that arbors as employed on the gear-cutting machines may be held for testing gears mounted upon them. The countershaft for either machine has a forward and reverse drive, and runs at 350 revolutions per minute. There is a wood-lined brake on the

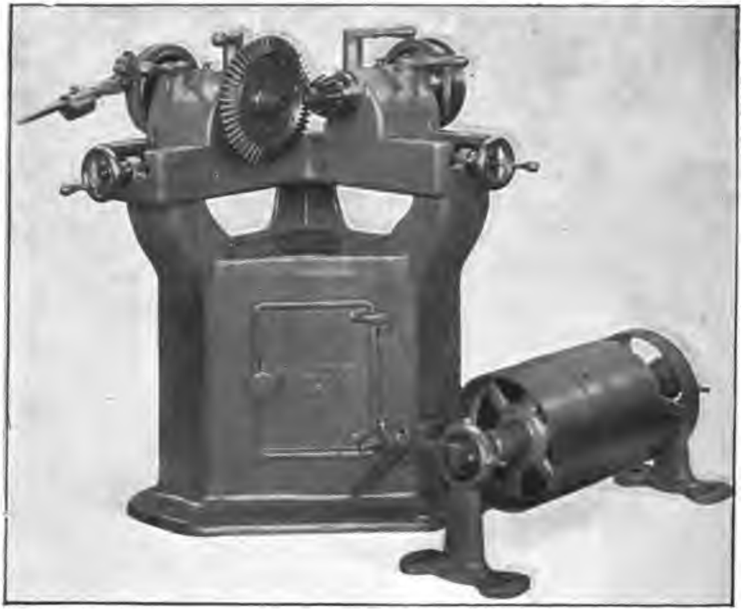


Fig. 364.—Gleason Testing Machine for Bevel Gears.

spindle of each driven head, and a lever to apply varying pressure as desired, in order that the gears may be tested for running under load.

Noise of Gears.—With respect to the noise made by gears, the experiments made by Sir Charles A. Parsons demonstrated that this is due to slight inaccuracies in the teeth, and that these arise from the worm gear, by which the blank is rotated. A microphone oscillograph record was obtained from a double-helical gear by suspending over the gear case a microphone connected with an oscillograph. Definite notes were produced and the frequency was found to be 160 times the number of revolutions of the wheel which had 160 teeth. The inaccuracies of the gear being cut were measured and found to be co-periodic with the worm-wheel teeth. To obtain a higher degree of accuracy in the teeth it was seen to be necessary either to increase the accuracy of the master gear, or to devise means of cutting which should not reproduce the errors and avoid periodicity in the residual errors. Sir C. Parsons arrived at the conclusion that since the driving worm wheel is attached permanently to the work-table, the errors produced will be some function of the angular position of the work, and will therefore lie in planes through the axis of rotation, and these planes may be expected to lie at equal angular intervals. If, therefore, the work is given a small, steady advance in relation to the table, the errors, instead of lying in planes through the axis, will lie in spirals around the wheel, which when put to work will be obliterated and leave a true wheel. To produce this result a secondary table was mounted on the original table and given a creep in advance of 1 per cent. in relation to it by means of a train of gearing. Actually the lower table was driven at 1 per cent. less speed to retain the correct rotational speed for the upper table. In this way the errors were reduced to about one-fifth of their original magnitude.

CHAPTER XVIII

THE STRENGTH OF GEARS

The Wilfred Lewis formulæ for the strength of gear-wheel teeth have been generally welcomed and utilized since their introduction in 1893. Some occasional criticisms and suggestions bearing thereon appear from time to time, but nothing of material importance has been offered to challenge the data as a whole. The general formula takes account of the variables which occur in gear-wheel transmissions. The formula is:—

$$W = s\,p\,f\,y.$$

Where

W=the working load in pounds.

s=the fibre stress.

p=the circular pitch.

f=the width of face in inches.

y=a factor of strength obtained by calculation, or embodied in a table.

The table on p. 358 gives the factors y for gears with different numbers of teeth. The width across the root is taken instead of the width at the pitch line. Three sets of values are given respectively for the common involute of 15° of obliquity, the involute of 20°, and for radial flanks. Another table (p. 357) gives the safe working stresses s for different speeds. The latter was based on experiences in gears used in machine-tool design. The tables are employed for bevel gears by an approximate formula:—

$$W = s\,p\,f\,y\,\frac{d}{D}$$

Where d=the small diameter of the bevel gear, D=the large diameter.

Mr Lewis assumed that in good machinery the load may be safely taken as being distributed across the whole width of the tooth, instead of being regarded as concentrated on one corner, as many writers have assumed it to be. This is often the case when gears are cast with taper, but not when they are cut accurately. On the other hand he did not consider that the load might be reckoned as divided between two teeth, but as concentrated on one. Treating a single tooth as a beam sustaining the whole of the load, he deduced the general formula given, after drawing various sizes of involute gears of 15° and 20° pressure angle, and also cycloidal gears, and taking the widths at the roots instead of at the pitch line. With regard to the factor of safety, he thought that for slow gearing, where no serious results can ensue from breakage, a factor as low as 3 might be used, but that for high speeds, and where great risks demand absolute security, a factor of 12 or even 15 should be adopted.

Some interesting facts were collected during the preliminary researches undertaken by Mr Lewis. He mentioned that a Mr Cooper had collected forty-eight well-established rules for horse-power and working strength, sanctioned by some twenty-four authorities, and differing from each other in extreme cases by about 500 per cent. No attempt was made in any formula to include the working stress, and no account was taken of the actual tooth forms, that is of the width at the roots, but only of the thickness at the pitch line. Mr Cooper selected a formula which he deemed passably correct:—

$$x=2000\ p\ f.$$

Where

x=breaking load of tooth in pounds.

p=pitch of teeth in inches.

f=face of teeth in inches.

WILFRED LEWIS TABLE FOR STRENGTH OF GEARS

SAFE WORKING STRESSES, s, FOR DIFFERENT SPEEDS

Speed of teeth .	100 or less	200	300	600	900	1,200	1,80	2,400
Cast iron . .	8,000	6,000	4,800	4,000	3,000	2,400	2,000	1,700
Steel . . .	20,000	15,000	12,000	10,000	7,500	6,000	5,000	4,300

FACTOR FOR STRENGTH, y

No. of Teeth.	Involute 20° Obliquity.	15° Involute and Cycloidal.	Radial Flanks.	No. of Teeth.	Involute 20° Obliquity.	15° Involute and Cycloidal.	Radial Flanks.
12	·078	·067	·052	27	·111	·100	·064
13	·083	·070	·053	30	·114	·102	·065
14	·088	·072	·054	34	·118	·104	·066
15	·092	·075	·055	38	·122	·107	·067
16	·094	·077	·056	43	·126	·110	·068
17	·096	·080	·057	50	·130	·112	·069
18	·098	·083	·058	60	·134	·114	·070
19	·100	·087	·059	75	·138	·116	·071
20	·102	·090	·060	100	·142	·118	·072
21	·104	·092	·061	150	·146	·120	·073
23	·106	·094	·062	300	·150	·122	·074
25	·108	·097	·063	Rack	·154	·124	·075

With regard to the influence of the width of tooth on its strength, it is now considered generally preferable to increase strength by increasing width in preference to employing coarse pitches. Fine pitches are preferred for several reasons, chiefly in order to keep the angle of pressure low, and to increase the number of teeth in gear. Increasing the width of face lessens the pressure per unit area of surface without increasing the friction which is proportionate to total pressure. The friction per unit area is therefore greater with narrow teeth than with wide ones.

The old objection to wide teeth when teeth were mostly cast was the inaccuracy due to taper, rapping and moulding, which often caused teeth to have contact only at one corner until they wore themselves to a continuous bearing. This consideration now has relatively little weight because of the greatly extended employment of good cut gears, in which uniform contact can be relied on. So that when desirable the width of face is made wider than prudence would sanction in the case of cast gears.

The case of bevel gears stands alone. The width cannot be increased much beyond the common standard without causing difficulties in the use of standard rotary cutters or of planing tools. Besides which the teeth become very weak at the small end.

The Gleason Works, of Rochester, N.Y., estimate the strength of gears as follows:—

Premising that the load a gear will carry depends on its finish, the quality of the material, the shape of the tooth, whether the load comes on one tooth or two, the velocity, the supports for the gear, and the steadiness of the load: they assume for a standard rule the case of cast-iron gears of 18 teeth or more, rigidly supported for an average steady load. The safe working stress for such gears at different speeds is:—

Speed of teeth in feet per minute .	500	1000	1500	2000	2500
Pounds per square inch of section at pitch line	480	400	340	290	240

To find approximate h.p. for cut iron gears over 1in. pitch, multiply the section of tooth at pitch line by its velocity and by its stress as above, and divide by 33,000.

Example for iron spur gear of 40 T, 3in. P, 10in. face at 100 R.P.M.

$$\frac{\overset{\text{Section}}{(1\tfrac{1}{2}\text{in.} \times 10\text{in.})} \times \overset{\text{Velocity in feet}}{(40\text{T} \times 3\text{in. P} \times 100 \text{ R.P.M.} \times 1\tfrac{1}{2}\text{in.})} \times \overset{\text{Stress}}{400\text{lb.}}}{33{,}000 \text{ lb.}} = 182 \text{ h.p.}$$

Hammering loads like those of rolling mills or saw mills require heavier gears than the above gives.

The Fellows Co. calculate the strength of a gear when at rest as follows:—Divide the strength factor for the nearest number of teeth in the Table No. 1 (p. 360) by the diametral pitch; multiply together this result with the safe stress of the material in pounds per square inch, and by the width of the face of the gear in inches. The product gives the allowable load in pounds at the pitch line when the gear is still.

An increase of speed weakens the gear by an amount which varies with different materials. The usual rule is to multiply the load at pitch line by the speed factor given opposite the nearest pitch line velocity in Table No. 2 (p. 360). This gives the safe load at the pitch line for the given speed.

Example.—Find the strength of a 14 tooth, 6 pitch, 14½° pinion, 1¼in. face, running at a velocity of 1200 feet per minute at the pitch line, and made of a material which will safely sustain a dead load of 60,000lb. per square inch: $\frac{.237}{6} \times 60{,}000 \times 1\tfrac{1}{4} = 2962\text{lb.}$ = safe load when at rest. $2962 \times .30 = 888.6\text{lb.}$ = safe load at pitch line when running at a pitch line velocity of 1200 feet per minute.

The stub tooth gear is somewhat stronger, particularly for pinions of a small number of teeth. In the diagram, Fig. 365, the lower line

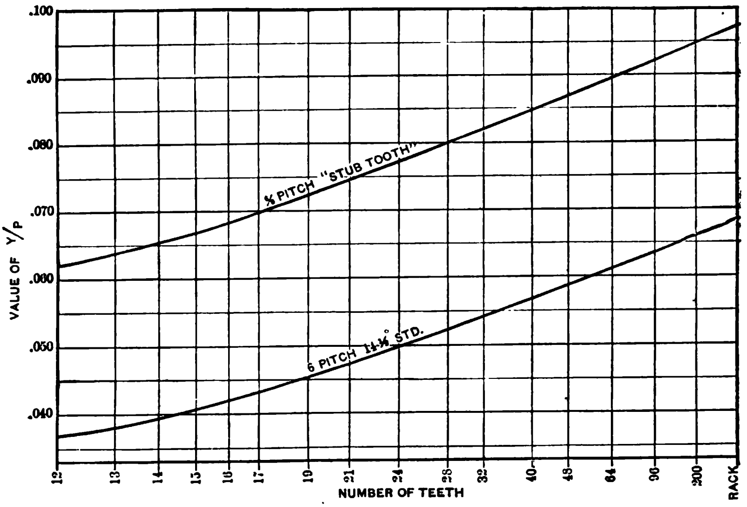


Fig. 365.

shows the values of the strength factor for an ordinary 6 pitch gear, as found by dividing the factors in Table No. 1 by 6. The upper line shows the corresponding values for a $\frac{6}{8}$ pitch Stub tooth gear.

TABLE No. 1

STRENGTH FACTORS (Y) FOR GEAR SHAPER GEARS 14½ DEGREE STD.

No. of Teeth.	Y.	No. of Teeth.	Y.
12	·222	28	·314
13	·229	32	·325
14	·237	40	·340
15	·245	48	·352
16	·254	64	·366
17	·263	96	·381
19	·275	200	·397
21	·286	Rack	·410
24	·300		

TABLE No. 2

VELOCITY FACTORS BASED ON MR LEWIS' CALCULATIONS

Ft. per Min.	Factor.	Ft. per Min.	Factor.
100 or less	1·00	900	·37
200	·75	1200	·30
300	·60	1800	·25
600	·50	2400	·22

The strength of gears is obtained by the gear-slide rule invented by Carl G. Barth, illustrated in Fig. 366. It utilizes the formula of

Wilfred Lewis (as modified by Mr Barth) as follows:—The proper working load on a spur gear is

$$W = \frac{600}{600 + v} \; s \, p \, f \, y,$$

in which

v=circumferential velocity of gear in feet per minute.

s=permissible stress on the material at the given speed.

p=circular pitch of gear.

f=face of gear.

y=strength factor found in Lewis's tables for a gear of the number of teeth and of the system under consideration.

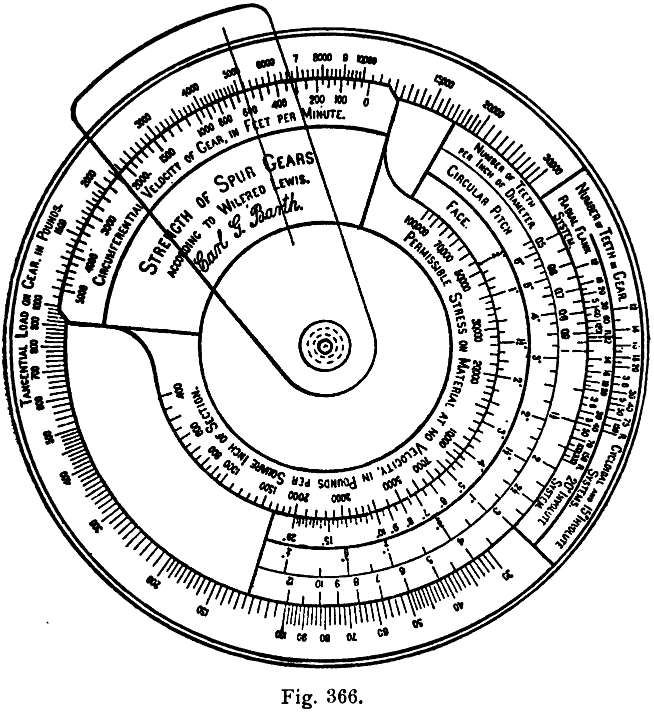

Strength of Spur Gears according to Wilfred Lewis.
Carl G. Barth.
Circumferential Velocity of Gear, in Feet per Minute.
Tangential Load on Gear, in Pounds.
Number of Teeth per Inch of Diameter
Circular Pitch
Face.
Permissible Stress on Material at No Velocity, in Pounds per Square Inch of Section.
Number of Teeth in Gear.
Radial Flank System.
20° Involute System.
Cycloidal and 15° Involute Systems.

Fig. 366.

Directions for Use.—First, in the proper system, on the bottom disc, set the rider to coincide with the number of teeth of the gear, the strength of which is to be found. Then set the intermediate disc (manipulated through the finger slot in the bottom disc) so that the pitch of the gear—diametral or circular, as the case may be—will coincide with the rider thus set, and then tighten the clamping nut at the centre. Next, set the rider to coincide with the face of the gear on the intermediate disc, and then loosen the clamping nut and set the top disc so that the permissible static stress on the material in the gear will coincide with the rider in this new position, and again tighten the clamping nut.

Finally, set the rider to coincide with the circumferential velocity of the gear on the top disc, and then read off the corresponding proper working load for the gear where the reading on the scale of loads on the bottom disc coincides with the rider in this last position.

Example.—What will be a proper working load for a spur gear of the 20° involute system, when it has 40 teeth, 1½ per in. of diameter, 5-in. face, and is made of cast iron the permissible static stress of which is 8000lb., and run at a circumferential velocity of 600ft. per minute. This is the example for which the instrument is set in the accompanying illustration, which shows a load of 5200lb.

Proportions for Arms

(Fig. 367)

Let P = circular pitch.
d.p. = diametral pitch.
A = breadth of face.
B = diameter of boss.
C = thickness of rim.
D = thickness of rib next rim.

Let E = thickness of rib next boss.
F = thickness of flat arm.
G = thickness of vertical rib.
H = width of arm next rim.
b = bore in inches.

P = Circular Pitch

Form of Arms.	A	B	C	D	E	F	G	H
Disc	3 P	$1\frac{7}{8}$ *b*	$\frac{1}{2}$ P	$\frac{9}{16}$ P	$\frac{5}{8}$ P	$\frac{1}{2}$ P	$\frac{1}{3}$ P	$2\frac{1}{8}$ P
O	"	"	"	"	"	"	"	"
T	"	"	"	"	"	"	"	"
I	"	"	"	"	"	"	"	"
+	"	"	"	"	"	"	"	"

d.p. = DIAMETRAL PITCH

Form of Arms.	A	B	C	D	E	F	G	H
Disc	9·5 d.p.	1⅞ *b*	1·5 d.p.	1·75 d.p.	2 d.p.	1·6 d.p.	1 d.p.	7 d.p.
O	,,	,,	,,	,,	,,	½ H	6·7 d.p.	,,
T	,,	,,	,,	,,	,,	,,	,,	,,
I	,,	,,	,,	,,	,,	,,	,,	,,
+	,,	,,	,,	,,	,,	,,	,,	,,

Conclusion.—It is clear from our study of gears that there can be no short cut to correct tooth design which will prove other than a snare if it does not take full account of fundamental principles of formation. Such short cuts have been given in the older books, which cannot be applied to an interchangeable set of gears. Any time-saving method must be based on one of the two tooth types which are universally accepted in workshops, and for which all cutters are made commercially. A slight departure from strict accuracy may be an advantage when it is on the side of clearance. Approximate methods of this character are not uncommon. But in most instances approximations are obtained by means of the odontograph scales, which give averages between a few limiting sizes of teeth. That is, so many teeth would be taken—fewer as the numbers are those of pinions, more as they are those of wheels;

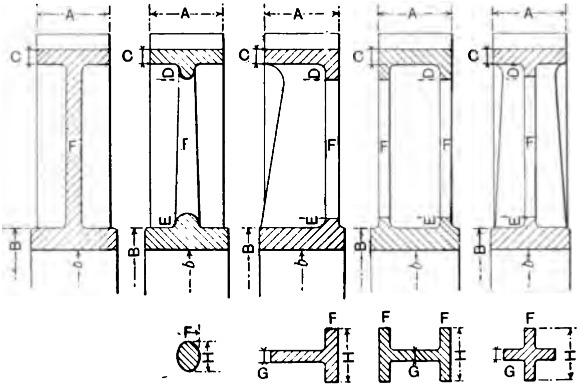


Fig. 367.

the curves for a tooth the mean of these are used for them all. The rapid growth of generating machines during recent years will in time react on all gear cutting. Designed in the first place to produce perfect bevels, they may be expected to extend to include all wheels, and to generate all standard tooth templets. They will then probably oust the odontograph scales, both instruments and tables, and remain the single source of evolution of all tooth shapes from a predetermined base. They will probably be the means in this and in other lesser and related ways of causing a great revolution in the entire practice of gear cutting.

Looking at the growing specialization of the machines now available for cutting gears, it stands to reason that as a secondary consideration the restriction of tasks of a given kind to one kind of machine must tend not only to prolong the durability of the machine, but to preserve its accuracy in a greater degree than that of a machine which is used for universal purposes. Specialization in manufacture has also a great tendency to produce a high degree of accuracy in machine construction, without rendering prices in the least degree prohibitive.

The very rapid developments of new and ever-modified forms of gear-cutting machines in recent years, while creating a sense of dissatisfaction with the old, tend to render the question of interchangeability in gearing a more complex one than it was when the choice of machines and methods was more limited. Gears cut on one kind of machine cannot be relied on to engage with those produced on one of another kind, even though the tooth proportions are identical. Again, the difference in the cost of cutting in different systems is a very material one, and that again is influenced not only by the kind of machine used, but also by the coarseness or fineness of pitch.

Further, the use of rigid standard cutters and the employment of cut gears without side clearance has already had the effect of developing a closer study of the conditions which govern the perfect mutual action of engaged teeth. There is little chance to fudge, as the old millwrights were able to do. Teeth must either be right or wrong, within a very minute degree, and the study of practicable curves has engaged the labours of some of the best machinists with excellent results. We see the effects of these things in the high quality of the gear cutting done now under the best shop systems, notwithstanding that the rate of cutting is fairly high, not, of course, by forcing cutters unduly, but by duplication of the work or of the cutters, and rapid automatic changes. Wheel teeth are pitched truly, cut truly, and with excellent finish. And hundreds of wheels will be all alike in every respect, because the forms of cutters do not change by regrinding, and the mechanism of the machine remains practically unaffected during many years.

Finally, the problems which deal with the forms of the teeth and the proportions of the wheels are seen to be essentially of a practical character. In almost any wheel the shapes imparted are not exactly those which would be deduced from strictly theoretical considerations. They might not even be those which would seem best for a particular pair of wheels, regarded alone. For almost every wheel has to be considered in regard to many others. The unit is related intimately to the system—to many other units—and is affected by methods of manufacture. Simplicity, interchangeabilty, and workshop and commercial conditions must be regarded, and the result is a compromise. This is the reason why, of so many suggested tooth forms, so few have survived in practice.

APPENDIX

TABLE OF TOOTH PARTS FOR DIAMETRAL PITCHES

Diametral Pitch.	Circular Pitch.	Thickness of Tooth on Pitch Line.	Depth to be Cut in Gear.	Addendum.
1	3·1416	1·5708	2·1571	1·0000
1¼	2·5133	1·2566	1·7257	·8000
1½	2·0944	1·0472	1·4381	·6666
1¾	1·7952	·8976	1·2326	·5714
2	1·5708	·7854	1·0785	·5000
2¼	1·3963	·6981	·9587	·4444
2½	1·2566	·6283	·8628	·4000
2¾	1·1424	·5712	·7844	·3636
3	1·0472	·5236	·7190	·3333
3½	·8976	·4488	·6163	·2857
4	·7854	·3927	·5393	·2500
5	·6283	·3142	·4314	·2000
6	·5236	·2618	·3595	·1666
7	·4488	·2244	·3081	·1429
8	·3927	·1963	·2696	·1250
9	·3491	·1745	·2397	·1111
10	·3142	·1571	·2157	·1000
11	·2856	·1428	·1961	·0909
12	·2618	·1309	·1798	·0833
14	·2244	·1122	·1541	·0714
16	·1963	·0982	·1348	·0625
18	·1745	·0873	·1198	.0555
20	·1571	·0785	·1079	·0500
22	·1428	·0714	·0980	·0455
24	·1309	·0654	·0898	·0417
26	·1208	·0604	·0829	·0385
28	·1122	·0561	·0770	·0357
30	·1047	·0524	·0719	·0333
32	·0982	·0491	·0674	·0312
36	·0873	·0436	·0599	·0278
40	·0785	·0393	·0539	·0250
48	·0654	·0327	·0449	·0208

TABLE OF TOOTH PARTS FOR CIRCULAR PITCHES

Circular Pitch.	Diametral Pitch.	Thickness of Tooth on Pitch Line.	Depth to be Cut in Gear.	Addendum.
Ins. 3	1·0472	1·5000	2·0598	·9550
2¾	1·1424	1·3750	1·8882	·8754
2½	1·2566	1·2500	1·7165	·7958
2¼	1·3963	1·1250	1·5449	·7162
2	1·5708	1·0000	1·3732	·6366
1⅞	1·6755	·9375	1·2874	·5968
1¾	1·7952	·8750	1·2016	·5570
1⅝	1·9333	·8125	1·1158	·5173
1½	2·0944	·7500	1·0299	·4775
1⅜	2·2848	·6875	·9441	·4377
1¼	2·5133	·6250	·8583	·3979
1⅛	2·7925	·5625	·7724	·3581
1	3·1416	·5000	·6866	·3183
15/16	3·3510	·4687	·6437	·2984
⅞	3·5904	·4375	·6007	·2785
13/16	3·8666	·4062	·5579	·2586
¾	4·1888	·3750	·5150	·2387
11/16	4·5696	·3437	·4720	·2189
⅝	5·0265	·3125	·4291	·1989
9/16	5·5851	·2812	·3862	·1790
½	6·2832	·2500	·3433	·1592
7/16	7·1808	·2187	·3003	·1393
⅜	8·3776	·1875	·2575	·1194
5/16	10·0531	·1562	·2146	·0995
¼	12·5664	·1250	·1716	·0796
⅛	25·1327	·0625	·0858	·0398
1/16	50·2655	·0312	·0429	·0199

RULES FOR OBTAINING DIAMETERS OF SPIRAL GEARS

(*David Brown & Sons* (*Huddersfield*), *Limited*)

Formula: Normal Diam. Pitch × Cos a = Circular Diam. Pitch.

Normal Diam. Pitch.	Circular Diam. Pitch Angles.			
	45°	20°	26° 35′	63° 25′
20	14·142	18·794	17·886	8·95
18	12·728	16·915	16·097	8·055
16	11·314	15·035	14·309	7·160
14	9·899	13·156	12·520	6·265
12	8·485	11·276	10·732	5·370
11	7·778	10·337	9·837	4·923
10	7·071	9·397	8·943	4·475
9	6·364	8·457	8·049	4·028
8	5·657	7·518	7·154	3·580
7	4·950	6·578	6·260	3·133
6	4·243	5·638	5·366	2·685
5	3·536	4·699	4·472	2·238
4	2·828	3·759	3·577	1·790
3	2·121	2·819	2·683	1·343
2½	1·768	2·349	2·236	1·119
2	1·414	1·879	1·789	·895
1¾	1·237	1·644	1·565	·783
1½	1·061	1·410	1·341	·671

$$\frac{\text{Number of Teeth}}{\text{Circul. Diam. Pitch}} = \text{Pitch Diameter.}$$

Pitch Diameter + $\frac{2}{\text{Normal Diam. Pitch}}$ = Whole Diameter of the blanks.

Example.—To find the pitch diam. and the whole diameter for a wheel with 50 teeth, if the Normal Diam. Pitch = 4 and the angle of the spiral = 45°.

$$\frac{50}{2{\cdot}828} = 17{\cdot}68'' = \text{Pitch Diameter.}$$

$$17{\cdot}68 + \tfrac{2}{4} = 18{\cdot}18'' = \text{Whole Diameter of the blank.}$$

RULES FOR OBTAINING DIAMETERS OF SPIRAL GEARS

(*David Brown & Sons* (*Huddersfield*), *Limited*)

Formula: Normal Pitch ÷ Cos. a = Circular Pitch.

Normal Pitch.		Circular Pitch Angles.			
Inches.	Divid.	45°	20°	26° 35′	63° 25′
¼	·08	·3536	·2660	·2795	·5587
⅜	·119	·5303	·3991	·4193	·8380
½	·159	·7071	·5321	·5591	1·1173
⅝	·199	·8839	·6651	·6989	1·3966
¾	·239	1·0607	·7981	·8386	1·6760
⅞	·279	1·2374	·9311	·9784	1·9553
1	·318	1·4142	1·0642	1·1182	2·2346
1⅛	·358	1·5910	1·1972	1·2580	2·5140
1¼	·398	1·7678	1·3302	1·3977	2·7933
1⅜	·438	1·9446	1·4632	1·5375	3·0726
1½	·477	2·1213	1·5963	1·6773	3·3519
1⅝	·517	2·2981	1·7293	1·8171	3·6313
1¾	·557	2·4749	1·8623	1·9568	3·9106
1⅞	·597	2·6517	1·9953	2·0966	4·1899
2	·637	2·8285	2·1283	2·2364	4·4693
2¼	·716	3·1820	2·3944	2·5160	5·0279
2	·796	3·5356	2·6604	2·7955	5·5866
2¾	·876	3·8891	2·9265	3·0751	6·1453
3	·955	4·2427	3·1925	3·3546	6·7039

$$\frac{\text{Number of Teeth} \times \text{Circular Pitch}}{3{\cdot}1416} = \text{Pitch Diameter.}$$

$$\text{Pitch Diameter} + 2 \times \frac{\text{Normal Pitch}}{3{\cdot}1416} = \text{Whole Diameter.}$$

If a pair of wheels have a ratio of 1 : 2

The Spiral of the wheel with the larger number of teeth is 26·35′.

,, ,, smaller ,, 63·25′.

Taking the above Spirals we find that the diameter of each wheel is nearly the same.

Example.—To find the pitch diameter, centre distance and whole diameter of the blanks for a pair of Spiral Wheels, as follows:—

(1) 40 Teeth, Normal Pitch ¾″, Spiral Angle 26° 35′; and

(2) 20 ,, ,, ¾″, ,, 63° 25′.

$$(1)\quad \frac{40 \times 0{\cdot}8386}{3{\cdot}1416} = 10{\cdot}67 = \text{Pitch Diameter.}$$

$$(2)\quad \frac{20 \times 1{\cdot}6760}{3{\cdot}1416} = 10{\cdot}67 = \text{Pitch Diameter.}$$

Centre Distance = 10·67 inches.

Whole Diam. of the blanks = 10·67 + 2 × 0·239 = 11·14 inches.

TABLE OF SPIRAL GEARS—DIAMETRAL PITCHES

(*Garvin Machine Company*)

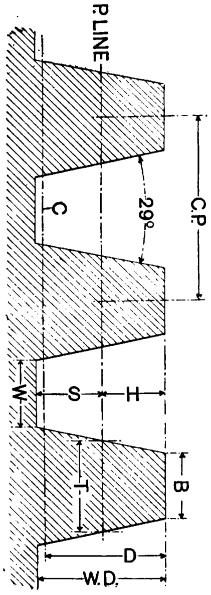
Diametral Pitch of Cutter.	Depth to be cut in Gear.	Angle—15°	Angle—30°	Angle—45°
		Corresponding Diametral Pitch of Spiral Gear.	Corresponding Diametral Pitch of Spiral Gear.	Corresponding Diametral Pitch of Spiral Gear.
24	·090 in.	·0431 in.	·0481 in.	·0589 in.
22	·098 ,,	·0470 ,,	·0525 ,,	·0643 ,,
20	·108 ,,	·0517 ,,	·0577 ,,	·0707 ,,
18	·120 ,,	·0575 ,,	·0641 ,,	·0785 ,,
16	·135 ,,	·0647 ,,	·0721 ,,	·0883 ,,
14	·154 ,,	·0739 ,,	·0824 ,,	·1010 ,,
12	·180 ,,	·0862 ,,	·0962 ,,	·1178 ,,
11	·196 ,,	·0941 ,,	·1049 ,,	·1285 ,,
10	·216 ,,	·1035 ,,	·1154 ,,	·1414 ,,
9	·240 ,,	·1150 ,,	·1283 ,,	·1571 ,,
8	·270 ,,	·1294 ,,	·1443 ,,	·1767 ,,
7	·308 ,,	·1479 ,,	·1649 ,,	·2020 ,,
6	·359 ,,	·1725 ,,	·1924 ,,	·2357 ,,
5	·431 ,,	·2070 ,,	·2309 ,,	·2828 ,,
4	·539 ,,	·2588 ,,	·2886 ,,	·3535 ,,
3	·719 ,,	·3451 ,,	·3849 ,,	·4714 ,,
2	1·078 ,,	·5176 ,,	·5773 ,,	·7071 ,,

WORMS AND WORM GEARING

(Bardons & Oliver)

C. P. Circular Pitch.	**pi.** Threads per Inch.	**D. P.** Diametrical Pitch.	**H.** Tooth above Pitch Line.	**D.** Working Depth of Tooth.	**C.** Clearance.	**S.** Depth of Space below Pitch Line.	**W. D.** Whole Depth of Tooth.	**T.** Thickness of Tooth on Pitch Line.	**W.** Width of Thread Tool at End.	**B.** Width of Thread at Top.
C. P. Inches.	$pi. = \frac{1}{C.P.}$	$D.\ P. = \frac{\pi}{C.\ P}$	$H. = \frac{1}{D.P.}$	$D. = 2 \times \frac{1}{D.P}$	$C. = \frac{1}{8} \times \frac{1}{D.P}$	S. = H. + C.	W. D. = D. + C.	$T. = \frac{C.P.}{2}$	W. = .3148 X C. P.	B. = .3354 X C. P.
2	1-2	1.5708	.6366	1.2732	.0795	.7161	1.3527	1.0000	.6296	.6708
1¾	4-7	1 7952	.5570	1.1141	.0696	.6266	1.1837	.8750	.5509	.5869
1½	2-3	2.0944	.4775	.9549	.0596	.5371	1.0145	.7500	.4722	.5031
1¼	4-5	2.5133	.3979	.7958	.0497	.4476	.8455	.6250	.3935	.4192
1	1	3.1416	.3183	.6366	.0397	.3580	.6763	.5000	.3148	.3354
3-4	1⅓	4.1888	.2387	.4775	.0298	.2685	.5073	.3750	.2361	.2515
2-3	1½	4.7124	.2122	.4244	.0265	.2387	.4509	.3333	.2098	.2236
1-2	2	6.2832	.1592	.3183	.0199	.1791	.3382	.2500	.1574	.1677
2-5	2½	7.8540	.1273	.2546	.0159	.1432	.2705	.2000	.1259	.1341
1-3	3	9.4248	.1061	.2122	.0132	.1193	.2254	.1666	.1049	.1118
2 7	3½	10.9956	.0909	.1819	.0113	.1022	.1932	.1429	.0899	.0958
1-4	4	12.5664	.0796	.1591	.0099	.0895	.1690	.1250	.0787	.0838
2-9	4½	14.1372	.0707	.1415	.0088	.0795	.1503	.1111	.0699	.0745
1-5	5	15.7080	.0637	.1273	.0079	.0716	.1352	.1000	.0629	.0670
1-6	6	18.8496	.0531	.1061	.0066	.0597	.1127	.0833	.0524	.0559
1-7	7	21.9911	.0455	.0910	.0056	.0511	.0966	.0714	.0449	.0479
1-8	8	25.1327	.0398	.0796	.0049	.0447	.0845	.0625	.0393	.0419
1-9	9	28.2743	.0354	.0707	.0044	.0398	.0752	.0555	.0349	.0372
1-10	10	31.4159	.0318	.0637	.0039	.0357	.0676	.0500	.0314	.0335
1-12	12	37.6992	.0265	.0530	.0033	.0298	.0563	.0416	.0262	.0279
1-14	14	43.9824	.0227	.0454	.0028	.0255	.0482	.0357	.0224	.0239
1-16	16	50.2655	.0199	.0398	.0024	.0223	.0422	.0312	.0196	.0209
1-18	18	56.5488	.0176	.0352	.0022	.0198	.0374	.0277	.0174	.0186

NOTE.—The above table refers to single threads only. For Multiple Threads, divide the sizes given in the table for the same pitch, by 2 for Double, 3 for Triple, 4 for Quadruple Threads, etc,



PITCH ANGLES OF BEVEL GEARS

(Brown & Sharpe Manufacturing Co.)

WHEEL.

PINION.

	72	71	70	69	68	67	66	65	64	63	62	61	60	59	58	57
12	80°33′ 9°27′	80°25′ 9°35′	80°16′ 9°44′	80°8′ 9°52′	79°59′ 10°1′	79°51′ 10°9′	79°42′ 10°18′	79°32′ 10°28′	79°23′ 10°37′	79°13′ 10°47′	79°3′ 10°57′	78°52′ 11°8′	78°41′ 11°19′	78°30′ 11°30′	78°19′ 11°41′	78°7′ 11°53′
13	79°46′ 10°14′	79°37′ 10°23′	79°29′ 10°31′	79°20′ 10°40′	79°11′ 10°49′	79°1′ 10°59′	78°51′ 11°9′	78°41′ 11°19′	78°31′ 11°29′	78°20′ 11°40′	78°9′ 11°51′	77°58′ 12°2′	77°46′ 12°14′	77°34′ 12°26′	77°22′ 12°38′	77°9′ 12°51′
14	79°0′ 11°0′	78°51′ 11°9′	78°41′ 11°19′	78°32′ 11°28′	78°22′ 11°38′	78°11′ 11°49′	78°1′ 11°59′	77°51′ 12°9′	77°40′ 12°20′	77°28′ 12°32′	77°17′ 12°43′	77°5′ 12°55′	76°52′ 13°8′	76°39′ 13°21′	76°26′ 13°34′	76°12′ 13°48′
15	78°14′ 11°46′	78°4′ 11°56′	77°54′ 12°6′	77°44′ 12°16′	77°34′ 12°26′	77°23′ 12°37′	77°12′ 12°48′	77°0′ 13°0′	76°48′ 13°12′	76°36′ 13°24′	76°24′ 13°36′	76°11′ 13°49′	75°58′ 14°2′	75°44′ 14°16′	75°30′ 14°30′	75°15′ 14°45′
16	77°28 12°32′	77°18′ 12°42′	77°7′ 12°53′	76°57′ 13°3′	76°45′ 13°15′	76°34′ 13°26′	76°22′ 13°38′	76°10′ 13°50′	75°58′ 14°2′	75°45′ 14°15′	75°32′ 14°28′	75°18′ 14°42′	75°4′ 14°56′	74°49′ 15°11′	74°35′ 15°25′	74°19′ 15°41′
17	76°43′ 13°17′	76°32′ 13°28′	76°21′ 13°39′	76°10′ 13°50′	75°58′ 14°2′	75°45′ 14°14′	75°33′ 14°27′	75°21′ 14°39′	75°8′ 14°52′	74°54′ 15°6′	74°40′ 15°20′	74°25′ 15°35′	74°11′ 15°49′	73°56′ 16°4′	73°40′ 16°20′	73°24′ 16°36′
18	75°58′ 14°2′	75°46′ 14°14′	75°35′ 14°25′	75°23′ 14°37′	75°10′ 14°50′	74°58′ 15°2′	74°45′ 15°15′	74°31′ 15°29′	74°17′ 15°43′	74°3′ 15°57′	73°49′ 16°11′	73°33′ 16°27′	73°18′ 16°42′	73°2′ 16°58′	72°45′ 17°15′	72°29′ 17°31′
19	75°13′ 14°47′	75°1′ 14°59′	74°49′ 15°11′	74°36′ 15°24′	74°23′ 15°37′	74°10′ 15°50′	73°56′ 16°4′	73°42′ 16°18′	73°28′ 16°32′	73°13′ 16°47′	72°58′ 17°2′	72°42′ 17°18′	72°20′ 17°34′	72°9′ 17°51′	71°52′ 18°8′	71°34′ 18°26′
20	74°29′ 15°31′	74°16′ 15°44′	74°3′ 15°57′	73°50′ 16°10′	73°37′ 16°23′	73°23′ 16°42′	73°9′ 16°51′	72°54′ 17°6′	72°39′ 17°21′	72°23′ 17°37′	72°7′ 17°53′	71°51′ 18°9′	71°34′ 18°26′	71°16′ 18°44′	70°59′ 19°1′	70°40′ 19°20′
21	73°45′ 16°15′	73°32′ 16°28′	73°18′ 16°42′	73°4′ 16°56′	72°50′ 17°10′	72°36′ 17°24′	72°21′ 17°39′	72°6′ 17°54′	71°50′ 18°10′	71°34′ 18°26′	71°17′ 18°43′	71°0′ 19°0′	70°43′ 19°17′	70°24′ 19°36′	70°6′ 19°54′	69°46′ 20°14′
22	73°1′ 16°59′	72°47′ 17°13′	72°33′ 17°27′	72°19′ 17°41′	72°4′ 17°56′	71°49′ 18°11′	71°34′ 18°26′	71°18′ 18°42′	71°2′ 18°58′	70°45′ 19°15′	70°28′ 19°34′	70°10′ 19°50′	69°52′ 20°8′	69°33′ 20°27′	69°13′ 20°47′	68°54′ 21°6′
23	72°17′ 17°43′	72°3′ 17°57′	71°49′ 18°11′	71°34′ 18°26′	71°19′ 18°41′	71°3′ 18°57′	70°47′ 19°13′	70°30′ 19°30′	70°14′ 19°46′	69°57′ 20°3′	69°39′ 20°21′	69°20′ 20°40′	69°2′ 20°58′	68°42′ 21°18′	68°22′ 21°38′	68°2′ 21°58′
24	71°34′ 18°26′	71°19′ 18°41′	71°5′ 18°55′	70°49′ 19°11′	70°34′ 19°26′	70°17′ 19°43′	70°1′ 19°59′	69°44′ 20°16′	69°26′ 20°34′	69°9′ 20°51′	68°50′ 21°10′	68°31′ 21°29′	68°12′ 21°48′	67°52′ 22°8′	67°31′ 22°29′	67°10′ 22°50′
25	70°51′ 19°9′	70°36′ 19°24′	70°21′ 19°39′	70°5′ 19°55′	69°49′ 20°11′	69°32′ 20°28′	69°15′ 20°45′	68°57′ 21°3′	68°40′ 21°20′	68°21′ 21°39′	68°3′ 21°57′	67°43′ 22°17′	67°23′ 22°37′	67°2′ 22°58′	66°41′ 23°19′	66°19′ 23°41′
26	70°9′ 19°51′	69°53′ 20°7′	69°37′ 20°23′	69°21′ 20°39′	69°4′ 20°56′	68°48′ 21°12′	68°30′ 21°30′	68°12′ 21°48′	67°54′ 22°6′	67°34′ 22°26′	67°15′ 22°45′	66°55′ 23°5′	66°34′ 23°26′	66°13′ 23°47′	65°51′ 24°9′	65°29′ 24°31′
27	69°27′ 20°33′	69°10′ 20°50′	68°54′ 21°6′	68°38′ 21°22′	68°20′ 21°40′	68°3′ 21°57′	67°45′ 22°15′	67°26′ 22°34′	67°8′ 22°52′	66°48′ 23°12′	66°28′ 23°32′	66°7′, 23°53′	65°46′ 24°14′	65°25′ 24°35′	65°2′ 24°58′	64°39′ 25°21′
28	68°45′ 21°15′	68°29′ 21°31′	68°12′ 21°48′	67°55′ 22°5′	67°37′ 22°23′	67°19′ 22°41′	67°1′ 22°59′	66°42′ 23°18′	66°22′ 23°38′	66°2′ 23°58′	65°42′ 24°18′	65°21′ 24°39′	64°59′ 25°1′	64°37′ 25°23′	64°14′ 25°46′	63°50′ 26°10′
29	68°4′ 21°56′	67°47′ 22°13′	67°30′ 22°30′	67°12′ 22°48′	66°54′ 23°6′	66°36′ 23°24′	66°17′ 23°43′	65°57′ 24°3′	65°37′ 24°23′	65°16′ 24°44′	64°55′ 25°5′	64°34′ 25°26′	64°12′ 25°48′	63°50′ 26°10′	63°26′ 26°34′	63°2′ 26°58′
30	67°23′ 22°37′	67°6′ 22°54′	66°48′ 23°12′	66°30′ 23°30′	66°12′ 23°48′	65°52′ 24°8′	65°33′ 24°27′	65°14′ 24°46′	64°53′ 25°7′	64°32′ 25°28′	64°10′ 25°50′	63°49′ 26°11′	63°26′ 26°34′	63°3′ 26°57′	62°39′ 27°21′	62°14′ 27°46′
31	66°42′ 23°18′	66°25′ 23°35′	66°6′ 23°54′	65°48′ 24°12′	65°29′ 24°31′	65°10′ 24°50′	64°50′ 25°10′	64°30′ 25°30′	64°9′ 25°51′	63°48′ 26°12′	63°26′ 26°34′	63°3′ 26°57′	62°40′ 27°20′	62°18′ 27°42′	61°53′ 28°7′	61°28′ 28°32′
32	66°2′ 23°58′	65°44′ 24°16′	65°26′ 24°34′	65°7′ 24°53′	64°48′ 25°12′	64°28′ 25°32′	64°8′ 25°52′	63°47′ 26°13′	63°26′ 26°34′	63°4′ 26°56′	62°42′ 27°18′	62°19′ 27°41′	61°56′ 28°4′	61°32′ 28°28′	61°7′ 28°53′	60°41′ 29°19′
33	65°23′ 24°37′	65°4′ 24°56′	64°45′ 25°15′	64°26′ 25°34′	64°7′ 25°53′	63°47′ 26°13′	63°26′ 26°34′	63°5′ 26°55′	62°43′ 27°17′	62°21′ 27°39′	61°58′ 28°2′	61°35′ 28°25′	61°11′ 28°49′	60°47′ 29°13′	60°21′ 29°39′	59°56′ 30°4′
34	64°43′ 25°17′	64°25′ 25°35′	64°5′ 25°55′	63°46′ 26°14′	63°26′ 26°34′	63°5′ 26°55′	62°45′ 27°15′	62°23′ 27°37′	62°1′ 27°59′	61°38′ 28°22′	61°15′ 28°45′	60°52′ 29°8′	60°28′ 29°32′	60°3′ 29°57′	59°37′ 30°23′	59°11′ 30°49′
35	64°5′ 25°55′	63°45′ 26°15′	63°26′ 26°34′	63°6′ 26°54′	62°46′ 27°14′	62°25′ 27°35′	62°4′ 27°56′	61°42′ 28°18′	61°19′ 28°41′	60°57′ 29°3′	60°33′ 29°27′	60°9′ 29°51′	59°45′ 30°15′	59°19′ 30°41′	58°53′ 31°7′	58°27′ 31°33′
36	63°26′ 26°34′	63°7′ 26°53′	62°47′ 27°13′	62°27′ 27°33′	62°6′ 27°54′	61°45′ 28°15′	61°23′ 28°37′	61°1′ 28°59′	60°38′ 29°22′	60°15′ 29°45′	59°51′ 30°9′	59°27′ 30°33′	59°2′ 30°58′	58°37′ 31°23′	58°10′ 31°50′	57°43′ 32°17′
37	62°48′ 27°12′	62°28′ 27°32′	62°8′ 27°52′	61°48′ 28°12′	61°27′ 28°33′	61°5′ 28°55′	60°44′ 29°16′	60°21′ 29°39′	59°58′ 30°2′	59°35′ 30°25′	59°10′ 30°50′	58°46′ 31°14′	58°20′ 31°40′	57°54′ 32°6′	57°28′ 32°32′	57°1′ 32°59′
38	62°11′ 27°49′	61°51′ 28°9′	61°30′ 28°30′	61°9′ 28°51′	60°48′ 29°12′	60°26′ 29°34′	60°4′ 29°56′	59°41′ 30°19′	59°18′ 30°42′	58°54′ 31°6′	58°30′ 31°30′	58°5′ 31°55′	57°39′ 32°21′	57°13′ 32°47′	56°46′ 33°14′	56°19′ 33°41′
39	61°33′ 28°27′	61°13′ 28°47′	60°53′ 29°7′	60°31′ 29°29′	60°10′ 29°50′	59°48′ 30°12′	59°25′ 30°35′	59°2′ 30°58′	58°39′ 31°21′	58°14′ 31°46′	57°50′ 32°10′	57°24′ 32°36′	56°58′ 33°2′	56°32′ 33°28′	56°6′ 33°54′	55°37′ 34°23′
40	60°57′ 29°3′	60°36′ 29°24′	60°15′ 29°45′	59°53′ 30°7′	59°32′ 30°28′	59°10′ 30°50′	58°47′ 31°13′	58°24′ 31°36′	58°0′ 32°0′	57°35′ 32°25′	57°10′ 32°50′	56°44′ 33°16′	56°19′ 33°41′	55°52′ 34°8′	55°24′ 34°35′	54°57′ 35°3′
41	60°20′ 29°40′	60°0′ 30°0′	59°39′ 30°21′	59°17′ 30°43′	58°55′ 31°5′	58°32′ 31°28′	58°9′ 31°51′	57°45′ 32°15′	57°21′ 32°39′	56°57′ 33°3′	56°32′ 33°28′	56°6′ 33°54′	55°39′ 34°21′	55°12′ 34°48′	54°44′ 35°16′	54°16′ 35°44′
42	59°45′ 30°15′	59°24′ 30°36′	59°3′ 30°57′	58°40′ 31°20′	58°18′ 31°42′	57°55′ 32°5′	57°32′ 32°28′	57°8′ 32°52′	56°43′ 33°17′	56°19′ 33°41′	55°53′ 34°7′	55°27′ 34°33′	55°0′ 35°0′	54°33′ 35°27′	54°5′ 35°55′	53°37′ 36°23′

For bevel gears with axes at right angles.
Angle for wheel above, for pinion below.

PITCH ANGLES

WHEEL.

PINION.

	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42
12	77°54′ 12°6′	77°42′ 12°18′	77°28′ 12°32′	77°15′ 12°45′	77°0′ 13°0′	76°46′ 13°14′	76°30′ 13°30′	76°14′ 13°46′	75°58′ 14°2′	75°41′ 14°19′	75°23′ 14°37′	75°4′ 14°56′	74°45′ 15°15′	74°25′ 15°35′	74°3′ 15°57′
13	76°56′ 13°4′	76°42′ 13°18′	76°28′ 13°32′	76°13′ 13°47′	75°58′ 14°2′	75°42′ 14°18′	75°26′ 14°34′	75°8′ 14°52′	74°51′ 15°9′	74°32′ 15°28′	74°13′ 15°47′	73°53′ 16°7′	73°32′ 16°28′	73°11′ 16°49′	72°48′ 17°12′
14	75°58′ 14°2′	75°43′ 14°17′	75°28′ 14°32′	75°12′ 14°48′	74°56′ 15°4′	74°39′ 15°21′	74°21′ 15°39′	74°3′ 15°57′	73°44′ 16°16′	73°25′ 16°35′	73°4′ 16°56′	72°43′ 17°17′	72°21′ 17°39′	71°58′ 18°2′	71°34′ 18°26′
15	75°0′ 15°0′	74°44′ 15°16′	74°29′ 15°31′	74°12′ 15°48′	73°55′ 16°5′	73°37′ 16°23′	73°18′ 16°42′	72°59′ 17°1′	72°39′ 17°21′	72°18′ 17°42′	71°56′ 18°4′	71°34′ 18°26′	71°10′ 18°50′	70°46′ 19°14′	70°21′ 19°39′
16	74°3′ 15°57′	73°47′ 16°13′	73°30′ 16°30′	73°12′ 16°48′	72°54′ 17°6′	72°35′ 17°25′	72°15′ 17°45′	71°55′ 18°5′	71°34′ 18°26′	71°12′ 18°48′	70°49′ 19°11′	70°26′ 19°34′	70°1′ 19°59′	69°35′ 20°25′	69°9′ 20°51′
17	73°7′ 16°53′	72°49′ 17°11′	72°31′ 17°29′	72°13′ 17°47′	71°54′ 18°6′	71°34′ 18°26′	71°13′ 18°47′	70°52′ 19°8′	70°30′ 19°30′	70°7′ 19°53′	69°43′ 20°17′	69°17′ 20°43′	68°52′ 21°8′	68°26′ 21°34′	67°58′ 22°2′
18	72°11′ 17°49′	71°53′ 18°7′	71°34′ 18°26′	71°15′ 18°45′	70°54′ 19°6′	70°33′ 19°27′	70°12′ 19°48′	69°50′ 20°10′	69°26′ 20°34′	69°3′ 20°57′	68°38′ 21°22′	68°12′ 21°48′	67°45′ 22°15′	67°17′ 22°43′	66°48′ 23°12′
19	71°15′ 18°45′	70°57′ 19°1′	70°37′ 19°23′	70°17′ 19°43′	69°56′ 20°4′	69°34′ 20°26′	69°12′ 20°48′	68°48′ 21°12′	68°25′ 21°35′	67°59′ 22°1′	67°34′ 22°26′	67°6′ 22°54′	66°38′ 23°22′	66°10′ 23°50′	65°39′ 24°21′
20	70°21′ 19°39′	70°1′ 19°59′	69°41′ 20°19′	69°19′ 20°41′	68°57′ 21°3′	68°35′ 21°25′	68°12′ 21°48′	67°48′ 22°12′	67°23′ 22°37′	66°57′ 23°3′	66°30′ 23°30′	66°2′ 23°58′	65°33′ 24°27′	65°3′ 24°57′	64°32′ 25°28′
21	69°26′ 20°34′	69°6′ 20°54′	68°45′ 21°15′	68°23′ 21°37′	68°0′ 22°0′	67°37′ 22°23′	67°13′ 22°47′	66°48′ 23°12′	66°22′ 23°38′	65°55′ 24°5′	65°28′ 24°32′	64°59′ 25°1′	64°29′ 25°31′	63°58′ 26°2′	63°26′ 26°34′
22	68°33′ 21°27′	68°12′ 21°48′	67°50′ 22°10′	67°27′ 22°33′	67°4′ 22°56′	66°40′ 23°20′	66°15′ 23°45′	65°49′ 24°11′	65°23′ 24°37′	64°55′ 25°5′	64°26′ 25°34′	63°57′ 26°3′	63°26′ 26°34′	62°54′ 27°6′	62°21′ 27°39′
23	67°41′ 22°19′	67°18′ 22°42′	66°55′ 23°5′	66°32′ 23°28′	66°8′ 23°52′	65°44′ 24°16′	65°18′ 24°42′	64°51′ 25°9′	64°24′ 25°36′	63°55′ 26°5′	63°26′ 26°34′	62°56′ 27°4′	62°24′ 27°36′	61°52′ 28°8′	61°18′ 28°42′
24	66°48′ 23°12′	66°26′ 23°34′	66°2′ 23°58′	65°38′ 24°22′	65°14′ 24°46′	64°48′ 25°12′	64°22′ 25°38′	63°54′ 26°6′	63°26′ 26°34′	62°57′ 27°3′	62°27′ 27°33′	61°56′ 28°4′	61°23′ 28°37′	60°50′ 29°10′	60°15′ 29°45′
25	65°57′ 24°3′	65°33′ 24°27′	65°9′ 24°51′	64°45′ 25°15′	64°20′ 25°40′	63°53′ 26°7′	63°26′ 26°34′	62°58′ 27°2′	62°29′ 27°31′	61°59′ 28°1′	61°29′ 28°31′	60°57′ 29°3′	60°24′ 20°36′	59°50′ 30°10′	59°14′ 30°46′
26	65°6′ 24°54′	64°42′ 25°18′	64°18′ 25°42′	63°52′ 26°8′	63°26′ 26°34′	62°59′ 27°1′	62°31′ 27°29′	62°3′ 27°57′	61°33′ 28°27′	61°3′ 28°57′	60°31′ 29°29′	59°59′ 30°1′	59°25′ 30°35′	58°50′ 31°10′	58°14′ 31°46′
27	64°16′ 25°44′	63°51′ 26°9′	63°26′ 26°34′	63°0′ 27°0′	62°34′ 27°26′	62°6′ 27°54′	61°38′ 28°22′	61°8′ 28°52′	60°38′ 29°22′	60°7′ 29°53′	59°35′ 30°25′	59°2′ 30°58′	58°28′ 31°32′	57°53′ 32°7′	57°16′ 32°44′
28	63°26′ 26°34′	63°1′ 26°59′	62°36′ 27°24′	62°9′ 27°51′	61°42′ 28°18′	61°14′ 28°46′	60°45′ 29°15′	60°15′ 29°45′	59°45′ 30°15′	59°13′ 30°47′	58°40′ 31°20′	58°7′ 31°53′	57°32′ 32°28′	56°56′ 33°4′	56°19′ 33°41′
29	62°37′ 27°23′	62°12′ 27°48′	61°45′ 28°15′	61°19′ 28°41′	60°51′ 29°9′	60°23′ 29°37′	59°53′ 30°7′	59°23′ 30°37′	58°52′ 31°8′	58°19′ 31°41′	57°46′ 32°14′	57°12′ 32°48′	56°37′ 33°23′	56°0′ 34°0′	55°23′ 34°37′
30	61°49′ 28°11′	61°23′ 28°37′	60°57′ 29°3′	60°29′ 29°31′	60°1′ 29°59′	59°32′ 30°28′	59°2′ 30°58′	58°32′ 31°28′	58°0′ 32°0′	57°27′ 32°33′	56°53′ 33°7′	56°19′ 33°41′	55°43′ 34°17′	55°5′ 34°55′	54°28′ 35°32′
31	61°2′ 28°58′	60°36′ 29°24′	60°6′ 29°54′	59°41′ 30°19′	59°12′ 30°48′	58°42′ 31°18′	58°12′ 31°48′	57°41′ 32°19′	57°8′ 32°52′	56°36′ 33°24′	56°1′ 33°59′	55°26′ 34°34′	54°50′ 35°10′	54°12′ 35°48′	53°34′ 36°26′
32	60°15′ 29°45′	59°48′ 30°12′	59°21′ 30°39′	58°52′ 31°8′	58°34′ 31°26′	57°54′ 32°6′	57°23′ 32°37′	56°52′ 33°8′	56°19′ 33°41′	55°45′ 34°15′	55°11′ 34°49′	54°35′ 35°25′	53°58′ 36°2′	53°21′ 36°39′	52°42′ 37°18′
33	59°29′ 30°31′	59°2′ 30°58′	58°34′ 31°26′	58°5′ 31°55′	57°36′ 32°24′	57°6′ 32°54′	56°34′ 33°26′	56°2′ 33°58′	55°30′ 34°30′	54°56′ 35°4′	54°21′ 35°39′	53°45′ 36°15′	53°8′ 36°52′	52°29′ 37°31′	51°50′ 38°9′
34	58°44′ 31°16′	58°16′ 31°44′	57°48′ 32°12′	57°19′ 32°41′	56°49′ 33°11′	56°19′ 33°41′	55°47′ 34°13′	55°15′ 34°45′	54°41′ 35°19′	54°7′ 35°53′	53°32′ 36°28′	52°52′ 37°8′	52°18′ 37°42′	51°40′ 38°20′	51°0′ 39°0′
35	58°0′ 32°0′	57°32′ 32°28′	57°3′ 32°57′	56°33′ 33°27′	56°3′ 33°57′	55°32′ 34°28′	55°0′ 35°0′	54°28′ 35°32′	53°54′ 36°6′	53°20′ 36°40′	52°44′ 37°16′	52°8′ 37°52′	51°30′ 38°30′	50°51′ 39°9′	50°12′ 39°48′
36	57°16′ 32°44′	56°48′ 33°12′	56°19′ 33°41′	55°49′ 34°11′	55°18′ 34°42′	54°47′ 35°13′	54°15′ 35°45′	53°42′ 36°18′	53°8′ 36°52′	52°33′ 37°27′	51°57′ 38°3′	51°20′ 38°40′	50°43′ 39°17′	50°4′ 39°56′	49°24′ 40°36′
37	56°32′ 33°28′	56°4′ 33°56′	55°35′ 34°25′	55°5′ 34°55′	54°34′ 35°26′	54°2′ 35°58′	53°30′ 36°30′	52°56′ 37°4′	52°23′ 37°37′	51°47′ 38°13′	51°12′ 38°48′	50°35′ 39°25′	49°56′ 40°4′	49°17′ 40°43′	48°37′ 41°23′
38	55°51′ 34°9′	55°21′ 34°39′	54°52′ 35°8′	54°23′ 35°37′	53°51′ 36°9′	53°18′ 36°42′	52°46′ 37°14′	52°12′ 37°48′	51°38′ 38°22′	51°3′ 38°57′	50°27′ 39°33′	49°49′ 40°11′	49°11′ 40°49′	48°32′ 41°28′	47°52′ 42°8′
39	55°9′ 34°51′	54°39′ 35°21′	54°10′ 35°50′	53°39′ 36°21′	53°7′ 36°53′	52°36′ 37°24′	52°3′ 37°57′	51°29′ 38°31′	50°54′ 39°6′	50°19′ 39°41′	49°42′ 40°18′	49°5′ 40°55′	48°27′ 41°33′	47°48′ 42°12′	47°7′ 42°53′
40	54°28′ 35°32′	53°58′ 36°2′	53°28′ 36°32′	52°58′ 37°2′	52°26′ 37°34′	51°54′ 38°6′	51°20′ 38°40′	50°46′ 39°14′	50°12′ 39°48′	49°36′ 40°24′	48°59′ 41°1′	48°22′ 41°38′	47°44′ 42°16′	47°5′ 42°55′	46°24′ 43°36′
41	53°48′ 36°12′	53°17′ 36°43′	52°48′ 37°12′	52°16′ 37°44′	51°45′ 38°15′	51°12′ 38°48′	50°39′ 39°21′	50°5′ 39°55′	49°30′ 40°30′	48°54′ 41°6′	48°17′ 41°43′	47°40′ 42°20′	47°1′ 42°59′	46°22′ 43°38′	45°41′ 44°19′
42	53°8′ 36°52′	52°38′ 37°22′	52°8′ 37°52′	51°36′ 38°24′	51°4′ 38°56′	50°32′ 39°28′	49°58′ 40°2′	49°24′ 40°36′	48°49′ 41°11′	48°13′ 41°47′	47°36′ 42°24′	46°59′ 43°1′	46°20′ 43°40′	45°40′ 44°20′	**45°**

For bevel gears with axes at right angles.
Angle for wheel above, for pinion below.

PITCH ANGLES

WHEEL.

PINION.

	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27
12	73°41' 16°19'	73°18' 16°42'	72°54' 17°6'	72°28' 17°32'	72°2' 17°58'	71°34' 18°26'	71°5' 18°55'	70°34' 19°26'	70°1' 19°59'	69°26' 20°34'	68°50' 21°10'	68°12' 21°48'	67°31' 22°29'	66°48' 23°12'	66°2' 23°58'
13	72°25' 17°35'	71°59' 18°1'	71°34' 18°26'	71°7' 18°53'	70°39' 19°21'	70°9' 19°51'	69°37' 20°23'	69°5' 20°55'	68°30' 21°30'	67°53' 22°7'	67°15' 22°45'	66°34' 23°26'	65°51' 24°9'	65°6' 24°54'	64°17' 25°43'
14	71°9' 18°51'	70°43' 19°17'	70°15' 19°45'	69°46' 20°14'	69°16' 20°44'	68°45' 21°15'	68°12' 21°48'	67°37' 22°23'	67°0' 23°0'	66°23' 23°37'	65°42' 24°18'	64°59' 25°1'	64°14' 25°46'	63°26' 26°34'	62°36' 27°24'
15	69°54' 20°6'	69°26' 20°34'	68°58' 21°2'	68°28' 21°32'	67°56' 22°4'	67°23' 22°37'	66°48' 23°12'	66°12' 23°48'	65°33' 24°27'	64°53' 25°7'	64°10' 25°50'	63°26' 26°34'	62°39' 27°21'	61°49' 28°11'	60°57' 29°3'
16	68°41' 21°19'	68°12' 21°48'	67°42' 22°18'	67°10' 22°50'	66°37' 23°23'	66°2' 23°58'	65°26' 24°34'	64°48' 25°12'	64°8' 25°52'	63°26' 26°34'	62°42' 27°18'	61°56' 28°4'	61°7' 28°53'	60°15' 29°45'	59°21' 30°39'
17	67°29' 22°31'	66°58' 23°2'	66°27' 23°33'	65°54' 24°6'	65°19' 24°41'	64°43' 25°17'	64°6' 25°54'	63°26' 26°34'	62°45' 27°15'	62°1' 27°59'	61°15' 28°45'	60°28' 29°32'	59°37' 30°23'	58°44' 31°16'	57°48' 32°12'
18	66°18' 23°42'	65°46' 24°14'	65°14' 24°46'	64°39' 25°21'	64°4' 25°56'	63°26' 26°34'	62°47' 27°13'	62°6' 27°54'	61°23' 28°37'	60°38' 29°22'	59°51' 30°9'	59°2' 30°58'	58°10' 31°50'	57°16' 32°44'	56°19' 33°41'
19	65°8' 24°52'	64°36' 25°24'	64°2' 25°58'	63°26' 26°34'	62°49' 27°11'	62°10' 27°50'	61°30' 28°30'	60°48' 29°12'	60°4' 29°56'	59°18' 30°42'	58°30' 31°30'	57°39' 32°21'	56°46' 33°14'	55°51' 34°9'	54°52' 35°8'
20	64°0' 26°0'	63°26' 26°34'	62°51' 27°9'	62°14' 27°46'	61°37' 28°23'	60°57' 29°3'	60°15' 29°45'	59°32' 30°28'	58°47' 31°13'	58°0' 32°0'	57°10' 32°50'	56°19' 33°41'	55°24' 34°36'	54°28' 35°32'	53°28' 36°32'
21	62°53' 27°7'	62°18' 27°42'	61°42' 28°18'	61°4' 28°56'	60°25' 29°35'	59°45' 30°15'	59°2' 30°58'	58°18' 31°42'	57°32' 32°28'	56°43' 33°17'	55°53' 34°7'	55°0' 35°0'	54°5' 35°55'	53°7' 36°53'	52°8' 37°52'
22	61°47' 28°13'	61°11' 28°49'	60°34' 29°26'	59°56' 30°4'	59°15' 30°45'	58°34' 31°26'	57°51' 32°9'	57°6' 32°54'	56°19' 33°41'	55°29' 34°31'	54°38' 35°22'	53°45' 36°15'	52°49' 37°11'	51°50' 38°10'	50°49' 39°11'
23	60°42' 29°18'	60°6' 29°54'	59°28' 30°32'	58°49' 31°11'	58°8' 31°52'	57°25' 32°35'	56°41' 33°19'	55°55' 34°5'	55°7' 34°53'	54°18' 35°42'	53°26' 36°34'	52°31' 37°29'	51°35' 38°25'	50°36' 39°24'	49°34' 40°26'
24	59°39' 30°21'	59°2' 30°58'	58°23' 31°37'	57°44' 32°16'	57°2' 32°58'	56°19' 33°41'	55°33' 34°27'	54°47' 35°13'	53°58' 36°2'	53°8' 36°52'	52°15' 37°45'	51°20' 38°40'	50°23' 39°37'	49°24' 40°36'	48°22' 41°38'
25	58°38' 31°22'	58°0' 32°0'	57°20' 32°40'	56°40' 33°20'	55°57' 34°3'	55°13' 34°47'	54°28' 35°32'	53°40' 36°20'	52°51' 37°9'	52°0' 38°0'	51°7' 38°53'	50°12' 39°48'	49°14' 40°46'	48°14' 41°46'	47°12' 42°48'
26	57°37' 32°23'	56°58' 33°2'	56°19' 33°41'	55°37' 34°23'	54°54' 35°6'	54°10' 35°50'	53°24' 36°36'	52°36' 37°24'	51°46' 38°14'	50°54' 39°6'	50°1' 39°59'	49°5' 40°55'	48°7' 41°53'	47°7' 42°53'	46°5' 43°55'
27	56°38' 33°22'	55°59' 34°1'	55°18' 34°42'	54°36' 35°24'	53°53' 36°7'	53°7' 36°53'	52°21' 37°39'	51°33' 38°27'	50°43' 39°17'	49°51' 40°9'	48°57' 41°3'	48°0' 42°0'	47°3' 42°57'	46°2' 43°58'	45°
28	55°40' 34°20'	55°0' 35°0'	54°19' 35°41'	53°37' 36°23'	52°53' 37°7'	52°8' 37°52'	51°20' 38°40'	50°32' 39°28'	49°41' 40°19'	48°49' 41°11'	47°55' 42°5'	46°58' 43°2'	46°0' 44°0'	45°	
29	54°44' 35°16'	54°3' 35°57'	53°22' 36°38'	52°39' 37°21'	51°55' 38°5'	51°9' 38°51'	50°21' 39°39'	49°32' 40°28'	48°41' 41°19'	47°49' 42°11'	46°54' 43°6'	45°58' 44°2'	45°		
30	53°48' 36°12'	53°7' 36°53'	52°26' 37°34'	51°42' 38°18'	50°58' 39°2'	50°12' 39°48'	49°24' 40°36'	48°35' 41°25'	47°43' 42°17'	46°51' 43°9'	45°56' 44°4'	45°			
31	52°54' 37°6'	52°13' 37°47'	51°31' 38°29'	50°48' 39°12'	50°2' 39°58'	49°16' 40°44'	48°28' 41°32'	47°39' 42°21'	46°47' 43°13'	45°54' 44°6'	45°				
32	52°2' 37°58'	51°20' 38°40'	50°38' 39°22'	49°54' 40°6'	49°9' 40°51'	48°22' 41°38'	47°34' 42°26'	46°44' 43°16'	45°53' 44°7'	45°					
33	51°10' 38°50'	50°29' 39°31'	49°46' 40°14'	49°2' 40°58'	48°16' 41°44'	47°29' 42°31'	46°41' 43°19'	45°51' 44°9'	45°						
34	50°20' 39°40'	49°38' 40°22'	48°55' 41°5'	48°11' 41°49'	47°25' 42°35'	46°38' 43°22'	45°50' 44°10'	45°							
35	49°31' 40°29'	48°48' 41°12'	48°5' 41°55'	47°21' 42°39'	46°35' 43°25'	45°48' 44°12'	45°								
36	48°43' 41°17'	48°0' 42°0'	47°17' 42°43'	46°33' 43°27'	45°47' 44°13'	45°									
37	47°56' 42°4'	47°14' 42°46'	46°30' 43°30'	45°46' 44°14'	45°										
38	47°10' 42°50'	46°28' 43°32'	45°45' 44°15'	45°											
39	46°26' 43°34'	45°43' 44°17'	45°												
40	45°42' 44°18'	45°													
41	45°														

For bevel gears with axes at right angles.
Angle for wheel above, for pinion below.

PITCH ANGLES

WHEEL.

PINION.

	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12
12	65°14' 24°46'	64°22' 25°38'	63°26' 26°34'	62°27' 27°33'	61°23' 28°37'	60°15' 29°45'	59°2' 30°58'	57°44' 32°16'	56°19' 33°41'	54°47' 35°13'	53°7' 36°53'	51°20' 38°40'	49°24' 40°36'	47°17' 42°43'	45°
13	63°26' 26°34'	62°31' 27°29'	61°33' 28°27'	60°31' 29°29'	59°25' 30°35'	58°14' 31°46'	56°58' 33°2'	55°37' 34°23'	54°10' 35°50'	52°36' 37°24'	50°54' 39°6'	49°5' 40°55'	47°7' 42°53'	45°	
14	61°42' 28°18'	60°45' 29°15'	59°45' 30°15'	58°40' 31°20'	57°32' 32°28'	56°19' 33°41'	55°0' 35°0'	53°37' 36°23'	52°8' 37°52'	50°32' 39°28'	48°48' 41°12'	46°58' 43°2'	45°		
15	60°1' 29°50'	59°2' 30°58'	58°0' 32°0'	56°53' 33°7'	55°43' 34°17'	54°28' 35°32'	53°7' 36°53'	51°42' 38°18'	50°12' 39°48'	48°35' 41°25'	46°51' 43°9'	45°			
16	58°23' 31°37'	57°23' 32°37'	56°19' 33°41'	55°11' 34°49'	53°58' 36°2'	52°42' 37°18'	51°20' 38°40'	49°54' 40°6'	48°22' 41°38'	46°44' 43°16'	45°				
17	56°49' 33°11'	55°47' 34°13'	54°41' 35°19'	53°32' 36°28'	52°18' 37°42'	51°0' 39°0'	49°38' 40°22'	48°11' 41°49'	46°38' 43°22'	45°					
18	55°18' 34°42'	54°15' 35°45'	53°7' 36°53'	51°57' 38°3'	50°43' 39°17'	49°24' 40°36'	48°0' 42°0'	46°33' 43°27'	45°						
19	53°51' 36°9'	52°46' 37°14'	51°38' 38°22'	50°26' 39°34'	49°11' 40°49'	47°52' 42°8'	46°28' 43°32'	45°							
20	52°26' 37°34'	51°20' 38°40'	50°12' 39°48'	48°59' 41°1'	47°43' 42°17'	46°24' 43°36'	45°								
21	51°4' 38°56'	49°58' 40°2'	48°48' 41°12'	47°36' 42°24'	46°20' 43°40'	45°									
22	49°46' 40°14'	48°39' 41°21'	47°29' 42°31'	46°16' 43°44'	45°										
23	48°30' 41°30'	47°23' 42°37'	46°13' 43°47'	45°											
24	47°17' 42°43'	46°10' 43°50'	45°												
25	46°7' 43°53'	45°													
26	45°														

For bevel gears with axes at right angles.
Angle for wheel above, for pinion below.

ANGLE OF FACE OF BEVEL GEARS

90°—Angle given=Face Angle.

WHEEL.

PINION.	72	71	70	69	68	67	66	65	64	63	62	61	60	59	58	57
12	7°53′ 78°59′	8° 78°50′	8°7′ 78°39′	8°14′ 78°30′	8°21′ 78°19′	8°28′ 78°10′	8°35′ 77°59′	8°43′ 77°47′	8°51′ 77°37′	8°59′ 77°25′	9°7′ 77°13′	9°17′ 77°1′	9°26′ 76°48′	9°35′ 76°3′	9°45′ 76°23′	9°55′ 76°8′
13	8°40′ 78°12′	8°48′ 78°2′	8°54′ 77°52′	9°2′ 77°42′	9°9′ 77°31′	9°18′ 77°20′	9°26′ 77°8′	9°35′ 76°56′	9°43′ 76°45′	9°52′ 76°32′	10°1′ 76°19′	10°11′ 76°7′	10°21′ 75°53′	10°31′ 75°39′	10°42′ 75°26′	10°53′ 75°11′
14	9°26′ 77°26′	9°34′ 77°16′	9°42′ 77°4′	9°50′ 76°54′	9°59′ 76°43′	10°8′ 76°30′	10°16′ 76°18′	10°25′ 76°7′	10°35′ 75°55′	10°45′ 75°41′	10°54′ 75°28′	11°5′ 75°15′	11°16′ 75°	11°27′ 74°45′	11°39′ 74°31′	11°50′ 74°14′
15	10°12′ 76°40′	10°21′ 76°29′	10°30′ 76°18′	10°38′ 76°6′	10°47′ 75°55′	10°57′ 75°43′	11°6′ 75°30′	11°16′ 75°16′	11°27′ 75°3′	11°37′ 74°49′	11°47′ 74°35′	11°59′ 74°21′	12°11′ 74°7′	12°22′ 73°50′	12°35′ 73°35′	12°48′ 73°18′
16	10°59′ 75°55′	11°7′ 75°43′	11°17′ 75°31′	11°26′ 75°20′	11°37′ 75°7′	11°46′ 74°54′	11°56′ 74°40′	12°7′ 74°27′	12°17′ 74°13′	12°29′ 73°59′	12°40′ 73°44′	12°52′ 73°28′	13°5′ 73°13′	13°18′ 72°56′	13°30′ 72°40′	13°45′ 72°23′
17	11°44′ 75°10′	11°54′ 74°58′	12°4′ 74°46′	12°13′ 74°33′	12°24′ 74°20′	12°34′ 74°5′	12°46′ 73°52′	12°56′ 73°38′	13°7′ 73°23′	13°21′ 73°9′	13°32′ 72°52′	13°45′ 72°35′	13°59′ 72°21′	14°11′ 72°3′	14°26′ 71°45′	14°40′ 71°28′
18	12°29′ 74°25′	12°40′ 74°12′	12°50′ 74°	13° 73°46′	13°12′ 73°32′	13°23′ 73°19′	13°34′ 73°4′	13°47′ 72°49′	13°59′ 72°33′	14°12′ 72°18′	14°24′ 72°2′	14°38′ 71°44′	14°52′ 71°28′	15°6′ 71°10′	15°21′ 70°51′	15°36′ 70°34′
19	13°14′ 73°40′	13°25′ 73°27′	13°36′ 73°14′	13°48′ 73°	14° 72°46′	14°11′ 72°31′	14°24′ 72°16′	14°36′ 72°	14°49′ 71°45′	15°2′ 71°28′	15°15′ 71°11′	15°30′ 70°54′	15°44′ 70°30′	15°59′ 70°17′	16°15′ 69°59′	16°31′ 69°39′
20	13°59′ 72°57′	14°11′ 72°43′	14°23′ 72°29′	14°34′ 72°14′	14°46′ 72°	15°4′ 71°45′	15°11′ 71°29′	15°25′ 71°13′	15°39′ 70°56′	15°52′ 70°38′	16°7′ 70°21′	16°21′ 70°3′	16°37′ 69°45′	16°53′ 69°25′	17°8′ 69°6′	17°26′ 68°46′
21	14°43′ 72°13′	14°55′ 71°59′	15°8′ 71°44′	15°21′ 71°29′	15°33′ 71°13′	15°46′ 70°58′	15°59′ 70°41′	16°13′ 70°25′	16°28′ 70°8′	16°42′ 69°50′	16°58′ 69°32′	17°13′ 69°13′	17°28′ 68°54′	17°46′ 68°34′	18°2′ 68°14′	18°20′ 67°52′
22	15°27′ 71°29′	15°40′ 71°14′	15°53′ 70°59′	16°6′ 70°44′	16°20′ 70°28′	16°33′ 70°11′	16°47′ 69°55′	17°2′ 69°38′	17°16′ 69°20′	17°31′ 69°1′	17°49′ 68°43′	18°3′ 68°23′	18°20′ 68°4′	18°37′ 67°43′	18°56′ 67°22′	19°13′ 67°1′
23	16°12′ 70°46′	16°24′ 70°30′	16°38′ 70°16′	16°51′ 69°59′	17°5′ 69°43′	17°20′ 69°26′	17°34′ 69°8′	17°50′ 68°50′	18°5′ 68°33′	18°20′ 68°14′	18°36′ 67°54′	18°54′ 67°34′	19°10′ 67°14′	19°28′ 66°52′	19°48′ 66°32′	20°5′ 66°9′
24	16°55′ 70°3′	17°9′ 69°47′	17°22′ 69°32′	17°37′ 69°15′	17°51′ 68°59′	18°6′ 68°40′	18°21′ 68°23′	18°37′ 68°5′	18°53′ 67°45′	19°9′ 67°27′	19°26′ 67°6′	19°44′ 66°46′	20°1′ 66°25′	20°19 66°3′	20°39′ 65°41′	20°58′ 65°18′
25	17°39′ 60°21′	17°52′ 69°4′	18°6′ 68°48′	18°21′ 68°31′	18°36′ 68°14′	18°52′ 67°56′	19°7′ 67°37′	19°24′ 67°18′	19°40′ 67°	19°57′ 66°30′	20°14′ 66°20′	20°32′ 65°58′	20°51′ 65°37′	21°10′ 65°14′	21°29′ 64°51′	21°50′ 64°28′
26	18°21′ 68°39′	18°36′ 68°22′	18°51′ 68°5′	19°6′ 67°48′	19°22′ 67°30′	19°37′ 67°13′	19°53′ 66°53′	20°10′ 66°34′	20°26′ 66°14′	20°45′ 65°53′	21°2′ 65°32′	21°21′ 65°11′	21°41′ 64°49′	22° 64°26′	22°20′ 64°2′	22°41 63°39′
27	19°3′ 67°57′	19°19′ 67°39′	19°34′ 67°22′	19°49′ 67°5′	20°6′ 66°46′	20°22′ 66°28′	20°38′ 66°8′	20°56′ 65°48′	21°13′ 65°29′	21°32′ 65°8′	21°50′ 64°46′	22°10′ 64°24′	22°29′ 64°1′	22°49′ 63°39′	23°10′ 63°14′	23°31′ 62°49′
28	19°46′ 67°16′	20°1′ 66°59′	20°17′ 66°41′	20°32′ 66°22′	20°50′ 66°4′	21°6′ 65°44′	21°23′ 65°25′	21°41′ 65°5′	22° 64°44′	22°18′ 64°22′	22°37′ 64°1′	22°56′ 63°38′	23°17′ 63°15′	23°38′ 62°52′	23°59′ 62°27′	24°21′ 62°1′
29	20°27′ 66°35′	20°43′ 66°17′	20°59′ 65°59′	21°16′ 65°40′	21°33′ 65°21′	21°50′ 65°2′	22°8′ 64°42′	22°27′ 64°21′	22°45′ 63°59′	23°5′ 63°37′	23°25′ 63°15′	23°44′ 62°52′	24°4′ 62°28′	24°25′ 62°5′	24°48′ 61°40′	25°10′ 61°14′
30	21°9′ 65°55′	21°25′ 65°37′	21°42′ 65°18′	21°58′ 64°58′	22°15′ 64°39′	22°34′ 64°18′	22°52′ 63°58′	23°10′ 63°38′	23°30′ 63°16′	23°50′ 62°54′	24°10′ 62°30′	24°30′ 62°7′	24°51′ 61°43′	25°12′ 61°18′	25°36′ 60°54′	25°59′ 60°27′
31	21°50′ 65°14′	22°6′ 64°50′	22°24′ 64°36′	22°41′ 64°17′	22°59′ 63°57′	23°17′ 63°37′	23°35′ 63°15′	23°55′ 62°55′	24°14′ 62°32′	24°34′ 62°10′	24°54′ 61°46′	25°17′ 61°23′	25°38′ 60°58′	25°58′ 60°34′	26°22′ 60°8′	26°46′ 59°42′
32	22°31′ 64°35′	22°48′ 64°16′	23°4′ 63°56′	23°23′ 63°37′	23°40′ 63°16′	23°59′ 62°55′	24°18′ 62°34′	24°38′ 62°12′	24°58′ 61°50′	25°18′ 61°26′	25°39′ 61°3′	26°1′ 60°39′	26°23′ 60°15′	26°45′ 59°49′	27°9′ 59°23′	27°34′ 58°56′
33	23°10′ 63°56′	23°28′ 63°36′	23°46′ 63°16′	24°4′ 62°56′	24°22′ 62°36′	24°41′ 62°15′	25°1′ 61°53′	25°21′ 61°31′	25°42′ 61°8′	26°2′ 60°44′	26°24′ 60°20′	26°45′ 59°55′	27°9′ 59°31′	27°31′ 59°5′	27°56′ 58°38′	28°19′ 58°11′
34	23°51′ 63°17′	24°8′ 62°58′	24°27′ 62°37′	24°44′ 62°16′	25°4′ 61°56′	25°23′ 61°33′	25°42′ 61°12′	26°3′ 60°49′	26°24′ 60°26′	26°46′ 60°2′	27°7′ 59°37′	27°29′ 59°13′	27°52′ 58°48′	28°16′ 58°22′	28°40′ 57°54′	29°5′ 57°27′
35	24°29′ 62°39′	24°48′ 62°18′	25°6′ 61°58′	25°25′ 61°37′	25°44′ 61°16′	26°4′ 60°54′	26°24′ 60°32′	26°45′ 60°9′	27°6′ 59°44′	27°28′ 59°22′	27°50′ 58°56′	28°13′ 58°31′	28°36′ 58°6′	29°1′ 57°39′	29°25′ 57°11′	29°50′ 56°44′
36	25°9′ 62°1′	25°27′ 61°41′	25°45′ 61°20′	26°5′ 60°59′	26°24′ 60°36′	26°45′ 60°15′	27°5′ 59°51′	27°26′ 59°28′	27°48′ 59°4′	28°10′ 58°40′	28°33′ 58°15′	28°56′ 57°50′	29°20′ 57°24′	29°43′ 56°57′	30°9′ 56°29′	30°35′ 56°1′
37	25°47′ 61°23′	26°6′ 61°2′	26°25′ 60°41′	26°44′ 60°20′	27°4′ 59°58′	27°25′ 59°35′	27°45′ 59°13′	28°7′ 58°49′	28°29′ 58°25′	28°51′ 58°1′	29°15′ 57°35′	29°38′ 57°10′	30°2′ 56°42	30°27′ 56°15′	30°52′ 55°48′	31°18′ 55°20′
38	26°25′ 60°47′	26°44′ 60°26′	27°4′ 60°4′	27°24′ 59°42′	27°44′ 59°20′	28°4′ 58°56′	28°26′ 58°34′	28°47′ 58°9′	29°9′ 57°45′	29°33′ 57°21′	29°55′ 56°55′	30°20′ 56°30′	30°44′ 56°2′	31°9′ 55°35′	31°35′ 55°7′	32°1′ 54°29′
39	27°3′ 60°9′	27°22′ 59°48′	27°42′ 59°28′	28°2′ 59°4′	28°22′ 58°42′	28°43′ 58°19′	29°5′ 57°55′	29°27′ 57°31′	29°49′ 57°7′	30°13′ 56°41′	30°36′ 56°16′	31°1′ 55°49′	31°26′ 55°22′	31°50′ 54°54′	32°16′ 54°28′	32°43′ 53°57′
40	27°40′ 59°34′	28°2′ 59°14′	28°20′ 58°50′	28°41′ 58°27′	29°1′ 58°5′	29°22′ 57°42′	29°43′ 57°17′	30°6′ 56°54′	30°28′ 56°28′	30°52′ 56°2′	31°17′ 55°37′	31°42′ 55°10′	32°6′ 54°44′	32°31′ 54°15′	32°57′ 53°46′	33°24′ 53°18′
41	28°17′ 58°57′	28°37′ 58°37′	28°57′ 58°15′	29°18′ 57°52′	29°39′ 57°20′	30°1′ 57°4′	30°22′ 56°40′	30°45′ 56°15′	31°9′ 55°51′	31°31′ 55°25′	31°55′ 54°59′	32°20′ 54°32′	32°46′ 54°4′	33°12′ 53°36′	33°39′ 53°7′	34°6′ 52°38′
42	28°52′ 58°22′	29°12′ 58°1′	29°33′ 57°39′	29°55′ 57°15′	30°16′ 56°52′	30°38′ 56°28′	31° 56°4′	31°23′ 55°39′	31°47′ 55°13′	32°10′ 54°48′	32°35′ 54°21′	33° 53°54′	33°26′ 53°26′	33°52′ 52°58′	34°19′ 52°29′	34°46′ 52°

For bevel gears with axes at right angles.
Angle for wheel above, for pinion below.

ANGLE OF FACE

90°—Angle given=Face Angle.

WHEEL.

PINION.	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42
12	10°6′ 75°54′	10°16′ 75°40′	10°28′ 75°24′	10°39′ 75°9′	10°52′ 74°52′	11°3′ 74°37′	11°15′ 74°15′	11°30′ 73°58′	11°43′ 73°39′	11°58′ 73°20′	12°13′ 72°59′	12°29′ 72°37′	12°45′ 72°15′	13°1′ 71°51′	13°19′ 71°25′
13	11°4′ 74°56′	11°16′ 74°40′	11°28′ 74°24′	11°42′ 74°8′	11°54′ 73°50′	12°8′ 73°32′	12°20′ 73°12′	12°37′ 72°53′	12°51′ 72°33′	13°7′ 72°11′	13°23′ 71°49′	13°40′ 71°26′	13°58′ 71°2′	14°16′ 70°38′	14°35′ 70°11′
14	12°2′ 73°58′	12°16′ 73°42′	12°29′ 73°25′	12°43′ 73°7′	12°57′ 72°49′	13°11′ 72°29′	13°26′ 72°8′	13°42′ 71°48′	13°59′ 71°27′	14°15′ 71°5′	14°33′ 70°41′	14°51′ 70°17′	15°10′ 69°52′	15°30′ 69°26′	15°51′ 68°59′
15	13°1′ 73°1′	13°16′ 72°44′	13°28′ 72°26′	13°43′ 72°7′	13°59′ 71°49′	14°14′ 71°28′	14°30′ 71°6′	14°47′ 70°45′	15°5′ 70°23′	15°23′ 69°59′	15°42′ 69°34′	16°1′ 69°9′	16°22′ 68°42′	16°48′ 68°15′	17°5′ 67°47′
16	13°59′ 72°5′	14°13′ 71°47′	14°28′ 71°28′	14°44′ 71°8′	15°1′ 70°49′	15°17′ 70°27′	15°35′ 70°5′	15°52′ 69°42′	16°11′ 69°19′	16°30′ 68°54′	16°50′ 68°28′	17°10′ 68°2′	17°32′ 67°34′	17°56′ 67°6′	18°18′ 66°36′
17	14°57′ 71°9′	15°11′ 70°49′	15°28′ 70°30′	15°44′ 70°10′	16°1′ 69°49′	16°18′ 69°26′	16°37′ 69°3′	16°55′ 68°39′	17°15′ 68°15′	17°36′ 67°50′	17°57′ 67°23′	18°20′ 66°54′	18°43′ 66°27′	19°6′ 65°58′	19°31′ 65°27′
18	15°52′ 70°14′	16°7′ 69°53′	16°26′ 69°34′	16°42′ 69°12′	17°1′ 68°49′	17°20′ 68°26′	17°39′ 68°3′	17°58′ 67°38′	18°20′ 67°12′	18°41′ 66°47′	19°3′ 66°19′	19°27′ 65°51′	19°50′ 65°20′	20°18′ 64°50′	20°42′ 64°18′
19	16°49′ 69°19′	17°2′ 68°58′	17°23′ 68°37′	17°41′ 68°15′	18° 67°52′	18°21′ 67°29′	18°40′ 67°4′	19°1′ 66°37′	19°22′ 66°12′	19°46′ 65°44′	20°8′ 65°16′	20°34′ 64°46′	20°59′ 64°15′	21°24′ 63°44′	21°52′ 63°10′
20	17°44′ 68°26′	18°1′ 68°3′	18°19′ 67°41′	18°40′ 67°18′	19° 66°54′	19°20′ 66°30′	19°41′ 66°5′	20°2′ 65°38′	20°25′ 65°11′	20°49′ 64°43′	21°13′ 64°13′	21°39′ 63°43′	22°5′ 63°11′	22°32′ 62°38′	23° 62°4′
21	18°39′ 67°31′	18°57′ 67°9′	19°16′ 66°46′	19°37′ 66°23′	19°58′ 65°58′	20°19′ 65°33′	20°41′ 65°7′	21°3′ 64°39′	21°27′ 64°11′	21°52′ 63°42′	22°17′ 63°13′	22°43′ 62°41′	23°10′ 62°8′	23°38′ 61°34′	24°8′ 61°
22	19°32′ 66°38′	19°52′ 66°16′	20°12′ 65°52′	20°33′ 65°27′	20°55′ 65°3′	21°17′ 64°37′	21°40′ 64°10′	22°3′ 63°41′	22°17′ 63°13′	22°53′ 62°43′	23°19′ 62°11′	23°46′ 61°40′	24°15′ 61°7′	24°44′ 60°32′	25°14′ 59°56′
23	20°25′ 65°47′	20°47′ 65°23′	21°8′ 64°58′	21°29′ 64°33′	21°52′ 64°8′	22°13′ 63°41′	22°37′ 63°13′	23°2′ 62°44′	23°27′ 62°15′	23°54′ 61°44′	24°21′ 61°13′	24°49′ 60°41′	25°18′ 60°6′	25°47′ 59°31′	26°18′ 58°54′
24	21°19′ 64°55′	21°39′ 64°31′	22°1′ 64°5′	22°24′ 63°40′	22°46′ 63°14′	23°10′ 62°46′	23°36′ 62°19′	24° 61°48′	24°26′ 61°18′	24°53′ 60°47′	25°21′ 60°15′	25°49′ 59°41′	26°20′ 59°6′	26°51′ 58°31′	27°23′ 57°53′
25	22°11′ 64°5′	22°33′ 63°39′	22°56′ 63°14′	23°18′ 62°48′	23°41′ 62°21′	24°7′ 61°53′	24°32′ 61°24′	24°57′ 60°53′	25°24′ 60°22′	25°52′ 59°50′	26°20′ 59°18′	26°50′ 58°44′	27°21′ 58°9′	27°52′ 57°32′	28°26′ 56°54′
26	23°3′ 63°15′	23°25′ 62°49′	23°47′ 62°23′	24°13′ 61°56′	24°36′ 61°28′	25°1′ 60°59′	25°28′ 60°30′	25°53′ 59°59′	26°21′ 59°27′	26°49′ 58°55′	27°19′ 58°21′	27°49′ 57°47′	28°21′ 57°11′	28°54′ 56°34′	29°27′ 55°55′
27	23°53′ 62°25′	24°16′ 61°58′	24°40′ 61°32′	25°5′ 61°5′	25°29′ 60°37′	25°55′ 60°7′	26°22′ 59°38′	26°48′ 59°5′	27°17′ 58°33′	27°46′ 58°	28°16′ 57°26′	28°47′ 56°51′	29°19′ 56°15′	29°52′ 55°38′	30°27′ 54°59′
28	24°44′ 61°36′	25°7′ 61°9′	25°31′ 60°43′	25°56′ 60°14′	26°22′ 59°46′	26°48′ 59°16′	27°15′ 58°45′	27°43′ 58°13′	28°12′ 57°42′	28°42′ 57°8′	29°12′ 56°32′	29°43′ 55°57′	30°16′ 55°20′	30°50′ 54°42′	31°25′ 54°3′
29	25°33′ 60°47′	25°57′ 60°21′	26°22′ 59°52′	26°47′ 59°25′	27°14′ 58°56′	27°40′ 58°26′	28°8′ 57°54′	28°36′ 57°22′	29°5′ 56°49′	29°37′ 56°15′	30°8′ 55°40′	30°40′ 55°4′	31°13′ 54°27′	31°48′ 53°48′	32°23′ 53°9′
30	26°22′ 60°	26°47′ 59°33′	27°12′ 59°6′	27°38′ 58°36′	28°4′ 58°6′	28°32′ 57°36′	29° 57°4′	29°28′ 56°32′	29°58′ 55°58′	30°30′ 55°24′	31°2′ 54°48′	31°34′ 54°12′	32°8′ 53°34′	32°44′ 52°54′	33°19′ 52°15′
31	27°10′ 59°14′	27°34′ 58°46′	28°3′ 58°15′	28°27′ 57°49′	28°54′ 57°18′	29°23′ 56°47′	29°51′ 56°15′	30°20′ 55°42′	30°52′ 55°8′	31°22′ 54°34′	31°55′ 53°57′	32°29′ 53°21′	33°2′ 52°42′	33°39′ 52°3′	34°15′ 51°23′
32	27°58′ 58°28′	28°23′ 57°59′	28°49′ 57°31′	29°17′ 57°1′	29°33′ 56°41′	30°12′ 56°	30°42′ 55°28′	31°10′ 54°54′	31°42′ 54°20′	32°14′ 53°44′	32°46′ 53°8′	33°21′ 52°31′	33°56′ 51°52′	34°31′ 51°13′	35°8′ 50°32′
33	28°45′ 57°43′	29°10′ 57°14′	29°37′ 56°45′	30°5′ 56°15′	30°32′ 55°49′	31°1′ 55°13′	31°31′ 54°39′	32°1′ 54°5′	32°32′ 53°32′	33°4′ 52°56′	33°38′ 52°20′	34°12′ 51°42′	34°47′ 51°3′	35°24′ 50°22′	36° 49°41′
34	29°31′ 56°59′	29°57′ 56°29′	30°24′ 56°	30°51′ 55°29′	31°20′ 54°58′	31°49′ 54°27′	32°19′ 53°53′	32°50′ 53°20′	33°22′ 52°44′	33°54′ 52°8′	34°28′ 51°32′	35°6′ 50°50′	35°38′ 50°14′	36°15′ 49°35′	36°53′ 48°53′
35	30°15′ 56°15′	30°42′ 55°46′	31°10′ 55°16′	31°38′ 54°44′	32°7′ 54°13′	32°36′ 53°40′	33°7′ 53°7′	33°38′ 52°34′	34°10′ 51°58′	34°42′ 51°22′	35°17′ 50°45′	35°51′ 50°7′	36°27′ 49°27′	37°5′ 48°47′	37°42′ 48°6′
36	31° 55°32′	31°27′ 55°3′	31°55′ 54°33′	32°23′ 54°1′	32°53′ 53°28′	33°23′ 52°57′	33°53′ 52°23′	34°25′ 51°40′	34°57′ 51°13′	35°31′ 50°37′	36°5′ 49°59′	36°41′ 49°21′	37°16′ 48°42′	37°53′ 48°1′	38°32′ 47°20′
37	31°45′ 54°49′	32°12′ 54°20′	32°40′ 53°50′	33°8′ 53°18′	33°38′ 52°46′	34°9′ 52°13′	34°40′ 51°40′	35°12′ 51°4′	35°43′ 50°29′	36°18′ 49°52′	36°51′ 49°15′	37°27′ 48°37′	38°4′ 47°56′	38°42′ 47°16′	39°20′ 46°34′
38	32°27′ 54°9′	32°56′ 53°38′	33°24′ 53°8′	33°52′ 52°38′	34°22′ 52°4′	34°54′ 51°30′	35°24′ 50°56′	35°57′ 50°21′	36°29′ 49°45′	37°3′ 49°9′	37°38′ 48°32′	38°14′ 47°52′	38°51′ 47°13′	39°28′ 46°32′	40°7′ 45°51′
39	33°10′ 53°28′	33°39′ 52°57′	34°7′ 52°27′	34°36′ 51°54′	35°7′ 51°21′	35°37′ 50°49′	36°9′ 50°15′	36°41′ 49°39′	37°15′ 49°3′	37°48′ 48°26′	38°24′ 47°48′	39° 47°10′	39°36′ 46°30′	40°13′ 45°49′	40°53′ 45°7′
40	33°52′ 52°48′	34°21′ 52°17′	34°50′ 51°46′	35°18′ 51°14′	35°49′ 50°41′	36°20′ 50°8′	36°53′ 49°33′	37°25′ 48°57′	37°58′ 48°22′	38°33′ 47°45′	39°8′ 47°6′	39°41′ 46°28′	40°20′ 45°48′	40°58′ 45°8′	41°37′ 44°25′
41	34°33′ 52°9′	35°8′ 51°37′	35°31′ 51°7′	36°1′ 50°33′	36°31′ 50°1′	37°3′ 49°27′	37°35′ 48°53′	38°7′ 48°17′	38°41′ 47°41′	39°16′ 47°4′	39°51′ 46°25′	40°27′ 45°47′	41°5′ 45°7′	41°42′ 44°26′	41°22′ 43°44′
42	35°14′ 51°30′	35°43′ 50°59′	36°12′ 50°28′	36°42′ 49°54′	37°13′ 49°21′	37°44′ 48°48′	38°17′ 48°18′	38°49′ 47°37′	39°23′ 47°1′	39°58′ 46°24′	40°34′ 45°46′	41°9′ 45°7′	41°47′ 44°27′	42°26′ 43°46′	43°4′

For bevel gears with axes at right angles.
Angle for wheel above, for pinion below.

ANGLE OF FACE

90°—Angle given=Face Angle.

WHEEL.

PINION.

	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27
12	13°37' 70°59'	13°57' 70°33'	14°18' 70°6'	14°39' 69°35'	15°1' 69°5'	15°24' 68°32'	15°49' 67°59'	16°15' 67°23'	16°43' 66°45'	17°13' 66°5'	17°43' 65°23'	18°15' 64°39'	18°51' 63°53'	19°27' 63°3'	20°5' 62°9'
13	14°55' 69°45'	15°17' 69°15'	15°39' 68°47'	16°1' 68°15'	16°25' 67°43'	16°51' 67°9'	17°19' 66°33'	17°46' 65°56'	18°16' 65°16'	18°48' 64°34'	19°21' 63°51'	19°57' 63°5'	20°32' 62°14'	21°11' 61°23'	21°54' 60°28'
14	16°13' 68°31'	16°34' 68°0'	16°59' 67°29'	17°24' 66°56'	17°50' 66°22'	18°17' 65°47'	18°45' 65°9'	19°16' 64°30'	19°48' 63°48'	20°20' 63°6'	20°56' 62°20'	21°34' 61°32'	22°13' 60°41'	22°55' 59°47'	23°38' 53°50'
15	17°28' 67°16'	17°53' 66°45'	18°18' 66°14'	18°44' 65°40'	19°11' 65°3'	19°40' 64°26'	20°11' 63°49'	20°44' 63°8'	21°18' 62°24'	21°53' 61°39'	22°31' 60°51'	23°10' 60°2'	23°51' 59°9'	24°35' 58°13'	25°20' 57°14'
16	18°42' 66°4'	19°9' 65°33'	19°35' 64°59'	20°3' 64°23'	20°32' 63°46'	21°3' 63°7'	21°36' 62°28'	22°9' 61°45'	22°45' 61°01'	23°22' 60°14'	24°1' 59°25'	24°42' 58°34'	25°26' 57°40'	26°12' 56°42'	27°1' 55°43'
17	19°56' 64°54'	20°24' 64°20'	20°51' 63°45'	21°21' 63°9'	21°53' 62°31'	22°24' 61°50'	22°57' 61°9'	23°33' 60°25'	24°10' 59°40'	24°50' 58°52'	25°31' 58°1'	26°14' 57°10'	27°59' 56°13'	27°47' 55°15'	28°37' 54°13'
18	21°9' 63°45'	21°37' 63°9'	22°6' 62°34'	22°38' 61°56'	23°9' 61°17'	23°43' 60°35'	24°18' 59°52'	24°56' 59°8'	25°34' 58°20'	26°15' 57°31'	26°57' 56°39'	27°42' 55°46'	28°29' 54°49'	29°18' 53°50'	30°9' 52°47'
19	22°20' 62°36'	22°49' 62°1'	23°20' 61°24'	23°52' 60°44'	24°26' 60°4'	25°1' 59°21'	25°37' 58°37'	26°15' 57°51'	26°56' 57°4'	27°38' 56°14'	28°22' 55°22'	29°8' 54°26'	29°56' 53°28'	30°43' 52°28'	31°40' 51°24'
20	23°30' 61°30'	24°1' 60°53'	24°32' 60°14'	25°6' 59°34'	25°40' 58°54'	26°16' 58°10'	26°55' 57°25'	27°34' 56°38'	28°15' 55°49'	28°58' 54°58'	29°44' 54°4'	30°31' 53°9'	31°21' 52°9'	32°13' 51°9'	33°8' 50°4'
21	24°39' 60°25'	25°10' 59°46'	25°43' 59°7'	26°18' 58°26'	26°53' 57°43'	27°30' 57°0'	28°10' 56°14'	28°50' 55°26'	29°32' 54°36'	30°17' 53°43'	31°4' 52°50'	31°52' 51°52'	32°43' 50°53'	33°36' 49°50'	34°31' 48°47'
22	25°46' 59°20'	26°19' 58°41'	26°53' 58°1'	27°27' 57°19'	28°5' 56°35'	28°43' 55°51'	29°22' 55°4'	30°5' 54°17'	30°48' 53°26'	31°34' 52°32'	32°22' 51°38'	33°11' 50°41'	34°2' 49°41'	34°57' 48°37'	35°54' 47°32'
23	26°52' 58°16'	27°26' 57°38'	28°0' 56°56'	28°36' 56°14'	29°14' 55°30'	29°53' 54°43'	30°35' 53°57'	31°15' 53°8'	32°1' 52°15'	32°48' 51°24'	33°36' 50°28'	34°27' 49°29'	35°20' 48°30'	36°15' 47°27'	37°12' 46°20'
24	27°57' 57°15'	28°31' 56°35'	29°7' 55°53'	29°43' 55°11'	30°22' 54°26'	31°2' 53°40'	31°45' 52°51'	32°28' 52°2'	33°14' 51°10'	34°1' 50°5'	34°50' 49°20'	35°42' 48°22'	36°35' 47°21'	37°30' 46°18'	38°28' 45°12'
25	28°59' 56°15'	29°34' 55°34'	30°12' 54°52'	30°49' 54°9'	31°29' 53°23'	32°10' 52°36'	32°52' 51°48'	33°37' 50°57'	34°23' 50°5'	35°11' 49°11'	36°0' 48°14'	36°52' 47°16'	37°47' 46°15'	38°43' 45°11'	39°41' 44°5'
26	30°1' 55°15'	30°38' 54°34'	31°14' 53°52'	31°54' 53°8'	32°34' 52°22'	33°15' 51°35'	33°58' 50°46'	34°45' 49°55'	35°31' 49°3'	36°19' 48°7'	37°10' 47°12'	38°2' 46°12'	38°56' 45°10'	39°53' 44°7'	40°52' 43°2'
27	31°3' 54°19'	31°39' 53°37'	32°18' 52°54'	32°57' 52°9'	33°37' 51°23'	34°20' 50°34'	35°3' 49°45'	35°49' 48°55'	36°36' 48°2'	37°25' 47°7'	38°16' 46°10'	39°10' 45°10'	40°4' 44°10'	41°1' 43°5'	42°
28	32°2' 53°22'	32°39' 52°38'	33°18' 51°56'	33°57' 51°11'	34°39' 50°25'	35°21' 49°31'	36°7' 48°47'	36°52' 47°56'	37°40' 47°2'	38°29' 46°7'	39°21' 45°11'	40°16' 44°11'	41°9' 43°9'	42°7'	
29	32°59' 52°27'	33°38' 51°44'	34°17' 51°1'	34°58' 50°16'	35°39' 49°29'	36°23' 45°41'	37°8' 47°50'	37°54' 46°58'	38°42' 46°4'	39°32' 45°10'	40°24' 44°12'	41°18' 43°14'	42°13		
30	33°57' 51°33'	34°36' 50°50'	35°15' 50°7'	35°56' 49°20'	36°38' 48°34'	37°21' 47°45'	38°7' 46°55'	38°53' 46°3'	39°43' 45°9'	40°32' 44°14'	41°25' 43°17'	42°18'			
31	34°53' 50°41'	35°31' 49°57'	36°11' 49°13'	36°52' 48°28'	37°35' 47°39'	38°20' 46°52'	39°5' 46°1'	39°52' 45°10'	40°41' 44°15'	41°32' 43°20'	42°23'				
32	35°46' 49°50'	36°27' 49°7'	37°6' 48°22'	37°48' 47°36'	38°31' 46°49'	39°15' 45°59'	40°1' 45°9'	40°49' 44°17'	41°38' 43°24'	42°28'					
33	36°39' 45°59'	37°19' 48°17'	38°0' 47°32'	38°42' 46°46'	39°26' 45°58'	40°0' 45°8'	40°56' 44°18'	41°44' 43°26'	42°33'						
34	37°32' 48°12'	38°11' 47°27'	38°53' 46°43'	39°35' 45°57'	40°18' 45°8'	41°4' 44°20'	41°49' 43°29'	42°37'							
35	38°22' 47°24'	39°3' 46°39'	39°44' 45°54'	40°26' 45°8'	41°10' 44°20'	41°55' 43°31'	42°41'								
36	39°11' 46°37'	39°52' 45°52'	40°34' 45°8'	41°15' 44°21'	42°0' 43°34'	42°45'									
37	40°0' 45°52'	40°40' 45°8'	41°22' 44°22'	42°5' 43°37'	42°48'										
38	40°47' 45°7'	41°28' 44°24'	42°9' 43°39'	42°52'											
39	41°32' 44°24'	42°14' 43°40'	42°56'												
40	42°18' 43°42'	42°58'													
41	43°2'														

For bevel gears with axes at right angles.
Angle for wheel above, for pinion below.

ANGLE OF FACE

90°—Angle given=Face Angle.

WHEEL.

PINION.	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12
12	20°46′ 61°14′	21°31′ 60°15′	22°18′ 59°10′	23°8′ 58°2′	24°3′ 56°49′	25°2′ 55°32′	26°3′ 54°7′	27°11′ 52°39′	28°25′ 51°3′	29°43′ 49°17′	31°11′ 47°25′	32°44′ 45°24′	34°26′ 43°14′	36°16′ 40°50′	38°17′
13	22°37′ 59°29′	23°26′ 58°28′	24°15′ 57°21′	25°9′ 56°11′	26°6′ 54°56′	27°8′ 53°36′	28°14 52°10′	29°25′ 50°39′	30°42′ 49°2′	32°4′ 47°16′	33°34′ 45°22′	35°10′ 43°20′	36°55′ 41°9′	38°48′	
14	24°25′ 57°49′	25°16′ 56°46′	26°8′ 55°38′	27°5′ 54°25′	28°4′ 53°8′	29°9′ 51°47′	30°20′ 50°20′	31°33′ 48°47′	32°52′ 47°8′	34°18′ 45°12′	35°50′ 43°26′	37°28′ 41°24′	39°15′		
15	26°11′ 56°13′	27°3′ 55°7′	27°58′ 53°58′	28°58′ 52°44′	30°0′ 51°26′	31°6′ 50°2′	32°19′ 48°33′	33°36′ 47°0′	34°56′ 45°20′	36°23′ 43°33′	37°57′ 41°39′	39°38′			
16	27°52′ 54°38′	28°45′ 53°31′	29°43′ 52°21′	30°44′ 51°6′	31°50′ 49°46′	32°58′ 48°22′	34°12′ 46°52′	35°31′ 45°19′	36°54′ 43°38′	38°23′ 41°51′	39°57′				
17	29°30′ 53°8′	30°26′ 52°0′	31°26′ 50°48′	32°28′ 49°32′	33°35′ 48°11′	34°47′ 46°47′	36°0′ 45°16′	37°21′ 43°43′	38°45′ 42°1′	40°15′					
18	31°5′ 51°41′	32°2′ 50°32′	33°4′ 49°18′	34°8′ 48°2′	35°15′ 46°41′	36°28′ 45°16′	37°45′ 43°45′	39°5′ 42°11′	40°31′						
19	32°36′ 50°18′	33°36′ 49°8′	34°38′ 47°54′	35°49′ 46°36′	36°53′ 45°15′	38°6′ 43°50′	39°24′ 42°20′	40°45′							
20	34°5′ 48°57′	35°6′ 47°46′	36°8′ 46°32′	37°16′ 45°14′	38°26′ 43°52′	39°39′ 42°27′	40°57′								
21	35°31′ 47°39′	36°32′ 46°28′	37°37′ 45°13′	38°44′ 43°56′	39°54′ 42°34′	41°8′									
22	36°52′ 46°24′	37°55′ 45°13′	39°0′ 43°58′	40°8′ 42°40′	41°19′										
23	38°12′ 45°12′	39°15′ 44°1′	40°20′ 42°46′	41°28′											
24	39°29′ 44°3′	40°32′ 42°52′	41°38′												
25	40°43′ 42°57′	41°46′													
26	41°53′														

For bevel gears with axes at right angles.
Angle for wheel above, for pinion below.

NATURAL SINE

Deg	0′	10′	20′	30′	40′	50′	60′	
0	·00000	·00291	·00581	·00872	·01163	·01454	·01745	89
1	·01745	·02036	·02326	·02617	·02908	·03199	·03489	88
2	·03489	·03780	·04071	·04361	·04652	·04943	·05233	87
3	·05233	·05524	·05814	·06104	·06395	·06685	·06975	86
4	·06975	·07265	·07555	·07845	·08135	·08425	·08715	85
5	·08715	·09005	·09295	·09584	·09874	·10163	·10452	84
6	·10452	·10742	·11031	·11320	·11609	·11898	·12186	83
7	·12186	·12475	·12764	·13052	·13341	·13629	·13917	82
8	·13917	·14205	·14493	·14780	·15068	·15356	·15643	81
9	·15643	·15930	·16217	·16504	·16791	·17078	·17364	80
10	·17364	·17651	·17937	·18223	·18509	·18795	·19080	79
11	·19080	·19366	·19651	·19936	·20221	·20506	·20791	78
12	·20791	·21075	·21359	·21644	·21927	·22211	·22495	77
13	·22495	·22778	·23061	·23344	·23627	·23909	·24192	76
14	·24192	·24474	·24756	·25038	·25319	·25600	·25881	75
15	·25881	·26162	·26443	·26723	·27004	·27284	·27563	74
16	·27563	·27843	·28122	·28401	·28680	·28958	·29237	73
17	·29237	·29515	·29793	·30070	·30347	·30624	·30901	72
18	·30901	·31178	·31454	·31730	·32006	·32281	·32556	71
19	·32556	·32831	·33106	·33380	·33654	·33928	·34202	70
20	·34202	·34475	·34748	·35020	·35293	·35565	·35836	69
21	·35836	·36108	·36379	·36650	·36920	·37190	·37460	68
22	·37460	·37730	·37999	·38268	·38536	·38805	·39073	67
23	·39073	·39340	·39607	·39874	·40141	·40407	·40673	66
24	·40673	·40939	·41204	·41469	·41733	·41998	·42261	65
25	·42261	·42525	·42788	·43051	·43313	·43575	·43837	64
26	·43837	·44098	·44359	·44619	·44879	·45139	·45399	63
27	·45399	·45658	·45916	·46174	·46432	·46690	·46947	62
28	·46947	·47203	·47460	·47715	·47971	·48226	48481	61
29	·48481	·48735	·48989	·49242	·49495	·49747	·50000	60
30	·50000	·50251	·50503	·50753	·51004	·51254	·51503	59
31	·51503	·51752	·52001	·52249	·52497	·52745	·52991	58
32	·52991	·53238	·53484	·53730	·53975	·54219	·54463	57
33	·54463	·54707	·54950	·55193	·55436	·55677	·55919	56
34	·55919	·56160	·56400	·56640	·56880	·57119	·57357	55
35	·57357	·57595	·57833	·58070	·58306	·58542	·58778	54
36	·58778	·59013	·59248	·59482	·59715	·59948	·60181	53
37	·60181	·60413	·60645	·60876	·61106	·61336	·61566	52
38	·61566	·61795	·62023	·62251	·62478	·62705	·62932	51
39	·62932	·63157	·63383	·63607	·63832	·64055	·64278	50
40	·64278	·64501	·64723	·64944	·65165	·65386	·65605	49
41	·65605	·65825	·66043	·66262	·66479	·66696	·66913	48
42	·66913	·67128	·67344	·67559	·67773	·67986	·68199	47
43	·68199	·68412	·68624	·68835	·69046	·69256	·69465	46
44	·69465	·69674	·69883	·70090	·70298	·70504	·70710	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

NATURAL COSINE

NATURAL SINE

Deg	0′	10′	20′	30′	40′	50′	60′	
45	·70710	·70916	·71120	·71325	·71528	·71731	·71934	44
46	·71934	·72135	·72336	·72537	·72737	·72936	·73135	43
47	·73135	·73333	·73530	·73727	·73923	·74119	·74314	42
48	·74314	·74508	·74702	·74895	·75088	·75279	·75471	41
49	·75471	·75661	·75851	·76040	·76229	·76417	·76604	40
50	·76604	·76791	·76977	·77162	·77347	·77531	·77714	39
51	·77714	·77897	·78079	·78260	·78441	·78621	·78801	38
52	·78801	·78979	·79157	·79335	·79512	·79688	·79863	37
53	·79863	·80038	·80212	·80385	·80558	·80730	·80901	36
54	·80901	·81072	·81242	·81411	·81580	·81748	·81915	35
55	·81915	·82081	·82247	·82412	·82577	·82740	·82903	34
56	·82903	·83066	·83227	·83388	·83548	·83708	·83867	33
57	·83867	·84025	·84182	·84339	·84495	·84650	·84804	32
58	·84804	·84958	·85111	·85264	·85415	·85566	·85716	31
59	·85716	·85866	·86014	·86162	·86310	·86456	·86602	30
60	·86602	·86747	·86892	·87035	·87178	·87320	·87462	29
61	·87462	·87602	·87742	·87881	·88020	·88157	·88294	28
62	·88294	·88430	·88566	·88701	·88835	·88968	·89100	27
63	·89100	·89232	·89363	·89493	·89622	·89751	·89879	26
64	·89879	·90006	·90132	·90258	·90383	·90507	·90630	25
65	·90630	·90753	·90875	·90996	·91116	·91235	·91354	24
66	·91354	·91472	·91589	·91706	·91821	·91936	·92050	23
67	·92050	·92163	·92276	·92388	·92498	·92609	·92718	22
68	·92718	·92827	·92934	·93041	·93148	·93253	·93358	21
69	·93358	·93461	·93565	·93667	·93768	·93869	·93969	20
70	·93969	·94068	·94166	·94264	·94360	·94456	·94551	19
71	·94551	·94646	·94739	·94832	·94924	·95015	·95105	18
72	·95105	·95195	·95283	·95371	·95458	·95545	·95630	17
73	·95630	·95715	·95799	·95882	·95964	·96045	·96126	16
74	·96126	·96205	·96284	·96363	·96440	·96516	·96592	15
75	·96592	·96667	·96741	·96814	·96887	·96958	·97029	14
76	·97029	·97099	·97168	·97237	·97304	·97371	·97437	13
77	·97437	·97502	·97566	·97629	·97692	·97753	·97814	12
78	·97814	·97874	·97934	·97992	·98050	·98106	·98162	11
79	·98162	·98217	·98272	·98325	·98378	·98429	·98480	10
80	·98480	·98530	·98580	·98628	·98676	·98722	·98768	9
81	·98768	·98813	·98858	·98901	·98944	·98985	·99026	8
82	·99026	·99066	·99106	·99144	·99182	·99218	·99254	7
83	·99254	·99289	·99323	·99357	·99389	·99421	·99452	6
84	·99452	·99482	·99511	·99539	·99567	·99593	·99619	5
85	·99619	·99644	·99668	·99691	·99714	·99735	·99756	4
86	·99756	·99776	·99795	·99813	·99830	·99847	·99863	3
87	·99863	·99877	·99891	·99904	·99917	·99928	·99939	2
88	·99939	·99948	·99957	·99965	·99972	·99979	·99984	1
89	·99984	·99989	·99993	·99996	·99998	·99999	1 · 0000	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

NATURAL COSINE

NATURAL TANGENT

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	·00000	·00290	·00581	·00872	·01163	·01454	·01745	89
1	·01745	·02036	·02327	·02618	·02909	·03200	·03492	88
2	·03492	·03783	·04074	·04366	·04657	·04949	·05240	87
3	·05240	·05532	·05824	·06116	·06408	·06700	·06992	86
4	·06992	·07285	·07577	·07870	·08162	·08455	·08748	85
5	·08748	·09042	·09335	·09628	·09922	·10216	·10510	84
6	·10510	·10804	·11099	·11393	·11688	·11983	·12278	83
7	·12278	·12573	·12869	·13165	·13461	·13757	·14054	82
8	·14054	·14350	·14647	·14945	·15242	·15540	15838	81
9	·15838	·16136	·16435	·16734	·17033	·17332	·17632	80
10	·17632	·17932	·18233	·18533	·18834	·19136	·19438	79
11	·19438	·19740	·20042	·20345	·20648	·20951	·21255	78
12	·21255	·21559	·21864	·22169	·22474	·22780	·23086	77
13	·23086	·23393	·23700	·24007	·24315	·24624	·24932	76
14	·24932	·25242	·25551	·25861	·26172	·26483	·26794	75
15	·26794	·27106	·27419	·27732	·28046	·28360	·28674	74
16	·28674	·28989	·29305	·29621	·29938	·30255	·30573	73
17	·30573	·30891	·31210	·31529	·31850	·32170	·32492	72
18	·32492	·32813	·33136	·33459	·33783	·34107	·34432	71
19	·34432	·34758	·35084	·35411	·35739	·36067	·36397	70
20	·36397	·36726	·37057	·37388	·37720	·38053	·38386	69
21	·38386	·38720	·39055	·39391	·39727	·40064	·40402	68
22	·40402	·40741	·41080	·41421	·41762	·42104	·42447	67
23	·42447	·42791	·43135	·43481	·43827	·44174	·44522	66
24	·44522	·44871	·45221	·45572	·45924	·46277	·46630	65
25	·46630	·46985	·47341	·47697	·48055	·48413	·48773	64
26	·48773	·49133	·49495	·49858	·50221	·50586	·50952	63
27	·50952	·51319	·51687	·52056	·52427	·52798	·53170	62
28	·53170	·53544	·53919	·54295	·54672	·55051	·55430	61
29	·55430	·55811	·56193	·56577	·56961	·57347	·57735	60
30	·57735	·58123	·58513	·58904	·59297	·59690	·60086	59
31	·60086	·60482	·60880	·61280	·61680	·62083	·62486	58
32	·62486	·62892	·63298	·63707	·64116	·64528	·64940	57
33	·64940	·65355	·65771	·66188	·66607	·67028	·67450	56
34	·67450	·67874	·68300	·68728	·69157	·69588	·70020	55
35	·70020	·70455	·70891	·71329	·71769	·72210	·72654	54
36	·72654	·73099	·73546	·73996	·74447	·74900	·75355	53
37	·75355	·75812	·76271	·76732	·77195	·77661	·78128	52
38	·78128	·78598	·79069	·79543	·80019	·80497	·80978	51
39	·80978	·81461	·81946	·82433	·82923	·83415	·83910	50
40	·83910	·84406	·84906	·85408	·85912	·86419	·86928	49
41	·86928	·87440	·87955	·88472	·88992	·89515	·90040	48
42	·90040	·90568	·91099	·91633	·92169	·92709	·93251	47
43	·93251	·93796	·94345	·94896	·95450	·96008	·96568	46
44	·96568	·97132	·97699	·98269	·98843	·99419	1·0000	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

NATURAL COTANGENT

NATURAL TANGENT

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	1·0000	1·0058	1·0117	1·0176	1·0235	1·0295	1·0355	44
46	1·0355	1·0415	1·0476	1·0537	1·0599	1·0661	1·0723	43
47	1·0723	1·0786	1·0849	1·0913	1·0977	1·1041	1·1106	42
48	1·1106	1·1171	1·1236	1·1302	1·1369	1·1436	1·1503	41
49	1·1503	1·1571	1·1639	1·1708	1·1777	1·1847	1·1917	40
50	1·1917	1·1988	1·2059	1·2131	1·2203	1·2275	1·2349	39
51	1·2349	1·2422	1·2496	1·2571	1·2647	1·2723	1·2799	38
52	1·2799	1·2876	1·2954	1·3032	1·3111	1·3190	1·3270	37
53	1·3270	1·3351	1·3432	1·3514	1·3596	1·3680	1·3763	36
54	1·3763	1·3848	1·3933	1·4019	1·4106	1·4193	1·4281	35
55	1·4281	1·4370	1·4459	1·4550	1·4641	1·4733	1·4825	34
56	1·4825	1·4919	1·5013	1·5108	1·5204	1·5301	1·5398	33
57	1·5398	1·5497	1·5596	1·5696	1·5798	1·5900	1·6003	32
58	1·6003	1·6107	1·6212	1·6318	1·6425	1·6533	1·6642	31
59	1·6642	1·6753	1·6864	1·6976	1·7090	1·7204	1·7320	30
60	1·7320	1·7437	1·7555	1·7674	1·7795	1·7917	1·8040	29
61	1·8040	1·8164	1·8290	1·8417	1·8546	1·8676	1·8807	28
62	1·8807	1·8940	1·9074	1·9209	1·9347	1·9485	1·9626	27
63	1·9626	1·9768	1·9911	2·0056	2·0203	2·0352	2·0503	26
64	2·0503	2·0655	2·0809	2·0965	2·1123	2·1283	2·1445	25
65	2·1445	2·1609	2·1774	2·1943	2·2113	2·2285	2·2460	24
66	2·2460	2·2637	2·2816	2·2998	2·3182	2·3369	2·3558	23
67	2·3558	2·3750	2·3944	2·4142	2·4342	2·4545	2·4750	22
68	2·4750	2·4959	2·5171	2·5386	2·5604	2·5826	2·6050	21
69	2·6050	2·6279	2·6510	2·6746	2·6985	2·7228	2·7474	20
70	2·7474	2·7725	2·7980	2·8239	2·8502	2·8770	2·9042	19
71	2·9042	2·9318	2·9600	2·9886	3·0178	3·0474	3·0776	18
72	3·0776	3·1084	3·1397	3·1715	3·2040	3·2371	3·2708	17
73	3·2708	3·3052	3·3402	3·3759	3·4123	3·4495	3·4874	16
74	3·4874	3·5260	3·5655	3·6058	3·6470	3·6890	3·7320	15
75	3·7320	3·7759	3·8208	3·8667	3·9136	3·9616	4·0107	14
76	4·0107	4·0610	4·1125	4·1653	4·2193	4·2747	4·3314	13
77	4·3314	4·3896	4·4494	4·5107	4·5736	4·6382	4·7046	12
78	4·7046	4·7728	4·8430	4·9151	4·9894	5·0658	5·1445	11
79	5·1445	5·2256	5·3092	5·3955	5·4845	5·5763	5·6712	10
80	5·6712	5·7693	5·8708	5·9757	6·0844	6·1970	6·3137	9
81	6·3137	6·4348	6·5605	6·6911	6·8269	6·9682	7·1153	8
82	7·1153	7·2687	7·4287	7·5957	7·7703	7·9530	8·1443	7
83	8·1443	8·3449	8·5555	8·7768	9·0098	9·2553	9·5143	6
84	9·5143	9·7881	10·078	10·385	10·711	11·059	11·430	5
85	11·430	11·826	12·250	12·706	13·196	13·726	14·300	4
86	14·300	14·924	15·604	16·349	17·169	18·075	19·081	3
87	19·081	20·205	21·470	22·904	24·541	26·431	28·636	2
88	28·636	31·241	34·367	38·188	42·964	49·103	57·290	1
89	57·290	68·750	85·939	114·58	171·88	343·77	∞	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

NATURAL COTANGENT

INCHES TO MILLIMETRES—FRACTIONS

Inches				m/m	Inches.				m/m
		$\frac{1}{64}$	·015625	·3969			$\frac{33}{64}$	·515625	13·0969
	$\frac{1}{32}$		·03125	·7937		$\frac{17}{32}$		·53125	13·4937
		$\frac{3}{64}$	·046875	1·1906			$\frac{35}{64}$	·546875	13·8906
$\frac{1}{16}$			·0625	1·5875	$\frac{9}{16}$			·5625	14·2875
		$\frac{5}{64}$	·078125	1·9844			$\frac{37}{64}$	·578125	14·6844
	$\frac{3}{32}$		·09375	2·3812		$\frac{19}{32}$		·59375	15·0812
		$\frac{7}{64}$	·109375	2·7781			$\frac{39}{64}$	·609375	15·4781
$\frac{1}{8}$			·125	3·1750	$\frac{5}{8}$			·625	15·8750
		$\frac{9}{64}$	·140625	3·5719			$\frac{41}{64}$	·640625	16·2719
	$\frac{5}{32}$		·15625	3·9687		$\frac{21}{32}$		·65625	16·6687
		$\frac{11}{64}$	·171875	4·3656			$\frac{43}{64}$	·671875	17·0656
$\frac{3}{16}$			·1875	4·7625	$\frac{11}{16}$			·6875	17·4625
		$\frac{13}{64}$	·203125	5·1594			$\frac{45}{64}$	·703125	17·8594
	$\frac{7}{32}$		·21875	5·5562		$\frac{23}{32}$		·71875	18·2562
		$\frac{15}{64}$	·234375	5·9531			$\frac{47}{64}$	·734375	18·6531
$\frac{1}{4}$			·25	6·3500	$\frac{3}{4}$			·75	19·0500
		$\frac{17}{64}$	·265625	6·7469			$\frac{49}{64}$	·765625	19·4469
	$\frac{9}{32}$		·28125	7·1437		$\frac{25}{32}$		·78125	19·8437
		$\frac{19}{64}$	·296875	7·5406			$\frac{51}{64}$	·796875	20·2406
$\frac{5}{16}$			·3125	7·9375	$\frac{13}{16}$			·8125	20·6375
		$\frac{21}{64}$	·328125	8·3344			$\frac{53}{64}$	·828125	21·0344
	$\frac{11}{32}$		·34375	8·7312		$\frac{27}{32}$		·84375	21·4312
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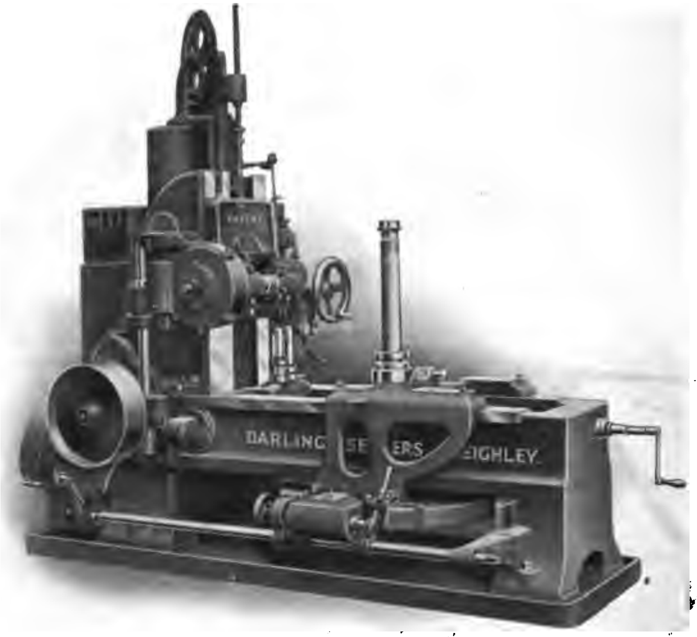

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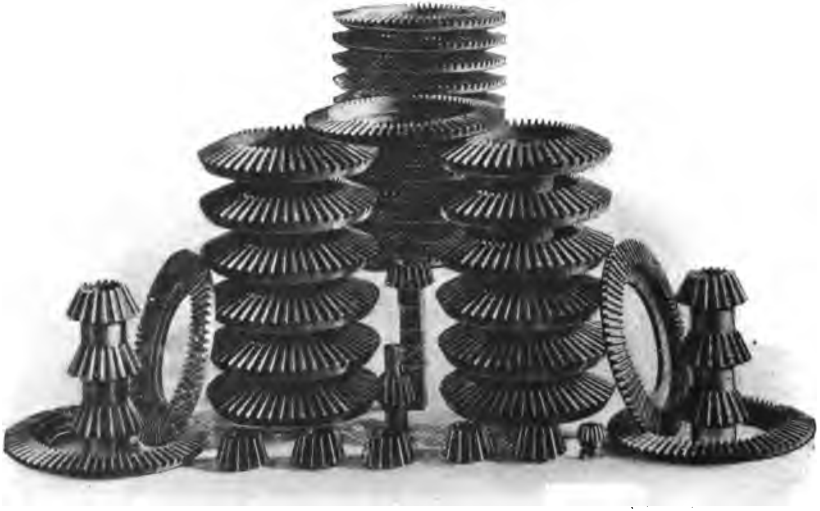

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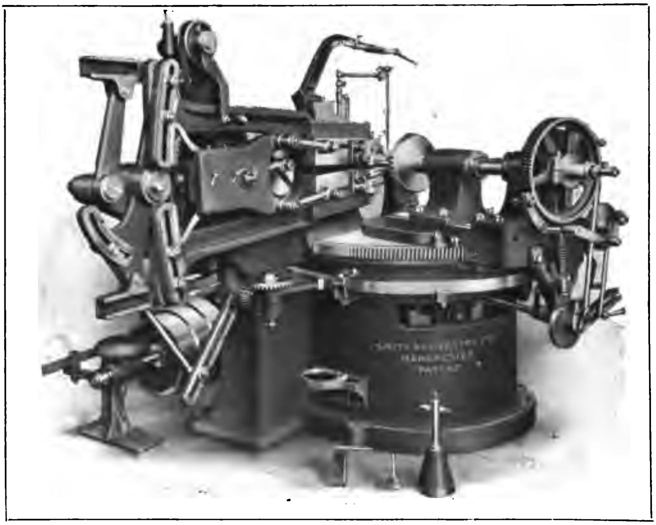

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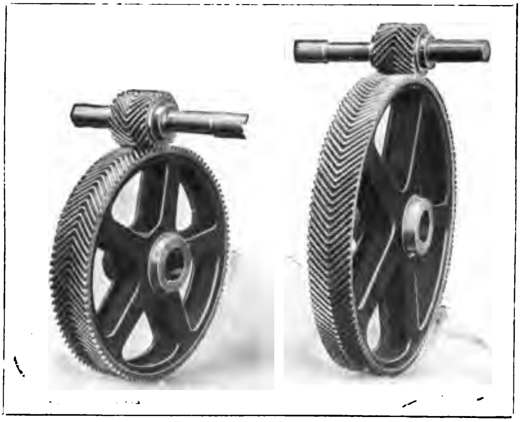

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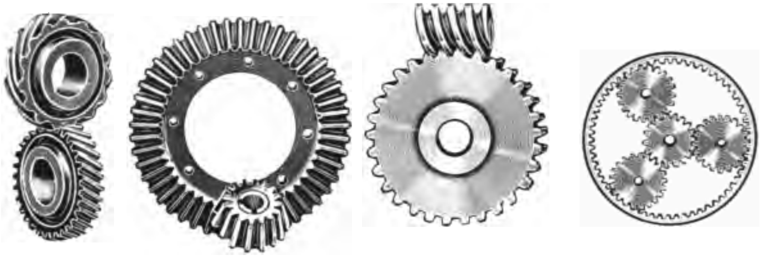

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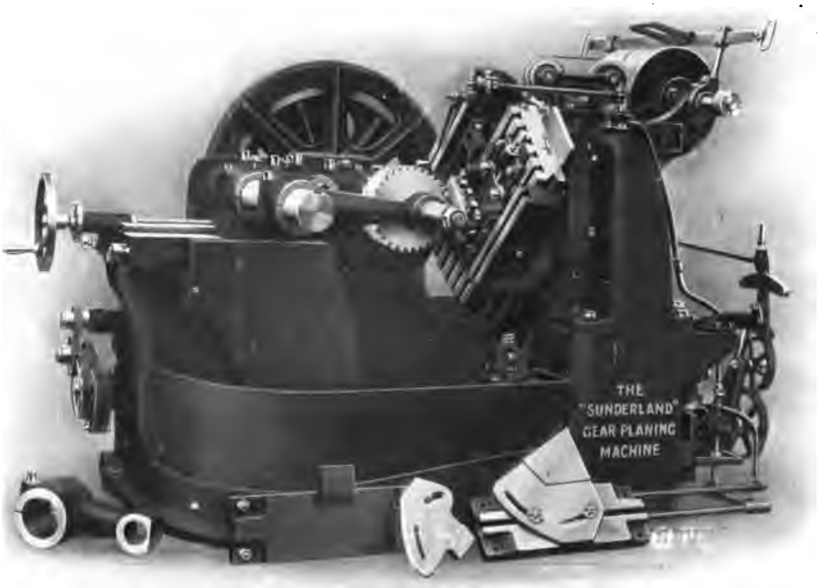

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